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Skewness and Kurtosis Adjusted Black-Scholes Model: A Note on Hedging Performance

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Abstract

This article investigates the delta hedging performance of the skewness and kurtosis adjusted Black-Scholes model of Corrado and Su (1996) and Brown and Robinson (2002). The empirical tests in the FTSE 100 index option market show that the more sophisticated skewness and kurtosis adjusted model performs worse than the simplistic Black-Scholes model in terms of delta hedging. The hedging errors produced by the skewness and kurtosis adjusted model are consistently larger than the Black-Scholes hedging errors, regardless of the moneyness and maturity of the options and the length of the hedging horizon.

Keywords: Option Hedging, Delta Hedging, Skewness, Kurtosis
JEL classification: G10, G13

1. INTRODUCTION

The Black-Scholes (1973) model assumes the asset price dynamics to be described by a geometric Brownian motion, and thus, implies constant volatility and Gaussian log-returns. However, both the assumptions of constant volatility and Gaussian returns are obviously violated in financial markets. It has been recognized for a long time that asset return distributions tend to be leptokurtic. More recently, a vast literature has documented volatility to be time-varying. In addition, there seems to be a tendency for changes in stock prices to be negatively correlated with changes in volatility, due to which stock return distributions tend to be negatively skewed.

Given the possible misspecifications of the Black-Scholes model, a substantial literature has devoted to the development of option pricing models which account for the observed empirical violations, such as the volatility smile. The Black-Scholes constant volatility assumption is relaxed in stochastic volatility models such as Hull and White (1987) and Heston (1993a), in deterministic volatility models by Dupire (1994), Derman and Kani (1994), and Rubinstein (1994), and in ARCH models of Engle and Mustafa (1992), Duan (1995), and Heston and Nandi (2000). The assumption of lognormal terminal price distribution is relaxed, e.g., in skewness and kurtosis adjusted models of Jarrow and Rudd (1982), and Corrado and Su (1996), log-gamma model of Heston (1993b), lognormal mixture model by Melick and Thomas (1997), and hyberbolic model of Eberlein et al. (1998).

This article focuses on the delta hedging performance of the skewness and kurtosis adjusted Black-Scholes model of Corrado and Su (1996) and Brown and Robinson (2002). Despite the intensive empirical research on different option pricing models, surprisingly little is known about the hedging performance beyond the Black-Scholes model. Previous studies on hedging performance have focused on different time-varying volatility option pricing models. The hedging performance of stochastic volatility option pricing models is investigated, e.g., in Bakshi et al. (1997, 2000), Nandi (1998), and Lim and Guo (2000) whereas Dumas et al. (1998), Engle and Rosenberg (2000), Coleman et al. (2001), and Lim and Zhi (2002) examine option hedging under deterministic volatility models. A bit surprisingly, previous studies indicate that although time-varying volatility option pricing models clearly outperform the Black-Scholes model in terms of pricing [see e.g., Bakshi et al.

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(1997)], such models do not necessarily improve the hedging performance. Apparently, the hedging performance of option pricing models which relax the normality assumption has not been investigated. This article aims to fill this void by examining the delta hedging performance of the skewness and kurtosis adjusted Black-Scholes model.

The remainder of the article is organized as follows. The skewness and kurtosis adjusted Black-Scholes model is presented in Section 2. In section 3, the FTSE 100 index option data used in the empirical analysis are described. Section 4 presents the methodology applied in the article. Empirical findings are reported in Section 5. Finally, concluding remarks are offered in Section 6.

2. SKEWNESS AND KURTOSIS ADJUSTED BLACK-SCHOLES MODEL

Corrado and Su (1996) and Brown and Robinson (2002) use a Gram-Charlier series expansion of the standard normal density function to derive an expanded Black-Scholes option pricing formula with explicit adjustment terms for nonnormal skewness and kurtosis. The skewness and kurtosis adjusted price of a call option is

$$c = SN(d) - Ke^{-rT}N(d-\sigma\sqrt{T}) + \mu_3 Q_3 + (\mu_4 - 3)Q_4 \tag{1}$$

where

$$Q_3 = \frac{1}{3!}S\sigma\sqrt{T}\left[\left(2\sigma\sqrt{T}-d\right)n(d) + \sigma^2 TN(d)\right]$$

$$Q_4 = \frac{1}{4!}S\sigma\sqrt{T}\left[\left(d^2 - 1 - 3\sigma\sqrt{T}\left(d-\sigma\sqrt{T}\right)\right)n(d) + \sigma^3 T^{3/2}N(d)\right]$$

$$d = \frac{\ln\left(S/K\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

and $N(\cdot)$ denotes the cumulative standard normal distribution function, $n(\cdot)$ is the standard normal density function, *S* is the price of the underlying asset, *K* is the strike price of the option, σ is the volatility of the underlying asset, *r* is the risk-free interest rate, *T* is the time to maturity of the option, and μ_3 and μ_4 denote the standardized coefficients of skewness and kurtosis, respectively. The first two terms of equation (1) constitute the Black-Scholes (1973) option pricing formula whereas the additional terms $\mu_3 Q_3$ and $(\mu_4 - 3)Q_4$ measure the effects of nonnormal skewness and kurtosis on the option price, respectively.

The skewness and kurtosis adjusted Black-Scholes model in equation (1) is particularly convenient from a hedging point of view since it yields closed-form solutions for the hedge ratios. By definition, the delta is obtained by taking the first partial derivative of *c* with respect to *S*. Thus, the skewness and kurtosis adjusted delta can be written as

$$\delta = \frac{\partial c}{\partial S} = N\left(d\right) + \mu_3 q_3 + \left(\mu_4 - 3\right)q_4 \tag{2}$$

where

$$q_3 = \frac{\partial Q_3}{\partial S} = \frac{1}{3!}\left[\sigma^3 T^{3/2}N\left(d\right) + \left(\frac{\phi_1 d}{\sigma\sqrt{T}} + \sigma^2 T - 1 - \phi_1\right)n\left(d\right)\right]$$

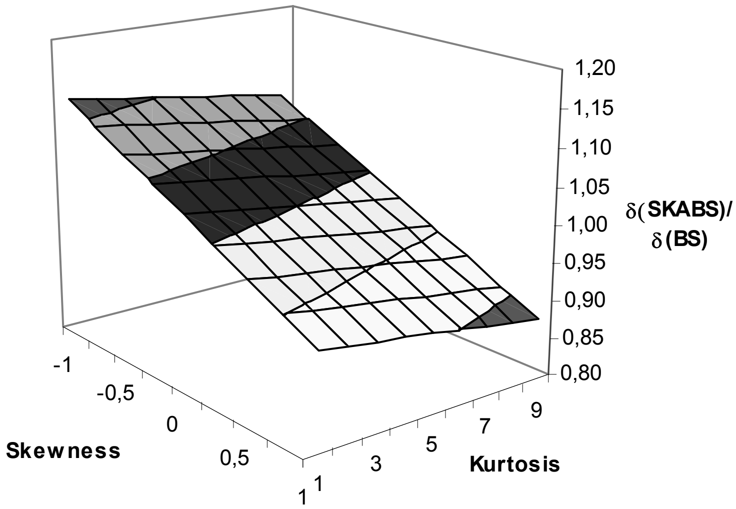
$$q_4 = \frac{\partial Q_4}{\partial S} = \frac{1}{4!}\left[\sigma^4 T^2 N(d) + \sigma^3 T^{3/2} n(d) + \frac{n(d)}{\sigma\sqrt{T}}\left(\phi_2 - 2\sigma^2 T + 2rT + 2\ln(S/K)\right) - \frac{\phi_2 dn(d)}{\sigma^2 T}\right]$$

$$\phi_1 = rT - (3/2)\sigma^2 T + \ln(S/K)$$

$$\phi_2 = r^2T^2 - 2r\sigma^2T^2 + (7/4)\sigma^4T^2 - \sigma^2 T + \ln(S/K)\left(2rT - 2\sigma^2 T + \ln(S/K)\right).$$

The skewness and kurtosis adjusted delta in equation (2) consists of the Black-Scholes delta, $N(d)$, and two additional terms, $\mu_3 q_3$ and $(\mu_4 - 3)q_4$, which measure the effects of nonnormal skewness and kurtosis, respectively. Figure 1 illustrates the impact of skewness and kurtosis on the delta of an at-the-money call option. It can be noted that the delta is a decreasing function of both skewness and kurtosis.

Figure 1. Impact of Skewness and Kurtosis on the Delta



Notes: The figure plots the ratio of skewness and kurtosis adjusted delta (SKABS) to Black-Scholes delta (BS) of an at-the-money call option for different levels of skewness and kurtosis.

3. DATA

The data used in this article contain settlement prices of the European-style FTSE 100 index options traded at the London International Financial Futures and Options Exchange (LIFFE). The sample period extends from January 2, 2001 to December 28, 2001. The settlement prices for the FTSE 100 index options and the closing prices for the underlying implied index futures are obtained from the LIFFE. The risk-free rate needed for the calculation of the deltas and for the delta hedging experiment is proxied by the three-month LIBOR (London Interbank Offered Rate) rate.

Two exclusionary criteria are applied to the complete FTSE 100 index option sample to construct the sample used in the empirical analysis. First, options with fewer than 5 or more than 120 trading days to maturity are eliminated. This choice avoids any expiration-related unusual price fluctuations and minimizes the liquidity problems often affecting the prices of long-term options. Second, options with moneyness greater than 1.10 or less than 0.90 are eliminated. Moneyness is defined as the ratio of futures price to strike price for call options and strike price to futures price for put options. The moneyness criterion is applied because deep out-of-the-money and in-the-money options tend to be thinly traded.

The final sample contains 35 180 settlement prices on options with 5 to 120 trading days to maturity and moneyness between 0.90 and 1.10. This sample is considered to be a representative sample of the most actively traded index option contracts. The sample is partitioned into three moneyness and two time to maturity categories. An option is said to be out-of-the-money (OTM) if the moneyness ratio is less than 0.97, at-the-money (ATM) if the ratio is larger than 0.97 and less than 1.03, and in-the-money (ITM) if the ratio is greater than 1.03. An option is said to be short-term if it has less than 40 trading days to expiration and long-term otherwise.

4. METHODOLOGY

A central issue in the empirical testing of option pricing models is the estimation of the unobservable model parameters. Following the standard approach of simultaneous equations, the vector of model parameters, **Φ**, is estimated by minimizing the sum of squared deviations between the observed market prices and theoretical option prices

$$\min_{\Phi} \sum_{i=1}^{N} \left[c_i - \hat{c}_i(\Phi)\right]^2 \tag{3}$$

where N is the number of option price observations on a given day for a given maturity class, c and $\hat{c}$ are the observed and theoretical option prices, respectively, and $\Phi = \{\sigma\}$ for the Black-Scholes model and $\Phi = \{\sigma, \mu_3, \mu_4\}$ for the skewness and kurtosis adjusted model.

The delta hedging performance of the Black-Scholes and skewness and kurtosis adjusted models is investigated by constructing a self-financed delta-hedged portfolio with one unit short position in an option, δ units of the underlying asset, and B units of a risk-free bond. The value of the portfolio Π at time t is

$$\Pi_t = \delta_t S_t + B_t - c_t \tag{4}$$

At the beginning of the hedging horizon $B_0 = c_0 - \delta_0 S_0$, and thus, $\Pi_0 = 0$. The delta hedging performance of the two deltas is examined in one day and one week hedging horizons using daily rebalancing of the hedge portfolio. At each hedge-revision time t the hedge parameter is recomputed and the position in the bond is adjusted to

$$B_t = e^{rt} B_{t-1} + S_t(\delta_{t-1} - \delta_t) \tag{5}$$

The delta hedging error ε from hedge-revision time t-1 to t is calculated as

$$\varepsilon_t = \delta_{t-1} S_t - c_t + e^{rt} B_{t-1} \tag{6}$$

and the total hedging error during the hedging horizon τ is given as Π_T, the value of the portfolio at the end of the hedging horizon

$$\varepsilon_\tau = \sum_{t=1}^{T} \varepsilon_t = \Pi_T \tag{7}$$

Two commonly used error statistics, mean absolute hedging error (MAHE) and root mean squared hedging error (RMSHE) are used to analyze the delta hedging performance of the models. To avoid any distributional assumptions about the error statistics, bootstrapping is used to test whether the hedging errors from the two models are statistically significantly different.

5. EMPIRICAL RESULTS

Table 1 presents the summary statistics of the model parameter estimates. The estimated parameters of the skewness and kurtosis adjusted model indicate that the return distribution of the FTSE 100 index is fat-tailed and negatively skewed with mean skewness and kurtosis estimates of –0.14 and 3.82, respectively. Interestingly, the implied volatility estimates under the skewness and kurtosis adjusted model tend to be higher than the Black-Scholes implied volatilities.

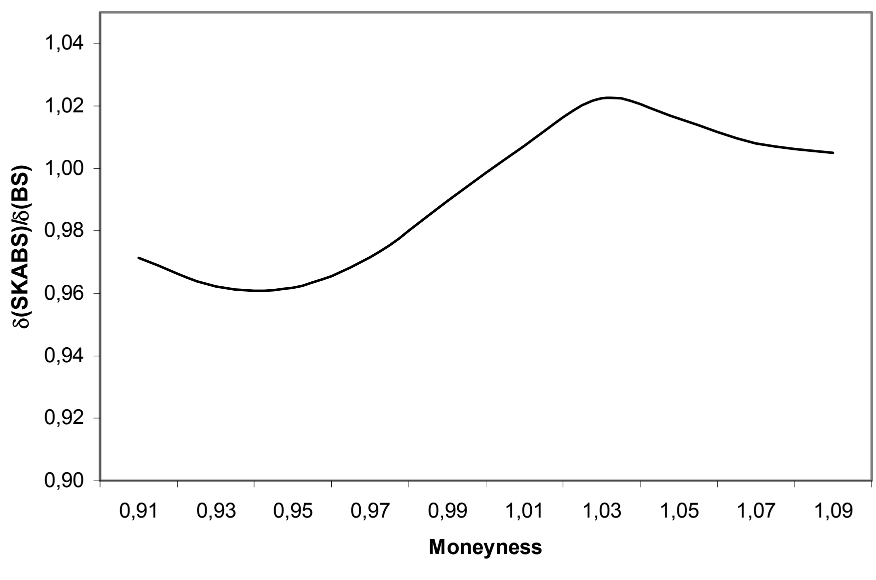
Table 1. Summary Statistics of the Estimated Model Parameters

	BS	SKABS		
	σ	σ	μ_3	μ_4
Mean	20.18	20.86	-0.14	3.82
Median	18.88	19.27	-0.11	3.84
Minimum	12.72	12.99	-1.27	0.17
Maximum	46.02	47.42	0.82	5.63
Std. Dev.	4.73	5.42	0.38	0.60

Notes: The table reports the summary statistics of the model parameter estimates. The model parameters are estimated by minimizing the sum of squared deviations between the observed market prices and theoretical option prices. The parameters σ, μ_3, and μ_4 represent volatility, skewness, and kurtosis, respectively. BS and SKABS denote the Black-Scholes and skewness and kurtosis adjusted option pricing models, respectively.

The ratio of skewness and kurtosis adjusted delta to Black-Scholes delta for different level of moneyness is presented in Figure 2. The ratio seems to vary with moneyness, being below one for OTM options and above one for ITM options, and thus, indicates that the skewness and kurtosis adjusted delta is smaller than the Black-Scholes delta for OTM calls and ITM puts but larger for ITM calls and OTM puts. For ATM options the difference in deltas is almost negligible. Similar delta structure for stochastic volatility models is documented in Bakshi et al. (2000).

Figure 2. The Ratio of Skewness and Kurtosis Adjusted Delta to Black-Scholes Delta



Notes: The figure plots the average ratio of skewness and kurtosis adjusted delta (SKABS) to Black-Scholes delta (BS) for different levels of moneyness. The deltas are calculated using the model parameter estimates summarized in Table I.

The results of the delta hedging experiment for the one day hedging horizon are reported in Table 2. The reported numbers are mean absolute hedging errors (MAHE), root mean squared hedging errors (RMSHE), and the mean difference between the error statistics of the two models. The results in Table 2 indicate that the skewness and kurtosis adjusted model performs worse than the Black-Scholes model in terms of delta hedging. The hedging errors produced by the skewness and kurtosis adjusted model are consistently larger than the Black-Scholes hedging errors, regardless of the moneyness and maturity of the options. The difference in hedging performance appears to be most distinct for long-term ATM options. All differences reported in Table 2 are statistically significant at the 1 % level.

Table 2. Hedging Errors for One Day Hedging Horizon

Moneyness	Time to Maturity	MAHE			RMSHE		
		BS	SKABS	Difference	BS	SKABS	Difference
Full Sample	All	4.45	4.73	-0.27*	7.46	8.02	-0.55*
	Short	4.49	4.71	-0.22*	8.06	8.51	-0.46*
	Long	4.42	4.74	-0.32*	6.84	7.51	-0.67*
OTM	All	4.12	4.35	-0.23*	6.87	7.35	-0.48*
	Short	3.91	4.09	-0.17*	7.12	7.47	-0.35*
	Long	4.32	4.61	-0.28*	6.62	7.23	-0.62*
ATM	All	5.13	5.49	-0.36*	8.66	9.32	-0.66*
	Short	5.61	5.93	-0.32*	9.82	10.42	-0.59*
	Long	4.66	5.05	-0.39*	7.35	8.11	-0.75*
ITM	All	4.18	4.41	-0.23*	6.82	7.34	-0.52*
	Short	4.04	4.21	-0.17*	7.08	7.48	-0.40*
	Long	4.30	4.59	-0.29*	6.57	7.20	-0.63*

Notes: The reported numbers for each maturity-moneyness category are (i) the mean absolute hedging error (MAHE), (ii) the root mean squared hedging error (RMSHE), and (iii) the mean difference between the errors of the two models. BS and SKABS denote the Black-Scholes delta and skewness and kurtosis adjusted delta, respectively. * significant at the 0.01 level and ” significant at the 0.05 level.

Table 3. Hedging Errors for One Week Hedging Horizon

Moneyness	Time to Maturity	MAHE			RMSHE		
		BS	SKABS	Difference	BS	SKABS	Difference
Full Sample	All	13.67	14.12	-0.45*	20.56	21.34	-0.78*
	Short	12.71	13.12	-0.41*	21.37	22.12	-0.75*
	Long	14.59	15.08	-0.49*	19.77	20.58	-0.81*
OTM	All	11.74	12.05	-0.31*	17.28	17.73	-0.45*
	Short	10.03	10.31	-0.28"	16.69	16.99	-0.31
	Long	13.39	13.72	-0.33*	17.83	18.41	-0.58*
ATM	All	18.33	19.08	-0.75*	27.64	28.95	-1.31*
	Short	19.19	19.86	-0.67*	30.93	32.30	-1.37*
	Long	17.56	18.38	-0.82*	24.28	25.54	-1.26*
ITM	All	11.88	12.23	-0.36*	16.49	16.97	-0.48*
	Short	10.38	10.72	-0.34*	15.64	16.05	-0.41
	Long	13.33	13.70	-0.37*	17.27	17.81	-0.54*

Notes: The reported numbers for each maturity-moneyness category are (i) the mean absolute hedging error (MAHE), (ii) the root mean squared hedging error (RMSHE), and (iii) the mean difference between the errors of the two models. BS and SKABS denote the Black-Scholes delta and skewness and kurtosis adjusted delta, respectively. * significant at the 0.01 level and ” significant at the 0.05 level.

Table 3 presents the hedging results for the one week hedging horizon. The results seem very similar to the results for the one day hedging horizon. Again, the error statistics clearly indicate that delta hedging under the Black-Scholes models is more effective than under the skewness and kurtosis adjusted model. Regardless of the moneyness and maturity of the options, both error statistics are lower for the Black-Scholes delta. Most of the differences reported in Table III are highly statistically significant, with the only exceptions being short-term OTM and ITM options.

6. CONCLUSIONS

This article has investigated the delta hedging performance of the skewness and kurtosis adjusted Black-Scholes model of Corrado and Su (1996) and Brown and Robinson (2002). The empirical tests in the FTSE 100 index option market show that the more sophisticated skewness and kurtosis adjusted model performs worse than the simplistic Black-Scholes model in terms of delta hedging. The hedging errors produced by the skewness and kurtosis adjusted model are consistently larger than the Black-Scholes hedging errors, regardless of the moneyness and maturity of the options and the length of the hedging horizon. At the first sight, these results may seem rather surprising. The results are, however, consistent with the empirical tests on the hedging performance of time-varying volatility models [see e.g., Bakshi et al. (1997), Dumas et al. (1998), Nandi (1998), Lim and Guo (2000)], and thus, provide further support for the view that a good option pricing model is not necessarily a good model for hedging. A potential explanation might be that, although the Black-Scholes delta is likely to be biased, the estimation error in the delta is relatively small due to the simplicity of the model.

REFERENCES

Bakshi, G., C. Cao and Z. Chen (1997) Empirical Performance of Alternative Option Pricing Models, *Journal of Finance*, **52**, 203-2049.

Bakshi, G., C. Cao and Z. Chen (2000) Pricing and Hedging Long-Term Options, *Journal of Econometrics,* **94**, 277-318.

Black, F. and M. Scholes (1973) The Pricing of Options and Corporate Liabilities, *Journal of Political Economy,* **81**, 637-659.

Brown, C. and D. Robinson (2002) Skewness and Kurtosis Implied by Option Prices: A Correction, *Journal of Financial Research,* **25**, 279-282.

Coleman, T. F., Y. Kim, Y. Li and A. Verma (2001) Dynamic Hedging with a Deterministic Local Volatility Function Model, *Journal of Risk,* **4**, 63-89.

Corrado, C. and T. Su (1996) Skewness and Kurtosis in S&P 500 Index Returns Implied by Option Prices, *Journal of Financial Research,* **19**, 175–192.

Derman, E. and I. Kani (1994) Riding on a Smile, *Risk,* **7**, 32-39.

Duan, J.-C. (1995) The GARCH Option Pricing Model, *Mathematical Finance,* **5**, 13-32.

Dumas, B., J. Fleming and R.E. Whaley (1998) Implied Volatility Functions: Empirical Tests, *Journal of Finance,* **53**, 2059-2106.

Dupire, B. (1994) Pricing with a Smile, *Risk***,** **7**, 18-20.

Eberlein, E., U. Keller and K. Prause (1998) New Insights into Smile, Mispricing and Value at Risk: The Hyperbolic Model, *Journal of Business,* **71**, 371-405.

Engle, R.F. and C. Mustafa (1992) Implied ARCH Models from Option Prices, *Journal of Econometrics,* **52**, 289-311.

Engle, R.F. and J. Rosenberg (2000) Testing the Volatility Term Structure Using Option Hedging Criteria, *Journal of Derivatives,* **8**, 10-28.

Heston, S. (1993a) A Closed Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, *Review of Financial Studies,* **6**, 237-243.

Heston, S. (1993b) Invisible Parameters in Option Prices, *Journal of Finance,* **48**, 933-947.

Heston, S. and S. Nandi (2000) A Closed-form GARCH Option Valuation Model, *Review of Financial Studies,* **13**, 585-625.

Hull, J. and A. White (1987) The Pricing of Options on Assets with Stochastic Volatilities, *Journal of Finance,* **42**, 281-300.

Jarrow, R. and A. Rudd (1982) Approximate Option Valuation for Arbitrary Stochastic Processes, *Journal of Financial Economics,* **10**, 347-369.

Lim, K. and X. Guo (2000) Pricing American Options with Stochastic Volatility: Evidence from S&P 500 Futures Options, *Journal of Futures Markets,* **20**, 625-659.

Lim, K. and D. Zhi (2002) Pricing Options Using Implied Trees: Evidence from FTSE-100 Options, *Journal of Futures Markets,* **22**, 601-626.

Melick, W. and C. Thomas (1997) Recovering an Asset's Implied PDF from Option Prices: An Application to Crude Oil During the Gulf Crisis, *Journal of Financial and Quantitative Analysis,* **32**, 91-115.

Nandi, S. (1998) How Important is the Correlation between Returns and Volatility in Stochastic Volatility Model? Empirical Evidence from Pricing and Hedging in the S&P 500 Index Options Market, *Journal of Banking and Finance,* **22**, 589-610.

Rubinstein, M. (1994) Implied Binomial Trees, *Journal of Finance,* **69: 3**, 771-818.