

Introduction to Mathematical Economics, ORMS1030**Exercise 1, week 3** (Jan 13–17, 2014)

R1	ma	10–12	D115	R5	ti	14–16	C209
R2	ma	14–16	D115	R6	to	12–14	C209
R3	ti	08–10	D115	R7	pe	08–10	D115
R4	ti	12–14	A201	R8	pe	10–12	D115

1. Solve the equations

$$\text{a) } x + 3 = -9x + 8 \quad \text{b) } 6(x + 2) - x = 5x + 10 \quad \text{c) } \frac{3x + 1}{2} = x + 1$$

$$\begin{aligned} \text{a) } \quad & x + 3 = -9x + 8 && \text{move terms} \\ \Leftrightarrow & x + 9x = 8 - 3 && \text{simplify} \\ \Leftrightarrow & 10x = 5 && \text{divide by 10} \\ \Leftrightarrow & x = 0,5 && \text{Answer : one root } x = 0,5 \end{aligned}$$

$$\begin{aligned} \text{b) } \quad & 6(x + 2) - x = 5x + 10 && \text{remove parenthesis} \\ \Leftrightarrow & 6x + 12 - x = 5x + 10 && \text{move} \\ \Leftrightarrow & 6x - x - 5x = 10 - 12 && \text{simplify} \\ \Leftrightarrow & 0 = -2 && \text{False!} \\ & && \text{Answer : no root.} \end{aligned}$$

$$\begin{aligned} \text{c) } \quad & \frac{3x + 1}{2} = x + 1 && * 2, \text{ remove denominator} \\ \Leftrightarrow & 3x + 1 = 2x + 2 && \text{move and simplify} \\ \Leftrightarrow & x = 1 && \text{Answer : one root } x = 1 \end{aligned}$$

Some comment: a) also

2. Solve the equations

$$\text{a) } 2x^2 + 5x + 3 = 0 \quad \text{b) } 5x^2 + x = 4x^2 - 2x + 5 \quad \text{c) } (x + 2)(x - 3) = (x + 2)$$

(Hint: The roots in equation b) are not integers or rational numbers. You need a calculator to find them. Check out your answer by substituting roots to the equations.)

$$\begin{aligned} \text{a) } \quad & 2x^2 + 5x + 3 = 0 && \text{standard form ok, use formula} \\ \Leftrightarrow & x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} \\ \Leftrightarrow & x = \frac{-5 \pm 1}{4} \\ \Leftrightarrow & x = -1 \text{ or } x = -1,5 && \text{Answer1 : two roots } x = -1 \text{ or } x = -1,5 \\ & && \text{Answer2 : two roots } x_1 = -1 \text{ and } x_2 = -1,5 \end{aligned}$$

$$\begin{aligned} \text{b) } & 5x^2 + x = 4x^2 - 2x + 5 \\ \Leftrightarrow & 5x^2 - 4x^2 + x + 2x - 5 = 0 \\ \Leftrightarrow & x^2 + 3x - 5 = 0 \end{aligned}$$

find standard form

simplify

form ok, use formula

$$\Leftrightarrow x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-5)}}{2 \cdot 1}$$

$$\Leftrightarrow x = \frac{-3 \pm \sqrt{29}}{2}$$

Ans : $x = -4,1926$ or $x = 1,1926$

c) The equation is quadratic. So if we find two different roots, they will do. Clearly the value of the RHS (Right Hand Side) and the value of the LHS (Left Hand Side) of the equation are equal if $x - 3 = 1$ (LSH = $(x + 2) =$ RHS) or $x + 2 = 0$ (LHS = $0 =$ RHS). So

$$\text{equation is true } \Leftrightarrow x - 3 = 1 \text{ or } x + 2 = 0$$

$$\Leftrightarrow x = 4 \text{ or } x = -2$$

3. By calculator find the values of the following expressions

$$\text{a) } \sqrt{\frac{17,28}{3,25}} \quad \text{b) } 2^{10/3} \quad \text{c) } \ln(10) \quad \text{d) } \frac{2 \cdot 1,25 + 5 \cdot 3,15}{1,02^{3/12} + 1}$$

(In the exercise session you must be able to calculate the values by your own calculator. The right values are: a) 2,31 b) 10,079 c) 2,3026 d) 9,10)

a)

$$\sqrt{\frac{17,28}{3,25}} = \sqrt{(17,28/3,25)}$$

$$\sqrt{\text{ (17,28 / 3,25) }} \text{ = } \rightarrow \text{ 2,305845415 }, \text{ Answer: 2,31.}$$

b)

$$2^{10/3} = 2^{(10/3)}$$

$$2 \text{ ^ } \text{ (10 / 3) } \text{ = } \rightarrow \text{ 10,0793684 }, \text{ Answer: 10,079.}$$

c)

$$\ln 10 \text{ = } \rightarrow \text{ 2,302585093 }, \text{ Answer: 2,3026.}$$

d)

$$\frac{2 \cdot 1,25 + 5 \cdot 3,15}{1,02^{3/12} + 1} = (2 \cdot 1,25 + 5 \cdot 3,15) / (1,02^{(3/12)} + 1)$$

$$\text{ (2 } \times \text{ 1,25 } + \text{ 5 } \times \text{ 3,15) / (1,02 } \wedge \text{ (3 / 12) } + \text{ 1) } \text{ = } \rightarrow \text{ 9,102412674 }, \text{ Answer: 9,10.}$$

4. Solve the equations (Check out the roots by calculator.)

$$\text{a) } 2^x = 10 \quad \text{b) } \ln(x-1) = 2 \quad \text{c) } \frac{2x+1}{x-1} = 3$$

a) $2^x = 10$ natural logarithm of both sides
 $\Leftrightarrow \ln(2^x) = \ln(10)$ ($\ln(a^b) = b \cdot \ln(a)$)
 $\Leftrightarrow x \ln(2) = \ln(10)$ rest as usual
 $\Leftrightarrow x = \frac{\ln(10)}{\ln(2)} = 3,321928$ Ans : $x = 3,3219$

b) $\ln(x-1) = 2$ ($\ln(a) = b \Leftrightarrow a = e^b$)
 $\Leftrightarrow x-1 = e^2$
 $\Leftrightarrow x = e^2 + 1 = 8,389056$ Ans : $x = 8,389056$

c) $\frac{2x+1}{x-1} = 3$ $\cdot (x-1)$
 $\Leftrightarrow 2x+1 = 3(x+1)$ remove parenthesis
 $\Leftrightarrow 2x+1 = 3x+3$ move and simplify
 $\Leftrightarrow -x = 2$ Ans : $x = -2$

5. Solve a) $x-1 \leq 3x+1$ b) $4(x-1) \geq x+2$.

a) $x-1 \leq 3x+1$ move
 $\Leftrightarrow x-3x \leq 1+1$ simplify
 $\Leftrightarrow -2x \leq 2$ divide by -2
 $\Leftrightarrow \frac{-2x}{-2} \geq \frac{2}{-2}$ simplify
 $\Leftrightarrow x \geq -1$ Ans : $x \geq -1$

b) $4(x-1) \geq x+2$ remove parenthesis
 $\Leftrightarrow 4x-4 \geq x+2$ move
 $\Leftrightarrow 4x-x \geq 2+4$ simplify
 $\Leftrightarrow 3x \geq 6$ divide by 3
 $\Leftrightarrow \frac{3x}{3} \geq \frac{6}{3}$ simplify
 $\Leftrightarrow x \geq 2$ Ans : $x \geq 2$

6. In January the enterprise sells 256 products for price 25.20€ per item. The production cost of one product is 18.10€ (fixed cost not included). The January revenue is $R_1 = 256 \cdot 25.20€ = 6451.20€$ (Euro per month). The total cost of production (fixed costs not included) $C_1 = 256 \cdot 18.10€ = 4633.60€$ (Euro per month). The Margin in Euros is "Revenue - Cost": $K_1 = R_1 - C_1 = 6451.20€ - 4633.60€ = 1817.60€$ per month. The Margin per cent of sales is $K_1/R_1 \cdot 100\% = 28.17\%$.

In February the firm sells 300 product. The selling price do not change. But the production costs increase by 5,0%.

a) For February find the monthly Revenue R_2 , the total monthly production Cost C_2 , and the monthly Margin in Euros $K_2 = R_2 - C_2$.

b) For February find the Margin per cent ($K_2/R_2 \cdot 100\%$).

c) Find the percentage rate of change of the Revenue. ($\Delta R/R_1 \cdot 100\%$)

d) Find the percentage rate of change of the total production Cost. ($\Delta C/C_1 \cdot 100\%$)

e) Find the percentage rate of change of the Margin in Euros. ($\Delta K/K_1 \cdot 100\%$)

f) Find the change of the Margin per cent (percentage points) ($K_2/R_2 \cdot 100\% - K_1/R_1 \cdot 100\%$)

a)

$$R_2 = 300 \frac{\text{kpl}}{\text{kk}} \cdot 25,20 \frac{\text{€}}{\text{kpl}} = 7560 \frac{\text{€}}{\text{kk}}$$

$$C_2 = 300 \frac{\text{kpl}}{\text{kk}} \cdot (1,050 \cdot 18,10 \frac{\text{€}}{\text{kpl}}) = 5701,50 \frac{\text{€}}{\text{kk}}$$

$$K_2 = R_2 - C_2 = 7560 \frac{\text{€}}{\text{kk}} - 5701,50 \frac{\text{€}}{\text{kk}} = 1858,50 \frac{\text{€}}{\text{kk}}$$

b)

$$\text{Katepros}_2 = K_2/R_2 \cdot 100\% = \frac{1858,50}{7560} \cdot 100\% = 24,58\%$$

c)

$$\frac{(R_2 - R_1)}{R_1} \cdot 100\% = \frac{(7560 \frac{\text{€}}{\text{kk}} - 6451.20 \frac{\text{€}}{\text{kk}})}{6451.20 \frac{\text{€}}{\text{kk}}} \cdot 100\% = 17,19\%$$

d)

$$\frac{(C_2 - C_1)}{C_1} \cdot 100\% = \frac{(5701,50 \frac{\text{€}}{\text{kk}} - 4633.60 \frac{\text{€}}{\text{kk}})}{4633.60 \frac{\text{€}}{\text{kk}}} \cdot 100\% = 23,05\%$$

e)

$$\frac{(K_2 - K_1)}{K_1} \cdot 100\% = \frac{(1858,50 \frac{\text{€}}{\text{kk}} - 1817,60 \frac{\text{€}}{\text{kk}})}{1817,60 \frac{\text{€}}{\text{kk}}} \cdot 100\% = 2,3\%$$

e)

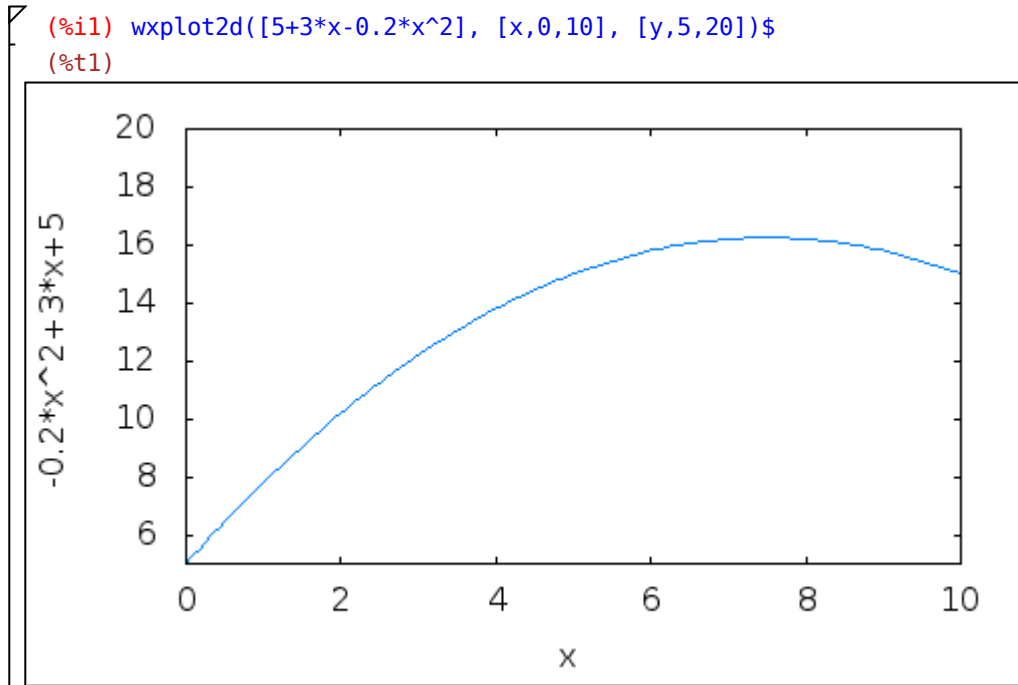
$$\text{Katepros}_2 - \text{Katepros}_1 = 24,58\% - 28,17\% = -3,6 \text{ prosenttiyksikköä}$$

Magrin increased 2.3%, but margin decreased 3.6 percentage points.

7. Draw the graph of the function $f(x) = 5 + 3x - 0.2x^2$, on the interval $0 \leq x \leq 10$. Where the function is increasing, and where the function is decreasing?
(Hint: If you remember the concept of the derivative, you can apply it to check out your answer.)

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The function is increasing on the interval $0 \leq x \leq 0.75$, and decreasing on the interval $0.75 \leq x \leq 10$.