UNIVERSITY OF VAASA
Economic Mathematics
Professor Ilkka Virtanen

OPERATIONS RESEARCH ORMS.1020

Exam 2.2.2008

1. A company wishes to minimise its combined costs of production and inventory over a 4-week time period. An item produced in a given week is available for consumption during that week, or it may be kept in inventory for use in later weeks. Initial inventory at the beginning of week 1 is 250 units. The minimum allowed inventory carried from one week to the next is 50 units. Unit production costs is 150 €, and the cost of storing a unit from one week to the next is 15 €. The following table shows production capacities and the demands that must be met during each week.

Week	Production capacity	Demand
1	800	900
2	700	600
3	600	800
4	800	600
Total	2900	2900

A minimum production of 500 items per week must be maintained. Inventory costs are not applied to items remaining at the end of the 4th production period, nor is the minimum inventory restriction applied after this final period.

Formulate this combined production-inventory problem into the form of a LP model. The object is to minimise the total costs caused by production and inventory. Take care of all the different types of constraints. The model needs not to be solved.

Hint. Let x_i be the number of units produced during the i-th week, i=1,2,3,4. The formulation is somewhat more manageable if we let d_i denote the number of items remaining at the end of each week (accounting for those held over from previous week, those produced during the current week, and those consumed during the current week). Note that the d_i are not decision variables, but will merely serve to simplify the written formulation.

You will get 2 extra bonus points (in addition to the six points normally available for each question) if you, after completion of the formulation described above, formulate the model using only the decision variables x_i (and the parameters of the model).

2. Consider a LP problem given in its canonical form with n decision variables and m constraints. The objective function is in the form of maximisation. Each of the statements from a to f below refers to the Simplex algorithm and its final (optimal) tableau. Fill in the blanks (marked _____) with an appropriate alternative from the set of conclusions 1 to 8.

a. If all slack and surplus variables are zero in the optimal solution, then _____

b. If there are m zeros in the top (0- or z-) row of the optimal tableau, then _____
c. If a basic variable has the value zero in the optimal solution, then _____
d. If an artificial variable is non-zero in the optimal solution, then _____
e. If all the coefficients in a pivot column of a non-basic variable are ≤ 0, then _____
f. If a non-basic variable has zero coefficients in the top (0- or z-) row of the optimal tableau, then _____

Completion alternatives:

1. the optimal solution is unique.
2. there are multiple optimal solutions.
3. the value of all the decision variables is zero.
4. the optimal solution is degenerate (contains two or more basic solutions).
5. the shadow prices are inverses of the dual variables.
6. no feasible solution exists.
7. the solution is unbounded.
8. all constraints are equalities at optimality.

3. Consider the following LP problem given in the canonical form:

$$\text{Max } z = 5x_1 + 2x_2$$

$$\begin{aligned} \text{St.} \quad 2x_1 + x_2 &\leq 100 \\ x_1 + x_2 &\leq 80 \\ x_1 &\leq 40 \\ x_1, \quad x_2 &\geq 0 \end{aligned}$$

a) Solve the model e.g. graphically.
b) Perform a sensitivity analysis for the coefficient of the variable x_1 in the objective function (value now = 5): determine how much the coefficient can vary without changing the original optimal solution; in which range does the value of the objective function then change?
c) Perform a sensitivity analysis for the right hand side of the first constraint (RHS now = 100): determine how much the RHS coefficient can vary without causing the optimal basis to change; how much do the optimal solution and objective function value change in this range?

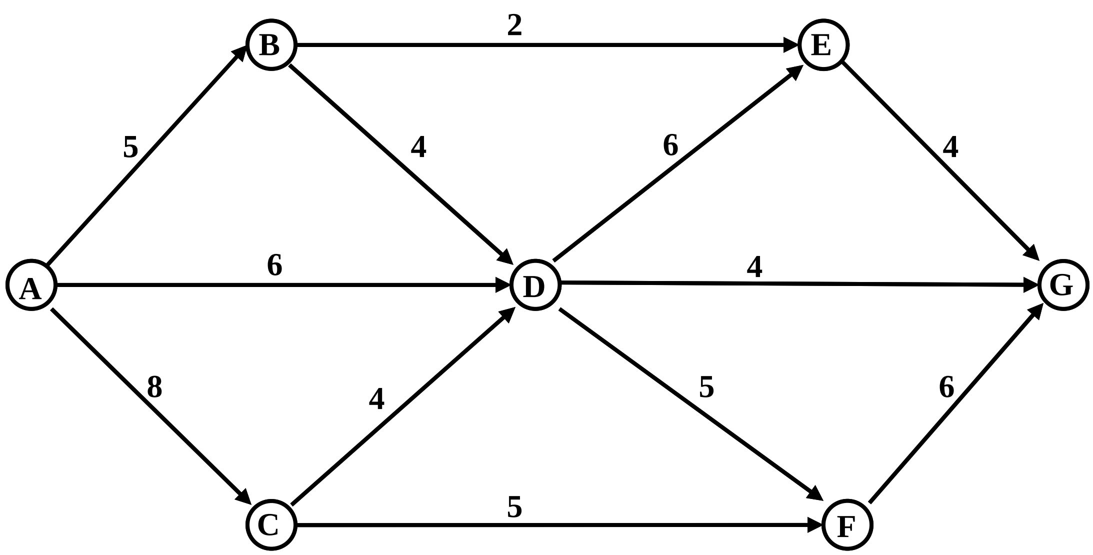
4. Four federally funded research projects are to be assigned to four existing research labs, with one project being allotted to each lab. The costs of each possible placement are given in the table below. Use the Hungarian method to determine the most economical allocation of projects.

Projekti	**Sandy Lab**	**Furmy Lab**	**Xenonne Lab**	**Liverly Lab**
Cryogenic cache memory	12	15	10	14
Spotted owl habitat	8	10	6	9
Pentium oxide depletion	20	22	18	12
Galactic genome mapping	10	12	8	16

<u>Note!</u> If you are not familiar with the Hungarian method, you can earn half of the maximum amount of points (= 3) if you formulate the problem in the form of a standard LP model (taking the special properties of the problem into account, however). This form of the model needs not to be solved.

5. Consider a data communication network in which processing nodes are connected by data links. In the figure below, data being collected or generated at site A must be transmitted through the network as quickly as possible to a destination processor at site G where the data can be archived or processed. Each data link has a capacity that effectively limits the flow of data through that link. Link capacities are shown as labels on the arcs.

Determine the maximum data flow that can be transmitted via the network.



You are allowed to have with you in the exam a usual calculator and a book containing the most frequently used mathematical formulas (e.g. MAOL)