University of Vaasa, spring 2014

Introduction to Mathematical Economics, ORMS1030

Exercise 2, week 4 (Jan 20–24, 2014)

R1	ma	10–12	D115	R5	ti	14–16	D218
R2	ma	14–16	**C209**	R6	to	12–14	C209
R3	ti	08–10	**D103**	R7	pe	08–10	**A201**
R4	ti	12–14	**B203**	R8	pe	10–12	D115

1. A factory needs carton as raw material for its production process. It purchases recycling material and separate usable stuff from waste. There is three possible source of recycling material (A, B, and C). All recycling material must be checked and the waste must be packed and transferred to final disposal. This all makes costs additional to the purchase prise. The costs are listed in the table below. For all three suppliers find the unit price of the raw material including all necessary checking and processing costs.

	A	B	C
price of the recycling material (€/ton)	100.00	125.00	95.00
rate of waste	6.0%	2.0%	10.0%
price of checking out material(€/ton)	10.0	11.0	12.0
price of processing waste (€/tonni)	20.0	15.0	5.0

Solution: There are three costs we face Purchasing cost c_1, cost of checking the recycling material c_2, and the cost of processing the waste. The unit price of the raw material is the total costs $c_1 + c_2 + c_3$ per the mass of the good raw material separated from the recycling stuff. If we purchase x ton of recycling material. Then

$$
\begin{aligned}
\text{unit price}_A &= \frac{\text{total costs}}{\text{what we got}} = \frac{100{,}00\frac{€}{\text{ton}} \cdot x + 10{,}0\frac{€}{\text{ton}} \cdot x + 20{,}0\frac{€}{\text{ton}} \cdot 0{,}06 \cdot x}{0{,}94 \cdot x} \\
&= \frac{111{,}20€/\text{ton}}{0{,}94} = 118{,}30\frac{€}{\text{ton}}
\end{aligned}
$$

$$
\begin{aligned}
\text{unit price}_B &= \frac{\text{total costs}}{\text{what we got}} = \frac{125{,}00\frac{€}{\text{ton}} \cdot x + 11{,}0\frac{€}{\text{ton}} \cdot x + 15{,}0\frac{€}{\text{ton}} \cdot 0{,}02 \cdot x}{0{,}98 \cdot x} \\
&= \frac{136{,}30€/\text{ton}}{0{,}98} = 139{,}08\frac{€}{\text{ton}}
\end{aligned}
$$

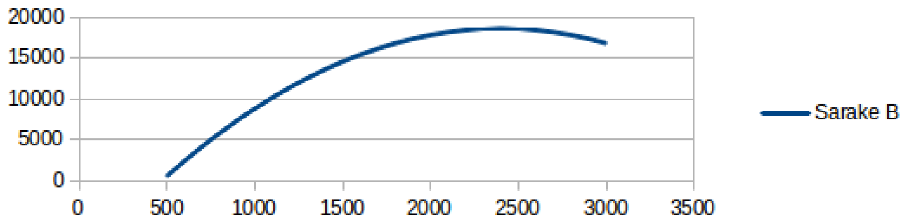
$$
\begin{aligned}
\text{unit price}_C &= \frac{\text{total costs}}{\text{what we got}} = \frac{95{,}00\frac{€}{\text{ton}} \cdot x + 12{,}0\frac{€}{\text{ton}} \cdot x + 5{,}0\frac{€}{\text{ton}} \cdot 0{,}10 \cdot x}{0{,}90 \cdot x} \\
&= \frac{107{,}50€/\text{ton}}{0{,}90} = 119{,}44\frac{€}{\text{ton}}
\end{aligned}
$$

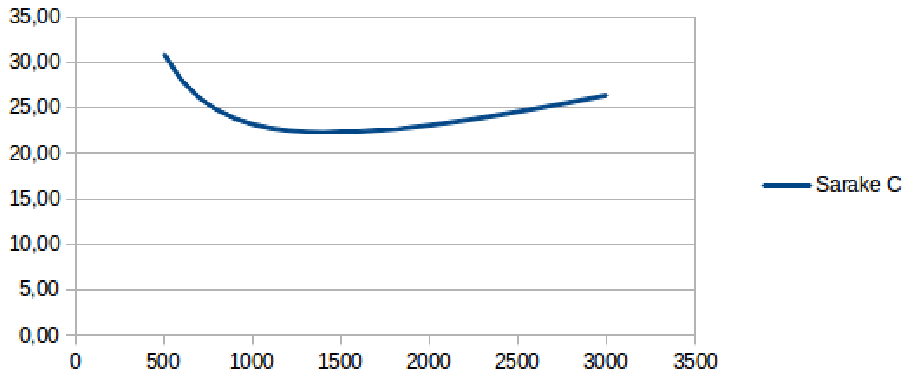
2. Nukia inc makes q products per week. The immediate production costs of one product are 8,00 €. So the weekly total production costs are $VC = 8q$ (€/week). The fixed costs are $FC = 10\,200$ (€/week). Due to problems in production equipments and stocks Nukia needs overtime work, which cause extra wage costs $LC = 0.005q^2$ (€/week). The total production costs are $TC(q) = FC + VC + LC = 10200 + 8q + 0.005q^2$. The selling price of the product is 32,00€, and the total revenue is thus $TR = 32q$ (€/week). The profit function is $P(q) = TR - TC = 24q - 0.005q^2 - 10200$ (€/week).
a) Draw the graphs of the profit function $P(q)$ and the average costs function $AC(q) = TC(q)/q$, when $0 < q < 3000$. b) Find the unit of $AC(q)$? c) What is the optimal production plan?

Solution: a) The functions we are drawing are:

$$
\begin{aligned}
P(q) &= 24q - 0.005q^2 - 10200, \\
AC(q) &= \frac{0.005q^2 + 8q + 10200}{q} = 0.005q + 8 + \frac{10200}{q}.
\end{aligned}
$$

	P	AC
500	550	30,90
600	2400	28,00
700	4150	26,07
800	5800	24,75
900	7350	23,83
1000	8800	23,20
1100	10150	22,77
1200	11400	22,50
1300	12550	22,35
1400	13600	22,29
1500	14550	22,30
1600	15400	22,38
1700	16150	22,50
1800	16800	22,67
1900	17350	22,87
2000	17800	23,10
2100	18150	23,36
2200	18400	23,64
2300	18550	23,93
2400	18600	24,25
2500	18550	24,58
2600	18400	24,92
2700	18150	25,28
2800	17800	25,64
2900	17350	26,02
3000	16800	26,40





b)

$$
[AC] = \frac{[TC]}{[q]} = \frac{\frac{€}{\text{week}}}{\frac{\text{product}}{\text{week}}} = \frac{€}{\text{week}} \cdot \frac{\text{week}}{\text{product}} = \frac{€}{\text{product}}
$$

c) Optimal production plan maximizes the profit. So optimal q is 2 400 products per week.

3. Determine the derivative of the profit function of the problem 2

$$P'(q) = \frac{dP}{dq}.$$

Where the derivative has zero value, $P'(q) = 0$? Check the answer by the graph drawn in the problem 1. Where the derivative has positive value, $P'(q) \geq 0$? Check the answer by the graph drawn in the problem 1.

Solution:

$$\begin{aligned} P(q) &= 24q - 0.005q^2 - 10200, \\ \Rightarrow P'(q) &= 24 - 0.01q. \end{aligned}$$

Answers of the questions:

$$\begin{aligned} P'(q) &= 0 \\ \Leftrightarrow 24 - 0.01q &= 0 \\ \Leftrightarrow -0.01q &= -24 \\ \Leftrightarrow q &= 2400 \qquad \text{(OK with the graph)} \end{aligned}$$

$$\begin{aligned} P'(q) &\geq 0 \\ \Leftrightarrow 24 - 0.01q &\geq 0 \\ \Leftrightarrow -0.01q &\geq -24 \\ \Leftrightarrow q &\leq 2400 \qquad \text{(OK with the graph)} \end{aligned}$$

4. Find the derivatives

$$\begin{array}{lll} a) & f'(3), & \text{when } f(x) = x^2 - 4x, \\ b) & g'(x), & \text{when } g(x) = 7x^2 + 5x - 3, \\ c) & h'(x), & \text{when } h(x) = 3x \cdot (x^2 - 5) \end{array}$$

Solution:

$$\begin{aligned} a) \qquad f(x) &= x^2 - 4x \\ \Rightarrow f'(x) &= 2x - 4 \\ \Rightarrow f'(3) &= 2 \cdot 3 - 4 = 2 \end{aligned}$$

$$\begin{aligned} b) \qquad g(x) &= 7x^2 + 5x - 3 \\ \Rightarrow g'(x) &= 14x + 5 \end{aligned}$$

$$\begin{aligned} c) \qquad h(x) &= 3x \cdot (x^2 - 5) \\ &= 3x^3 - 15x \\ \Rightarrow h'(x) &= 9x^2 - 15 \end{aligned}$$

5. Find the limits:

$$\text{a)}\ \lim_{x\to 2}\frac{2-x}{4-x^2}, \qquad \text{b)}\ \lim_{x\to 3}(x^2+2x-4).$$

Solution:

a) $$\lim_{x\to 2}\frac{2-x}{4-x^2}=\lim_{x\to 2}\frac{(2-x)\cdot 1}{(2+x)(2-x)}=\lim_{x\to 2}\frac{1}{(2+x)}=\frac{1}{(2+2)}=\frac{1}{4}=0.25$$

b) $$\lim_{x\to 3}(x^2+2x-4)=(3^2+2\cdot 3-4)=11$$

Exercise 1 problem 4. Solve the equations (Check out the roots by calculator.)

$$\text{a)}\ 2^x=10 \qquad \text{b)}\ \ln(x-1)=2 \qquad \text{c)}\ \frac{2x+1}{x-1}=3$$

a)

$$\begin{aligned} & 2^x=10 && \text{natural logarithm of both sides}\\ \Leftrightarrow\ & \ln(2^x)=\ln(10) && (\ln(a^b)=b\cdot\ln(a))\\ \Leftrightarrow\ & x\ln(2)=\ln(10) && \text{rest as usual}\\ \Leftrightarrow\ & x=\frac{\ln(10)}{\ln(2)}=3{,}321928 && \text{Ans}: x=3{,}3219 \end{aligned}$$

b)

$$\begin{aligned} & \ln(x-1)=2 && (\ln(a)=b \ \Leftrightarrow\ a=e^b)\\ \Leftrightarrow\ & x-1=e^2 && \\ \Leftrightarrow\ & x=e^2+1=8{,}389056 && \text{Ans}: x=8{,}389056 \end{aligned}$$

c)

$$\begin{aligned} & \frac{2x+1}{x-1}=3 && *\,(x-1)\\ \Leftrightarrow\ & 2x+1=3(x+1) && \text{remove parenthesis}\\ \Leftrightarrow\ & 2x+1=3x+3 && \text{move and simplify}\\ \Leftrightarrow\ & -x=2 && \text{Ans}: x=-2 \end{aligned}$$