Vaasan yliopisto, kevät 2018

Talousmatematiikan perusteet, ORMS1030

10. harjoitus, viikko 12 (19.3.–24.3.2018)

R1	ma	12–14	F453	R5	ti	14–16	F453
R2	ma	14–16	F453	R6	to	12–14	F425
R3	ti	08–10	F425	R7	pe	08–10	F453
R4	ti	12–14	F453	R8	pe	10–12	F453

1. Määritä a) rivioperaatioiden avulla b) adjungaatin avulla käänteismatriisi matriisille

$$\mathbf{A} = \begin{pmatrix} -1 & -1 & 0 \\ 1 & 1 & -2 \\ -1 & 4 & 3 \end{pmatrix}.$$

$$\left(\begin{array}{ccc|ccc} -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 & 1 & 0 \\ -1 & 4 & 3 & 0 & 0 & 1 \end{array}\right) \cdot 1 \quad \cdot(-1) \quad \cdot(-1)$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 1 & 0 \\ 0 & 5 & 3 & -1 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 5 & 3 & -1 & 0 & 1 \\ 0 & 0 & -2 & 1 & 1 & 0 \end{array}\right) \cdot(-0{,}2) \quad \cdot 0{,}2 \quad \cdot(-0{,}5)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -0{,}6 & -0{,}8 & 0 & -0{,}2 \\ 0 & 1 & 0{,}6 & -0{,}2 & 0 & 0{,}2 \\ 0 & 0 & 1 & -0{,}5 & -0{,}5 & 0 \end{array}\right) \cdot 0{,}6 \quad \cdot(-0{,}6)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1{,}1 & -0{,}3 & -0{,}2 \\ 0 & 1 & 0 & 0{,}1 & 0{,}3 & 0{,}2 \\ 0 & 0 & 1 & -0{,}5 & -0{,}5 & 0 \end{array}\right) \rightarrow A^{-1} = \begin{pmatrix} -1{,}1 & -0{,}3 & -0{,}2 \\ 0{,}1 & 0{,}3 & 0{,}2 \\ -0{,}5 & -0{,}5 & 0 \end{pmatrix}$$

Tark.

$$AA^{-1} = \begin{pmatrix} -1 & -1 & 0 \\ 1 & 1 & -2 \\ -1 & 4 & 3 \end{pmatrix} \begin{pmatrix} -1{,}1 & -0{,}3 & -0{,}2 \\ 0{,}1 & 0{,}3 & 0{,}2 \\ -0{,}5 & -0{,}5 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{Det}(A) = \begin{vmatrix} -1 & -1 & 0 \\ 1 & 1 & -2 \\ -1 & 4 & 3 \end{vmatrix}$$

$$= +(-1)\cdot\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix} - (-1)\cdot\begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix} + 0\cdot\begin{vmatrix} 1 & 1 \\ -1 & 4 \end{vmatrix}$$

$$= -1\left(1\cdot 3 - (-2)\cdot 4\right) + \left(1\cdot 3 - (-2)\cdot(-1)\right) + 0$$

$$= -(3+8) + (3-2) = \underline{-10}$$

Minorit

$$m_{11} = \begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix} = 11, \quad m_{12} = \begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix} = 1, \quad m_{13} = \begin{vmatrix} 1 & 1 \\ -1 & 4 \end{vmatrix} = 5$$

$$m_{21} = \begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} = -3, \quad m_{22} = \begin{vmatrix} -1 & 0 \\ -1 & 3 \end{vmatrix} = -3, \quad m_{23} = \begin{vmatrix} -1 & -1 \\ -1 & 4 \end{vmatrix} = -5$$

$$m_{31} = \begin{vmatrix} -1 & 0 \\ 1 & -2 \end{vmatrix} = 2, \quad m_{32} = \begin{vmatrix} -1 & 0 \\ 1 & -2 \end{vmatrix} = 2, \quad m_{33} = \begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix} = 0$$

$$A^{-1} = \frac{1}{\mathrm{det}(A)}\begin{pmatrix} m_{11} & -m_{21} & m_{31} \\ -m_{12} & m_{22} & -m_{32} \\ m_{13} & -m_{23} & m_{33} \end{pmatrix}$$

$$= \frac{1}{-10}\begin{pmatrix} (11) & -(-3) & (2) \\ -(1) & (-3) & -(2) \\ (5) & -(-5) & (0) \end{pmatrix}$$

$$= \begin{pmatrix} -1{,}1 & -0{,}3 & -0{,}2 \\ 0{,}1 & 0{,}3 & 0{,}2 \\ -0{,}5 & -0{,}5 & 0 \end{pmatrix}$$

2. Ratkaise Cramerin kaavoilla yhtälöryhmä.

$$\begin{cases} x + 3y - z = 0 \\ x + 2y + 3z = 5 \\ 2x + 3y + 2z = 0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 3 & -1 \\ 1 & 2 & 3 \\ 2 & 3 & 2 \end{vmatrix} = +1 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} - 3 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$$

$$= 1 \cdot (4-9) - 3 \cdot (2-6) - (3-4)$$

$$= -5 + 12 + 1$$

$$= 8$$

$$D_1 = \begin{vmatrix} 0 & 3 & -1 \\ 5 & 2 & 3 \\ 0 & 3 & 2 \end{vmatrix} = +0 - 5 \cdot \begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} + 0 = -45$$

$$D_2 = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 5 & 3 \\ 2 & 0 & 2 \end{vmatrix} = -0 + 5 \cdot \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} - 0 = 20$$

$$D_3 = \begin{vmatrix} 1 & 3 & 0 \\ 1 & 2 & 5 \\ 2 & 3 & 0 \end{vmatrix} = +0 - 5 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} + 0 = 15$$

$$\begin{cases} x = \dfrac{D_1}{D} = \dfrac{-45}{8} = -5{,}625 \\ y = \dfrac{D_2}{D} = \dfrac{20}{8} = 2{,}500 \\ z = \dfrac{D_3}{D} = \dfrac{15}{8} = 1{,}875 \end{cases}$$

tark.

$$\frac{-45}{8} + 3 \cdot \frac{20}{8} - \frac{15}{8} = 0 \quad \text{ok}$$

$$\frac{-45}{8} + 2 \cdot \frac{20}{8} + 3 \cdot \frac{15}{8} = \frac{40}{8} = 5 \quad \text{ok}$$

$$2 \cdot \left(\frac{-45}{8}\right) + 3 \cdot \frac{20}{8} + 2 \cdot \frac{15}{8} = 0 \quad \text{ok}$$

3. Laske determinantit

a) $\begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix}$ b) $\begin{vmatrix} 4 & 0 & 1 \\ 1 & 6 & 0 \\ 1 & 8 & 0 \end{vmatrix}$.

a) $\begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix} = 2\cdot 2 - 1\cdot(-3) = 4 + 3 = \underline{\underline{7}}$

b) $\begin{vmatrix} 4 & 0 & 1 \\ 1 & 6 & 0 \\ 1 & 8 & 0 \end{vmatrix} = +1\cdot\begin{vmatrix} 1 & 6 \\ 1 & 8 \end{vmatrix} - 0\cdot\begin{vmatrix} \cdot & \cdot \\ \cdot & \cdot \end{vmatrix} + 0\cdot\begin{vmatrix} \cdot & \cdot \\ \cdot & \cdot \end{vmatrix}$

$= 1\cdot(8-6) = \underline{2}$

4. Etsi jokin ei-triviaali ratkaisu yhtälöryhmälle

$$\begin{cases} 3x - 3y + z = 0 \\ \quad\;\; y - z = 0 \\ -3x + 4y - 2z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 3 & -3 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ -3 & 4 & -2 & 0 \end{array}\right) \cdot 1$$

$$\left(\begin{array}{ccc|c} 3 & -3 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \to \begin{cases} 3x - 3y + z = 0 \\ \quad\;\; y - z = 0 \end{cases}$$

kopio, pois

$$\Leftrightarrow \begin{cases} 3x - 3y = -z & (1) \\ \quad\;\; y = z & (2) \end{cases}$$

Valitaan $z = 1$

$(2) \to y = 1$

$(1) \to 3x - 3\cdot 1 = -1$

$\Leftrightarrow 3x = 2$

$\Leftrightarrow x = \frac{2}{3}$

V: $\begin{cases} x = 2/3 \\ y = 1 \\ z = 1 \end{cases}$

tark.

$$\begin{cases} 3\cdot\frac{2}{3} - 3\cdot 1 + 1 = 0 & \text{ok} \\ 1 - 1 = 0 & \text{ok} \\ -3\cdot\frac{2}{3} + 4\cdot 1 - 2\cdot 1 = 0 & \text{ok} \end{cases}$$

5. Onko $\boldsymbol{M}$ matriisi

$$\boldsymbol{M} = \begin{pmatrix} 2 & 3 & 5 \\ 1 & 1 & -2 \\ -1 & 2 & 3 \\ 4 & 5 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$

Jos yhtälöryhmällä $\boldsymbol{M}\vec{x} = \vec{b}$ on ratkaisu, niin se saadaan kaavalla $\vec{x} = \boldsymbol{M}^{\dagger}\vec{b}$. (ns. Pienimmän Neliösumman Ratkaisu), missä

$$\boldsymbol{M}^{\dagger} = (\boldsymbol{M}^T\boldsymbol{M})^{-1}\boldsymbol{M}^T$$

Käyttäen kaavaa, laske Exelillä kaavan mukainen ratkaisu, kun
a) $\vec{b} = (10 \ \ 0 \ \ 4 \ \ 7 \ \ 1)^T$, ja kun b) $\vec{b} = (9 \ \ 1 \ \ 5 \ \ 6 \ \ 2)^T$.
c) Ovatko a- ja b- kohdan PNS-ratkaisut todellisia ratkaisuja?

$$M^TM = \begin{pmatrix} 2 & 1 & -1 & 4 & 0 \\ 3 & 1 & 2 & 5 & -2 \\ 5 & -2 & 3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 3 & 5 \\ 1 & 1 & -2 \\ -1 & 2 & 3 \\ 4 & 5 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & 25 & -3 \\ 25 & 43 & 3 \\ -3 & 3 & 51 \end{pmatrix}$$

$$\mathrm{Det}(M^TM) = \begin{vmatrix} 22 & 25 & -3 \\ 25 & 43 & 3 \\ -3 & 3 & 51 \end{vmatrix}$$

$$= +22 \cdot \begin{vmatrix} 43 & 3 \\ 3 & 51 \end{vmatrix} - 25 \cdot \begin{vmatrix} 25 & 3 \\ -3 & 51 \end{vmatrix} + (-3) \cdot \begin{vmatrix} 25 & 43 \\ -3 & 3 \end{vmatrix}$$

$$= 48\,048 - 32\,100 - 612 = 15336$$

$$m_{11} = \begin{vmatrix} 43 & 3 \\ 3 & 51 \end{vmatrix} = 2184, \quad m_{12} = \begin{vmatrix} 25 & 3 \\ -3 & 51 \end{vmatrix} = 1284, \quad m_{13} = \begin{vmatrix} 25 & 43 \\ -3 & 3 \end{vmatrix} = 204$$

$$m_{21} = \begin{vmatrix} 25 & -3 \\ 3 & 51 \end{vmatrix} = 1284, \quad m_{22} = \begin{vmatrix} 22 & -3 \\ -3 & 51 \end{vmatrix} = 1113, \quad m_{23} = \begin{vmatrix} 22 & 25 \\ -3 & 3 \end{vmatrix} = 141$$

$$m_{31} = \begin{vmatrix} 25 & -3 \\ 43 & 3 \end{vmatrix} = 204, \quad m_{32} = \begin{vmatrix} 22 & -3 \\ 25 & 3 \end{vmatrix} = 141, \quad m_{33} = \begin{vmatrix} 22 & 25 \\ 25 & 43 \end{vmatrix} = 321$$

$$(M^TM)^{-1} = \frac{1}{15336} \begin{pmatrix} 2184 & -1284 & 204 \\ -1284 & 1113 & -141 \\ 204 & -141 & 321 \end{pmatrix}$$

$$(M^TM)^{-1}M^T = \frac{1}{15336}\begin{pmatrix} 2184 & -1284 & 204 \\ -1284 & 1113 & -141 \\ 204 & -141 & 321 \end{pmatrix}\begin{pmatrix} 2 & 1 & -1 & 4 & 0 \\ 3 & 1 & 2 & 5 & -2 \\ 5 & -2 & 3 & -2 & 3 \end{pmatrix}$$

$$= \frac{1}{15336}\begin{pmatrix} 1536 & 492 & -4140 & 1908 & 3180 \\ 66 & 111 & 3087 & 711 & -2649 \\ 1590 & -579 & 477 & -531 & 1245 \end{pmatrix}$$

a)

$(M^TM)^{-1}M^T\bar{b}_1 =$

$$\frac{1}{15336}\begin{pmatrix} 1536 & 492 & -4140 & 1908 & 3180 \\ 66 & 111 & 3087 & 711 & -2649 \\ 1590 & -579 & 477 & -531 & 1245 \end{pmatrix}\begin{pmatrix} 10 \\ 0 \\ 4 \\ 7 \\ 1 \end{pmatrix} =$$

$$\frac{1}{15336}\begin{pmatrix} 15336 \\ 15336 \\ 15336 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} 2\cdot 1 + 3\cdot 1 + 5\cdot 1 = 10 \\ 1\cdot 1 + 1\cdot 1 - 2\cdot 1 = 0 \\ -1\cdot 1 + 2\cdot 1 + 3\cdot 1 = 4 \\ 4\cdot 1 + 5\cdot 1 - 2\cdot 1 = 7 \\ 0\cdot 1 - 2\cdot 1 + 3\cdot 1 = 1 \end{cases}$$

on ratkaisu!

b) $(M^TM)^{-1}M^T\bar{b}_2 =$

$$\frac{1}{15336}\begin{pmatrix} 1536 & 492 & -4140 & 1908 & 3180 \\ 66 & 111 & 3087 & 711 & -2649 \\ 1590 & -579 & 477 & -531 & 1245 \end{pmatrix}\begin{pmatrix} 9 \\ 1 \\ 5 \\ 6 \\ 2 \end{pmatrix}$$

$$\frac{1}{15336}\begin{pmatrix} 11424 \\ 15108 \\ 15420 \end{pmatrix} \approx \begin{pmatrix} 0{,}744914 \\ 0{,}985133 \\ 1{,}005477 \end{pmatrix}$$

$$\begin{cases} 2\cdot 0{,}744914 + 3\cdot 0{,}985133 + 5\cdot 1{,}005477 = 9{,}472612 \\ 1\cdot 0{,}744914 + 1\cdot 0{,}985133 - 2\cdot 1{,}005477 = -0{,}280907 \\ -1\cdot 0{,}744914 + 2\cdot 0{,}985133 + 3\cdot 1{,}005477 = \underline{4{,}241784} \\ 4\cdot 0{,}744914 + 5\cdot 0{,}985133 - 2\cdot 1{,}005477 = 5{,}894367 \\ 0 \quad - 2\cdot 0{,}985133 + 3\cdot 1{,}005477 = 1{,}046171 \end{cases}$$

Ei ole ratkaisu

Harj 10 teht 5

	A	B	C	D	E	F	G	H	I
1									
2			2	3	5		10		9
3			1	1	-2		0		1
4		M =	-1	2	3	b1 =	4	b2 =	5
5			4	5	-2		7		6
6			0	-2	3		1		2
7									
8			2	1	-1	4	0		
9		M^T =	3	1	2	5	-2		
10			5	-2	3	-2	3		
11			=TRANSPOSE(C2:E6)						
12									
13			22	25	-3				
14		M^T*M =	25	43	3				
15			-3	3	51				
16			=MMULT(C8:G10;C2:E6)						
17									
18			0,14241	-0,08372	0,013302				
19		(M^T*M)^(-1) =	-0,08372	0,072574	-0,00919				
20			0,013302	-0,00919	0,020931				
21			=MINVERSE(C13:E15)						
22									
23			0,100156	0,032081	-0,26995	0,124413	0,207355		
24		Q=(M^T*M)^(-1)*M^T =	0,004304	0,007238	0,201291	0,046362	-0,17273		
25			0,103678	-0,03775	0,031103	-0,03462	0,081182		
26			=MMULT(C18:E20;C8:G10)						
27									
28			1			0,744914			
29		x1 = Q*b1 =	1		X2 = Q*b2 =	0,985133			
30			1			1,005477			
31			=MMULT(C23:G25;G2:G6)			=MMULT(C23:G25;I2:I6)			
32									
33			10			9,472613			
34			-2,2E-16			-0,28091			
35		M*x1 =	4	=b1	M*x2 =	4,241784	# b2		
36			7			5,894366			
37			1			1,046166			
38			=MMULT(C2:E6;C28:C30)			=MMULT(C2:E6;F28:F30)			

6. Alla on taulukossa esitettynä erään yrityksen tuotannon jakautuminen ja panosten käyttö tilikaudella. Laske tuotteiden omakustannusarvot. Teorian löydät kurssin verkkosivulta slaidilta L28b (tai opetusmonisteen sivuilta 238–247).

		Os1	Os2	Os3	myynti (kpl)	yhteensä (kpl)	hinta (€/kpl)
	Os1	50	100	0	850	1000	p_1
	Os2	10	40	50	400	500	p_2
	Os3	20	0	30	200	250	p_3
raaka-aine 1	tt1	300	0	700		1000	1.00
raaka-aine 2	tt2	400	400	0		800	15.00
työvoima 1	tt3	70	100	150		320	25.00
työvoima 2	tt4	10	15	10		35	50.00

Tarvittava kaava on: $\boldsymbol{p}^T = \boldsymbol{c}^T \boldsymbol{B}(\boldsymbol{I}-\boldsymbol{A})^{-1}$, missä

$$\boldsymbol{A} = \begin{pmatrix} 0{,}05 & 0{,}20 & 0{,}00 \\ 0{,}01 & 0{,}08 & 0{,}20 \\ 0{,}02 & 0{,}00 & 0{,}12 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} 0{,}30 & 0{,}00 & 2{,}80 \\ 0{,}40 & 0{,}80 & 0{,}00 \\ 0{,}07 & 0{,}20 & 0{,}60 \\ 0{,}01 & 0{,}03 & 0{,}04 \end{pmatrix}$$

$$\boldsymbol{I}-\boldsymbol{A} = \begin{pmatrix} 0{,}95 & -0{,}20 & 0{,}00 \\ -0{,}01 & 0{,}92 & -0{,}20 \\ -0{,}02 & 0{,}00 & 0{,}88 \end{pmatrix}, \qquad \boldsymbol{c} = \begin{pmatrix} 1{,}00 \\ 15{,}00 \\ 25{,}00 \\ 50{,}00 \end{pmatrix}$$

$$(\boldsymbol{I}-\boldsymbol{A})^{-1} = \begin{pmatrix} 1{,}056146942 & 0{,}229597161 & 0{,}052181173 \\ 0{,}016697975 & 1{,}090586516 & 0{,}247860572 \\ 0{,}024003340 & 0{,}005218117 & 1{,}137549572 \end{pmatrix}$$

Excel ks. seuraava sivu

→ omakustannusarvot ovat

$p_1 = 9{,}81$ (€/kpl)

$p_2 = 22{,}24$ (€/kpl)

$p_3 = 27{,}56$ (€/kpl)

Harj10 teht. 6

Sheet1

	A	B	C	D	E	F	G	H	I	J
1										
2		H10t6		5.555555555	5.555555555	5.555555555	xxxxx	xxxxx	xxxxx	xxxxx
3							myynti	yht.	hinta	
4				Os1	Os2	Os3	(kpl)	(kpl)	(€/kpl)	
5			Os1	50	100	0	850	1000	p1	
6			Os2	10	40	50	400	500	p2	
7			Os3	20	0	30	200	250	p3	
8		Raaka-aine 1	tt1	300	0	700		1000	1,00	
9		Raaka-aine 2	tt2	400	400	0		800	15,00	
10		Työvoima 1	tt3	70	100	150		320	25,00	
11		Työvoima 2	tt4	10	15	10		35	50,00	
12										
13				0,050	0,200	0,000		1	0	0
14			A =	0,010	0,080	0,200	I =	0	1	0
15				0,020	0,000	0,120		0	0	1
16				=D5:D7/H5	=E5:E7/H6	=F5:F7/H7				
17				0,300	0,000	2,800				
18			B =	0,400	0,800	0,000				
19				0,070	0,200	0,600				
20				0,010	0,030	0,040				
21				=D8:D11/H5	=E8:E11/H6	=F8:F11/H7				
22				0,950	-0,200	0,000				
23			(I-A) =	-0,010	0,920	-0,200				
24				-0,020	0,000	0,880				
25				=H13:J15-D13:F15						
26				1,056146942	0,229597161	0,052181173				
27			(I-A)^(-1) =	0,016697975	1,090586516	0,247860572				
28				0,02400334	0,005218117	1,137549572				
29				=MINVERSE(D22:F24)						
30				0,384053434	0,083489877	3,200793154				
31			B*(I-A)^(-1) =	0,435817157	0,964308078	0,219160927				
32				0,091671885	0,237319975	0,73575454				
33				0,012022542	0,035222292	0,053459612				
34				=MMULT(D17:F20;D26:F28)						
35										
36			p^T = c^T*B*(I-A)^(-1) =	9,81	22,24	27,56				
37				=MMULT(TRANSPOSE(I8:I11);D30:F33)						

6*. Miten edellisen tehtävän omakustannusarvot muuttuvat, jos toisen tuotannontekijän ('raaka-aine 2') yksikköhinta kasvaa eurolla (10,00€/kpl → 11,00€/kpl)? Jolloin siis

$$\boldsymbol{c}_{uusi} = \begin{pmatrix} 1{,}00 \\ 16{,}00 \\ 25{,}00 \\ 50{,}00 \end{pmatrix} \quad \text{ja} \quad \Delta\boldsymbol{c} = \boldsymbol{c}_{uusi} - \boldsymbol{c}_{alkup} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$p^T = c^T B (I - A)^{-1}$$

$$\Rightarrow \quad \Delta p^T = \Delta c^T B (I - A)^{-1}$$

$$\Rightarrow \quad \Delta p^T = (0\ 1\ 0\ 0) \begin{pmatrix} 0{,}384 & 0{,}083 & 3{,}201 \\ 0{,}436 & 0{,}964 & 0{,}219 \\ 0{,}092 & 0{,}237 & 0{,}736 \\ 0{,}012 & 0{,}035 & 0{,}053 \end{pmatrix}$$

$$= (0{,}436 \quad 0{,}964 \quad 0{,}219)$$

$$\Delta p_1 = 0{,}44 \ (\text{€/kpl})$$

$$\Delta p_2 = 0{,}96 \ (\text{€/kpl})$$

$$\Delta p_3 = 0{,}22 \ (\text{€/kpl})$$