

## ECONOMETRICS

### Exercises 6:

1. (Wooldridge 2&3 ed. 11.7) A *partial adjustment model* is

$$\begin{aligned}y_t^* &= \gamma_0 + \gamma_1 x_t + e_t \\y_t - y_{t-1} &= \lambda(y_t^* - y_{t-1}) + a_t,\end{aligned}$$

where  $y_t^*$  is the desired optimal level of  $y$ , and  $y_t$  is the actual (observed) level. For example,  $y_t^*$  is the desired growth in firm inventories, and  $x_t$  is the growth in firm sales. The parameter  $\gamma_1$  measures the effect of  $x_t$  on  $y_t^*$ . The second equation describes how the actual  $y$  adjusts depending on the relationship between desired  $y$  in time  $t$  and the actual  $y$  in time  $(t - 1)$ . The parameter  $\lambda$  measures the speed of adjustment and satisfies  $0 < \lambda < 1$ .

- a) Plug the equation for  $y_t^*$  into the second equation and show that we can write

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 x_t + u_t.$$

In particular, find  $\beta_j$  in terms of  $\gamma_j$  and  $\lambda$  and find  $u_t$  in terms of  $e_t$  and  $a_t$ . Therefore, the partial adjustment model leads to a model with a lagged dependent variable and a contemporaneous  $x$ .

- b) If  $\mathbb{E}[e_t | x_t, y_{t-1}, x_{t-1}, \dots] = \mathbb{E}[a_t | x_t, y_{t-1}, x_{t-1}, \dots] = 0$  and all series are weakly dependent, how would you estimate the  $\beta_j$ .
  - c) If  $\hat{\beta}_1 = 0.7$  and  $\hat{\beta}_2 = 0.2$ , what are the estimates of  $\gamma_1$  and  $\lambda$ ?
2. (Wooldridge 3 ed, C11.7 (modified)) Use `consump.xls` for this exercise. One version of the *permanent income hypothesis* (PIH) of consumption is that the *growth* in consumption is unpredictable. Let  $gc_t = \log(\text{cons}_t) - \log(\text{cons}_{t-1})$  be the growth in real per capita consumption (on nondurables and services). Then the (PIH) implies  $\mathbb{E}[gc_t | \mathcal{I}_{t-1}] = \mathbb{E}[gc_t]$ , where  $\mathcal{I}_{t-1}$  denotes information known at time  $t - 1$ .<sup>1</sup>

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<sup>1</sup>In statistical terms unpredictability means that the conditional expectation for the next period given the current information is constant, i.e., the information does not change our view about the future. Formally, if  $y_t$  is unpredictable then  $\mathbb{E}[y_{t+1} | \mathcal{I}_t] = \mathbb{E}[y_t]$ , the unconditional mean, which is a constant.

- a) Test the PIH by estimating

$$gc_t = \beta_0 + \beta_1 gc_{t-1} + u_t.$$

Clearly state the null and the alternative hypothesis. What is the information  $\mathcal{I}_{t-1}$  used in this model. What do you conclude from the estimation results?

- b) Add  $gy_{t-1}$  and  $i3_{t-1}$ , where  $gy_t = \log(\text{inc}_t) - \log(\text{inc}_{t-1})$  is the growth in real per capita income and  $i3_t$  is the interest rate on three-month T-bills (note that you need to use the lagged values in the model). What is your information now in the prediction? Are these additional variables significant?
- c) Does the above empirical analysis support PIH?
3. Download daily prices for Microsoft from `finance.yahoo.com` from January 1, 1990 to August 31, 2012. Calculate the log-returns as  $r_t = \log(P_t) - \log(P_{t-1})$  (you may multiply by 100 if you like, note also that EViews has a handy function `dlog(P_t)` to calculate the log-differences).
- a) Are the returns autocorrelated?
- b) Are the squared returns autocorrelated?
- c) Fit an ARCH(1) model to the returns. Investigate the standardized residuals. Is the model satisfactory?
- d) If ARCH(1) does not fit, experiment GARCH(1,1). Do the standardized residual checking.