Econometrics

Exercises 2:

1. Prove unbiasedness of $\hat\beta_0$ and formula (2.41) of the lecture notes. *Hint: Apply formula (2.7) and the properties of expectations and variances.*

2. (Wooldridge 3e, C3.6) Computer exercise: Use the dataset `wage2` behind the link on the web page for this problem. Estimate the following regressions

   a) Run the simple regression of IQ on $educ$ to obtain estimate $\tilde\delta$ for the the slope coefficient δ of $IQ = \delta_0 + \delta\, educ + u$

   b) Run the simple regression of $\log(wage) = \beta_0 + \beta_1 educ + v$ to obtain estimate $\tilde\beta_1$ for β_1.

   c) Run the multiple regression $\log(wage) = \beta_0 + \beta_1 educ + \beta_2 IQ + w$ to obtain estimates $\hat\beta_1$ and $\hat\beta_2$ for the slope coefficients.

   d) Verify that $\tilde\beta_1 = \hat\beta_1 + \tilde\delta\hat\beta_2$.

3. (W 3e, 3.13) Consider the simple regression model $y = \beta_0 + \beta_1 x + u$ satisfying the classical assumptions 1–5 (Gauss-Markov assumptions). For some function $g(x)$, for example $g(x) = x^2$ or $g(x) = \log(1 + x^2)$, define $z_i = g(x_i)$. Define the slope estimator

$$\tilde\beta_1 = \frac{\sum_{i=1}^n (z_i - \bar z)(y_i - \bar y)}{\sum_{i=1}^n (z_i - \bar z)(x_i - \bar x)} = \frac{\sum_{i=1}^n (z_i - \bar z)y_i}{\sum_{i=1}^n (z_i - \bar z)x_i}.$$

   a) Show that $\tilde\beta_1$ is linear and unbiased. (Remember, because $E[u|x] = E[u] = 0$ independent of x, you can consider $z_i = g(x_i)$ as non-random in your derivation.)

   b) Show that

$$\mathbb{V}\mathrm{ar}[\tilde\beta_1] = \sigma_u^2 \frac{\sum_{i=1}^n (z_i - \bar z)^2}{\left(\sum_{i=1}^n (z_i - \bar z)x_i\right)^2}.$$

   c) Show directly that under the classical assumptions.

$$\mathbb{V}\mathrm{ar}[\hat\beta_1] \le \mathbb{V}\mathrm{ar}[\tilde\beta],$$

   where $\hat\beta_1$ is the OLS estimator of β_1.

   Hint: Use the Cauchy-Schwarz inequality

$$\left|\sum_{i=1}^n (z_i - \bar z)(x_i - \bar x)\right| \le \sqrt{\sum_{i=1}^n (z_i - \bar z)^2}\sqrt{\sum_{i=1}^n (x_i - \bar x)^2}.$$

4. (W 3e, C4.1) The following model is used to study whether campaign expenditures affect election outcomes (dataset: `vote1`):

$$vote_A = \beta_0 + \beta_1 \log(expend_A) + \beta_2 \log(expend_B) + \beta_3 prtystr_A + u,$$

where $vote_A$ is the percent of the vote received by Candidate A, $expend_A$ and $expend_B$ are campaign expenditures by Candidates A and B, and $prtystr_A$ is a measure of party strength for Candidate A (the percent of recent presidential vote that went to A's party).

a) What is the interpretation of β_1?

b) In terms of parameters, state the null hypothesis that 1% increase in A's expenditures is offset by 1% increase in B's expenditures.

c) Estimate the given model using the data in *vote1.xls*. Do A's expenditures affect the outcome? What about B's expenditures?

d) Test the hypothesis in part b). What do you conclude?