ECONOMETRICS 1

Course homepage: `www.uwasa.fi/~bepa/Econometrics1.html`

Exercises 1:

1. a) Derive the algebraic properties (2.17)–(2.19) from the normal equations (2.5). *Hint: Start with proving (2.19).*

   b) Prove the decomposition (2.23). *Hint: Use (2.11) and (a).*

   c) Prove the result in Remark 2.4 of the lecture notes.

2. Let $y_1, \ldots, y_n$ be a random sample from a random variable Y with $\mathbb{E}[Y] = \mu$ and $\mathbb{V}\mathrm{ar}[Y] = \sigma^2$.

   Show that the sample mean

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

   is the OLS estimator of μ obtained by minimizing the squared errors. (*Hint.* Write $y_i = \mu + u_i$, where $u_i = y_i - \mu$ is the random error term.)

3. Find the rationale for the interpretations in Table 2.1 for the slope coefficients of the different regression specifications. I.e., when

   a) $y = \beta_0 + \beta_1 x + u$ (level-level)

   b) $y = \beta_0 + \beta_1 \log(x) + u$ (level-log)

   c) $\log(y) = \beta_0 + \beta_1 x + u$ (log-level)

   d) $\log(y) = \beta_0 + \beta_1 \log(x) + u$ (log-log)

   where β_0, β_1, and u generic symbols for the intercept, slope, and error term and of course are different in different models. *Hint: Consider the observation pairs* (x_1, y_1) *and* $(x_2, y_2) = (x_1 + \Delta x, y_1 + \Delta y)$ *and use*

$$\log(x_0 + \Delta x) - \log(x_0) = \log\left(\frac{x_0 + \Delta x}{x_0}\right) = \log\left(1 + \frac{\Delta x}{x_0}\right) \approx \frac{\Delta x}{x_0}.$$

4. (Wooldridge 3e, 2.9) Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the OLS estimators of intercept and slope coefficient of regression y_i on x_i, using n observations.

a) Let c_1 and $c_2 \neq 0$ be constants. Let $\tilde{\beta}_0$ and $\tilde{\beta}_1$ be the OLS estimators of the intercept and slope coefficients from the regression of $c_1 y_i$ on $c_2 x_i$. Show that $\tilde{\beta}_1 = (c_1/c_2)\hat{\beta}_1$ and $\tilde{\beta}_0 = c_1\hat{\beta}_0$. (Use the OLS formulas (2.6) and (2.7) to prove these.)

b) Now, let $\tilde{\beta}_0$ and $\tilde{\beta}_1$ be the OLS estimators of the intercept and slope coefficients from regression z_i on w_i, where $z_i = y_i + c_1$ and $w_i = x_i + c_2$. Show that $\tilde{\beta}_1 = \hat{\beta}_1$ and $\tilde{\beta}_0 = \hat{\beta}_0 + c_1 - c_2\hat{\beta}_1$.

c) Now, let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the OLS estimators of the regression coefficients from regressing $\log(y_i)$ on x_i, where we must assume that $y_i > 0$ for all i. For $c_1 > 0$, let $\tilde{\beta}_0$ and $\tilde{\beta}_1$ be the OLS estimators for the intercept and slope coefficient form regression of $\log(c_1 y_i)$ on x_i. Show that $\tilde{\beta}_1 = \hat{\beta}_1$ and $\tilde{\beta}_0 = \log(c_1) + \hat{\beta}_0$.