

Introduction to Mathematical Economics, ORMS1030

Exercise 5, week 7 (Feb 15–19, 2016)

R1	ma	10–12	F455	R5	ti	10–12	F119
R2	ma	14–16	F455	R6	to	12–14	F455
R3	ti	08–10	F455	R7	pe	08–10	F455
R4	ti	12–14	F455	R8	pe	10–12	F455

1. According an project agreement the Firm pays 1400€ on the date of the signature of the agreement. After the start of the project the Firm pays three times a payment of 1000€ by one year intervals. In addition to previous the Firm must also pay monthly 200€ for 2,5 years (30 months). The monthly payments starts 4 months after the signature of the agreement. Find the Present Value of the Cash Flow when the annual rate of discounting is 4,12%.

2. A person borrows 5000 euros from a bank and agrees to pay it off by equal payments at the end of each month for 21 months. If the effective annual rate of interest is 6.85% how much is each payment?

3. The price of a product (by one payment) is 25000€. A customer hire purchases the product with 15 months payment plan. The deposit is 5000 €, and the service charge is 800€. The effective annual rate of discounting is 6,25%. Find the monthly payment for the hire purchase payment plan.

4. The initial investment of the project is $H = 2800€$. The resulting continuous cash flow is $k = 100 €/\text{month}$. The cash flow starts at $t_1 = 0$ (year) and stops at $t_2 = 2,5$ (year). The residual value is $JA = +500€$. The discount rate is 6,0% (per annum) i.e. $\rho = \ln(1,06) \frac{1}{a}$. The Net Present Value by continuous compounding is

$$NPV = -H + \int_{t_1}^{t_2} e^{-\rho t} k(t) dt + e^{-\rho t_2} JA = -H + \frac{k}{\rho} (e^{-\rho t_1} - e^{-\rho t_2}) + e^{-\rho t_2} JA.$$

a) Find NPV, when $k = 1200€/a$, $\rho = \ln(1,06) \frac{1}{a}$, $t_1 = 0a$, $t_2 = 2,5a$.

b) Find NPV, when $k = 100€/\text{month}$, $\rho = \ln(1,06^{1/12}) \frac{1}{\text{month}}$, $t_1 = 0\text{month}$, $t_2 = 30\text{month}$.

c) What you can say about the internal rate of return?

5. Compare following two projects. The initial investments of project A is 2 000 €, and it creates an cash flow of 100€per month for two years. The initial investments of project B is 16 000 €, and it creates an cash flow of 200€per month for ten years. The cash flows include only the production costs and revenues. Financial costs are not included.

a) Find the Net Present Values of the projects when the discount rate is 8,0% (per annum). Are the projects profitable?

b) We define the Proportional Present Value (PPV) as:

$$PPV = \frac{\text{Present value of Income flow}}{\text{Present Value of Cost flow}}$$

Find the PPV for the projects A and B. Which one is the better one?

6. Using Excels IRR-function find the Internal Rate of Returns of the projects in the exercise 5. Which one is now better?

b) Find the values of "Return on Investment" ROI_{II} for the projects of the exercise 5. (The results are not necessarily reasonable. ROI is a sound measure only for a long project.) Which project is now better one?

7. Find the pay back periods for the projects of the exercise 5. Which one is now better?

Joitakin vastauksia:

4)a) $NPV_A = 217,29€$

4b) $PPV_A = 1,109$

6a) $IRR_A = 19,75\%$

6b) $ROI_{IIA} = 60,0\%$

7a) pay back period for A is 21,5month = 1,8year.

Kaavoja:

Korkolasku

simple interest rate: $K_t = (1 + it)K_0 = (1 + \frac{p}{100}t)K_0$, kun $0 < t < 1$

compounding interest: $K_t = (1 + i)^t K_0$, kun $t = 1, 2, 3, \dots$

continuous compounding: $K_t = (1 + i)^t K_0 = e^{\rho t} K_0$, kun $t > 1$ ja $(1 + i) = e^{\rho}$

Jaksolliset suoritukset prolongointitekijä, diskonttaustekijä, kuoletuskerroin

$$s_{n,i} = \frac{(1+i)^n - 1}{i}, \quad a_{n,i} = \frac{(1+i)^n - 1}{i(1+i)^n}, \quad c_{n,i} = \frac{i(1+i)^n}{(1+i)^n - 1}$$

Tasaerälaina ja osamaksukauppa

$$k = c_{n,i}K_0, \quad k = c_{n,i}(H - h + m)$$

$$\sum_{k=1}^n (a_1 + (k-1)d) = n \cdot \frac{(a_1 + a_n)}{2}, \quad \sum_{k=1}^n a_1 q^{k-1} = \frac{a_1(1 - q^n)}{1 - q}$$

Kassavirran nettonykyarvo

$$NPV = k_0 + \sum_{j=1}^n \frac{k_j}{(1+i)^j}$$

Projektin nettonykyarvo

$$NPV = -H + \sum_{j=1}^n \frac{k_j}{(1+i)^j}$$

Pääoman tuottoaste

$$ROI^I = \frac{\text{nettovuositulos}}{\text{keskimäärin sidottu pääoma}} \cdot 100\%$$

$$ROI^{II} = \frac{\text{nettovuositulos}}{\text{alussa sidottu pääoma}} \cdot 100\%$$

Takaisinmaksu-aika

$$n = \frac{\ln(k/(k - iH))}{\ln(1 + i)}$$