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Some Empirical Tests of the Arbitrage Pricing Theory Using Transformation Analysis[1]

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Abstract: The purpose of this paper is to test the Arbitrage Pricing Theory (APT) using monthly data for Finnish stock returns during the 1970–1986 period. The first stage involves estimating the systematic risks for each asset using factor analysis. The second stage involves testing by transformation analysis if the number and structure of factors which influence the security returns remain unchanged across various time periods. The third stage involves testing the implications of the APT using cross-sectional regression analysis.

1 Introduction

The Arbitrage Pricing Theory (APT) formulated by Ross (1976) is an elegant model for explaining the relationship between return and risk. The normal empirical procedure to test APT is the following: First, a factor analysis procedure is used to identify the number of factors and the factor loadings from daily, weekly or monthly stock return time series. Second, the estimated factor loadings are used to explain the cross-sectional variation of estimated expected returns.

Unfortunately, there are many problems in testing of APT. An intensively discussed problem is how to decide the correct number of priced factors. It has been found that the number of significant factors is an increasing function of the size of the groups analyzed (Dhrymes, Friend and Gultekin 1984, and Dhrymes, Friend, Gultekin and Gultekin 1985). There are also some additional methodological problems with the use of factor analysis (Elton and Gruber 1987: 343–354). First, the decision as how many factors to extract has been made subjectively. Second, there is no guarantee that factors are produced in a particular order. Third, the signs of the estimated parameters remain arbitrary.

In our opinion, the most relevant questions in testing the APT is neither the question as how to determine the "correct number" of the priced factors in dif-

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ferent samples nor the question in what kind of order factors are produced in those samples. We may get the same number of factors in different samples or in the same sample in different time periods but the content or empirical interpretation of the factors do not necessarily remain as the same in those groups. Therefore, it is very important to find such common factors which are the same across different samples during the same time period (cross-sectional studies) or across different time periods in the same sample (time-series studies) irrespective of what the number of those factors is or in what order the factors are produced. Transformation analysis offers us a versatile methodology with which it is possible to measure the stability and invariance existing among different factor structures (for business applications of transformation analysis see Yli-Olli and Virtanen 1990).

The purposes of this study are:

1. To describe briefly the APT,
2. To test the APT using monthly time series data of the Finnish firms quoted on the Helsinki Stock Exchange,
3. In testing the APT, the main effort is made to test, using transformation analysis, the stability of the factor structure over time. That means: transformation analysis tells us if the content of the factors remains the same in different time periods i.e. the factors are exactly the same over the two periods and the multiple betas are, up to a linear transformation, the same for each firm (the empirical content or interpretation of the factors could be for example market portfolio, unexpected inflation etc. Quite clearly, however, although transformation analysis can tell us that the content of the factors remains the same it can not show what the content explicitely is). Analogously, we can find if the same common factors (the empirical interpretation of the factors being the same) exist in different samples. In our analysis, therefore, it is not necessary that the factors in different samples are produced in a particular order.

2 The APT-Model

The Arbitrage Pricing Theory predicts that on perfectly competitive and frictionless stock markets the stock return is a linear function of a certain number, say k, economic factors. So, the APT starts with the assumption that returns on any stock, R_{it}, are generated by a k-factor model of the form (see e.g. Roll and Ross 1980, 1076–1082):

$$R_{it} = E(\underline{R}_i) + b_{i1}\delta_{1t} + b_{i2}\delta_{2t} + \ldots + b_{ik}\delta_{kt} + \varepsilon_{it} \ , \qquad (2.1)$$

where $E(\underset{\sim}{R}_i)$, $i = 1,2,\ldots,n$, is the expected return of the stock i, $\underset{\sim}{\delta}_j$, $j = 1,2,\ldots,k$, are unobserved economic factors, b_{ij} is the sensitivity of the security i to the economic factor j and $\underset{\sim}{\varepsilon}_i$ are the idiosyncratic risks of the stocks. In addition, it is assumed that $E(\underset{\sim}{\delta}_j) = 0$ for $j = 1,2,\ldots,k$, $E(\underset{\sim}{\varepsilon}_i) = 0$ for $i = 1,2,\ldots,n$, $E(\underset{\sim}{\varepsilon}_i\underset{\sim}{\varepsilon}_h) = 0$ for $i \neq h$, and $E(\underset{\sim}{\varepsilon}_i^2) = \sigma_i^2 < \infty$.

Ross (1976) has shown that if the number of stocks is sufficiently large the following linear risk-return relationship can be written:

$$E(\underset{\sim}{R}_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \ldots + \lambda_k b_{ik} \tag{2.2}$$

where λ_0 is a constant riskless rate of return (the common return on all zero-beta stocks), and λ_j, $j = 1,2,\ldots,k$, represents, in equilibrium, the risk premium for the jth factor.

In Equation (2.1) each stock i has a unique sensitivity b_{ij} to each factor $\underset{\sim}{\delta}_j$ but any factor $\underset{\sim}{\delta}_j$ has a value that is the same for all stocks. These common factors capture the systematic components of risk in Equation (2.1). Therefore, any $\underset{\sim}{\delta}_j$ affects necessarily more than one security return. In the other case it would have been compounded in the unsystematic component of the risk, i.e. in the residual term $\underset{\sim}{\varepsilon}_i$.

In order to test the Arbitrage Pricing Theory we have in principle two alternative approaches to test the model (2.1):

First, we could try to specify a priori, on the basis of the theory, the general factors that explain pricing in the stock market. Such macroeconomic variables could be e.g. the spread between long-term and short-term interest rates, expected and unexpected inflation, industrial production and spread between high- and low-grade bonds (see Chen, Roll and Ross 1986). In the thin Finnish stock market such variables could be e.g. aggregate future cash-flow of the firms, interest rate of bank deposits or return of the state bonds, the supply of money, and inflation (see Virtanen and Yli-Olli 1987). In the case we have factors based on economic theory the estimation procedure should be as follows. In the first stage, time-series regressions are run for each series of stocks (portfolios) to estimate each stock's (portfolio's) sensitivities b_{ij} to macroeconomic variables. Then the risk premia λ_j are estimated by running a cross-sectional regression for each time period examined. In every cross-sectional regression the average return of stocks is used as the dependent variable and the sensitivities of the securities as independent variables.

The more general and also much more problematic approach is to estimate the b_{ij} and unknown factors $\underset{\sim}{\delta}_j$ simultaneously by factor analysis (the coefficients b_{ij} are the factor loadings of stock returns on the factors). In that case the theory does not tell, a priori, what is the exact content or even the number of relevant factors. Without any theory, the decision how many factors to extract from the data has to be made subjectively or by statistical criteria. When we have obtained the systematic components of the risk, b_{ij}, the risk premia λ_j are again estimated using cross-sectional regressions.

In a factor analysis approach we have many methodological problems. First, there are no meaning to the signs of the factors produced by factor analysis. Second, the scaling of b_{ij}'s and λ_j's is arbitrary. Third, there is no guarantee that factors are produced in a particular order when analysis is performed on separate samples (see Elton and Gruber 1987:336–352). In addition, we have serious difficulties when we try to decide what is the correct number of priced factors. Dhrymes, Friend, Gultekin and Gultekin (1985) used samples of different sizes (30, 60 and 90 stocks) and they found that the number of significant factors is an increasing function of the size of the group analyzed.

In our opinion, a very important but non-discussed and non-analyzed problem is the question if the contents of the factor structures in different samples during the same time period or the contents of the factors in the same sample in different time periods are the same. In this paper we use a method which makes it possible to analyze the similarity of factor structures across different samples in the same time period or over different time periods in the same sample. It is not important in this method whether the factors in different samples are produced in a particular order or not. The only limitation is that we have the same number of factors in different samples. After what we can find such common factors which have the same contents, i.e. the same empirical interpretation for different samples.

In this study the matrices of factor loadings to be compared by transformation analysis consist of the coefficients of Equation (2.1).

3 Data and Statistical Methods

In Finland, on the Helsinki Stock Exchange, we have three commonly used indices: Unitas and KOP stock market indices published by two Finnish commercial banks and a return index developed by Berglund, Wahlroos and Grandell (1983). From the theoretical point of view, the return index developed by Berglund, Wahlroos and Grandell is the best measure and was therefore selected for our research. This index also includes the dividend component whereas Unitas and KOP indices are pure price indices.

The empirical verification of the APT and the stability analysis require both a large sample in terms of number of securities and also a long time period. We have in use monthly values of the selected indices from February 1970 to December 1986. The first sample consists of the shares of 30 firms (Appendix). The sample includes all shares which have been quoted on the Helsinki Stock Exchange during the entire sample period. For the stability analysis the whole period is divided into three subperiods: subperiod 1 includes years 1970–1975, subperiod 2 years 1976–1980 and subperiod 3 years 1981–1986. Stability analysis requires the same firms throughout the whole period to be examined. Therefore the firms in the first sample are the same for each subperiods. At the end of the period the total number of quoted firms was 59 (at the moment the number of different securities is much larger). The selection method for the first sample introduces a survival bias because the sample includes only the firms having continuous monthly data during the entire sample period. To escape this bias towards long-lasting firms another sample was also selected. This second sample includes the returns of 30 most traded stocks for each subperiod (the firms for the second sample are not given in Appendix, because the sample is a bit different for each subperiod). However, these two samples include in the main the same firms. Comparing the results from different samples it is possible to conclude how sensitive the results are to the choice of the firms in the sample.

The main statistical methods used in the study are factor analysis, regression analysis and transformation analysis. Factor analysis and regression analysis are usual techniques in business applications. Transformation analysis, on the contrary, has been mainly applied only in Finnish sociological and, in recent years, also in business research. Therefore, this paper contains a short description of this multivariate method.

The degree of stability in factor patterns has been traditionally measured with correlation or congruence coefficients (the same coefficients are used in measuring the stability of estimated betas; see e.g. Blume (1971)). Both of these measures give an index for the similarity of two different factor solutions in terms of the pattern of correlations among factor loadings across all variables in the reduced factor space. For the dissimilar part of these factor solutions these indices are, however, unable to describe and explain the reason for the non-invariant part prevailing in these factor solutions (see Yli-Olli and Virtanen 1985:25).

Originally transformation analysis was developed to compare factor solutions between two different groups of objects (Ahmavaara 1966). Yli-Olli (1983), Yli-Olli and Virtanen (1985 and 1990), Martikainen und Yli-Olli (1990) and Malkamäki, Martikainen und Perttunen (1990) have used the technique to compare two different factor solutions among the same group of objects, the two factor solutions being based on measurements made during two different time periods. In the following we sketch out the general idea behind transformation analysis (according to the papers of Yli-Olli and Virtanen 1985 and 1990).

Let us assume that we have two groups of observations G_1 and G_2 with the same variables, both by number and content. Let L_1 and L_2 be the factor loading matrices for G_1 and G_2, respectively. Let us further assume that the factor models used in deriving L_1 and L_2 are both orthogonal and have the same dimension, $p \times r$, say.

If there exists invariance between the two factor structures, there exists a non-singular $r \times r$-matrix T such that equation

$$L_2 = L_1 T_{12} \qquad (3.1)$$

holds. Matrix T_{12} is called the transformation matrix (between L_1 and L_2, or in direction $G_1 \rightarrow G_2$). If Equation (3.1) holds exactly, it means that the factor structures in groups G_1 and G_2 are, up to a linear transformation, invariant, all the variables have the same empirical meaning in different groups. Depending on the type of the transformation matrix T_{12}, the formation of the factors from the variables and thereby the interpretation of the factors either is preserved (T_{12} is the identity matrix I) or it changes (T_{12} has also non-zero off-diagonal elements).

In practice, situation (3.1) will not be reached, but, after matrix T_{12} has been estimated, we have $L_2 \neq L_1 T_{12}$. The goodness of fit criterion for the model (3.1) may be based on the residual matrix

$$E_{12} = L_1 T_{12} - L_2 \ . \tag{3.2}$$

Non-zero elements in E_{12} mean that the empirical meaning of the variables in question has changed. This is called abnormal transformation.

The main problem in transformation analysis is the estimation of the matrix T_{12}. The estimation methods are in general based on the minimization of the sum of squares of the residuals e_{ij} (the elements of the residual matrix E_{12}). This is the common method of least squares. The problem is to minimize

$$||E_{12}|| = ||L_1 T_{12} - L_2|| = \text{trace}\,((L_1 T_{12} - L_2)(L_1 T_{12} - L_2)') \ . \tag{3.3}$$

Depending on additional constrains set for the matrix T_{12}, we have three different estimation methods, i.e. three transformation analysis models (see e.g. Yli-Olli and Virtanen 1990). Of these three techniques, the symmetric transformation analysis is the most popular one. It is also applied in this study.

With correlation and congruence coefficients one can only measure the degree of similarity of two factor solutions (correlations or congruences among factor loadings). This is also possible via transformation analysis (coefficients of coincidence on the main diagonal of the transformation matrix). In addition to this we obtain a regression type model for shifting of variables from one factor to another (normal or explained transformation). This is revealed by non-zero off-diagonal elements in the transformation matrix and indicates interpretatively changes for the factors in question. And finally, large elements in the residual matrix indicate abnormal or unexplained transformation between the two factor solutions. This means that the empirical content of the corresponding variables has changed. Further, this abnormal transformation can be appointed to separate variables or to separate factors.

4 Empirical Results

At the beginning we examined some statistical properties of stock returns between different subperiods. First, the results showed that the mean and variance of the variables were not time invariant (these results are not reported in the paper). Especially the null hypotheses of equal variances between subperiods could be rejected in a large number of firms. However, the unambiguous tests of the Arbitrage Pricing Theory require, in principle, that the first two moments of return series are time invariant (see e.g. Diacogiannis (1986); the security return distributions at the London Stock Exchange were not intertemporally stationary). Second, equilibrium models of security markets are usually based on the assumption of normally distributed security returns. However, the empirical researches have shown that the distributions of individual daily returns are skewed. Intertemporal aggregation to monthly returns has reduced the skewness only slightly (see e.g. Roll and Ross 1980:1095–1096). The results also showed that the null hypotheses of normally distributed returns could be rejected among most firms examined. The non-stationarity and non-normality in return series indicate that the empirical results must be interpreted with relatively high caution.

The following step in our empirical analysis is to use factor analysis procedure to identify the number of factors affecting equilibrium returns. The estimation of factors can be carried out by different factor analytic methods. In this study we use the principal component method in the initial phase and varimax rotation thereafter.

The following step is to divide the whole sample period into three subperiods and to identify the number of factors affecting equilibrium returns during these subperiods. In this research we try to find such common factors which are stable for different subperiods (and among different groups). For this purpose we first extracted three-, four-, five-, six- and seven-factor solutions for each subperiod. Cumulative proportions of total variance explained (of the unrotated factor patterns) are presented in Table 1.

Table 1. Cumulative Proportions of Total Variance Explained

	Sample 1			Sample 2		
	1970–75	1976–80	1981–86	1970–75	1976–80	1981–86
Factor 1	0.382	0.335	0.302	0.402	0.306	0.304
Factor 2	0.473	0.451	0.405	0.475	0.441	0.393
Factor 3	0.533	0.543	0.482	0.532	0.537	0.468
Factor 4	0.589	0.608	0.551	0.586	0.600	0.531
Factor 5	0.638	0.663	0.611	0.633	0.651	0.588
Factor 6	0.680	0.709	0.656	0.678	0.694	0.637
Factor 7	0.717	0.744	0.698	0.718	0.732	0.677

Table 2. Number of Stocks (N = 30) Associated with Each Factor at the 5 Per Cent Level of Significance (Two-Tailed *t*-Test)

Subperiod	Number of factors	FACT1	FACT2	FACT3	FACT4	FACT5	FACT6	FACT7
1	4	24	18	15	7	–	–	–
	5	22	18	13	10	2	–	–
	6	19	18	13	10	5	1	–
	7	19	17	16	9	3	5	1
2	4	21	18	12	6	–	–	–
	5	19	15	14	7	5	–	–
	6	20	15	14	11	4	5	–
	7	16	12	12	8	9	5	5
3	4	15	20	16	11	–	–	–
	5	13	18	16	7	3	–	–
	6	15	17	13	8	8	4	–
	7	13	15	12	9	7	4	1

Cattell's scree tests (see Green 1978:365) showed that we can find 2–5 different factors for each subperiod. The *t*-tests presented in Table 2 seem to confirm the common factor interpretation. However, we have no absolute guarantee that the factors extracted have the same interpretation when the analysis is performed on separate subperiods, i.e. we can not yet be sure that the extracted factors are those common factors we try to identify.

In the following, only results obtained from sample 1 (the 30 shares quoted at the Helsinki Stock Exchange during the whole period examined) are presented. Sample 2 (containing 30 most traded stocks during each subperiod) was used to control a possible survival bias in the results of sample 1. The results were, however, very similar in both samples.

Next we measure, using transformation analysis, the stability of factor patterns over time. The conclusion about stability is based on the coefficients on the main diagonal of the transformation matrix provided that factors in different samples are produced in the same order. The numerical values of those coefficients are very close to one when the factor structure over time is stable. Tables 3

Table 3. Transformation Matrix Between the Factor Loadings of Stock Returns (Subperiod 1 vs. Subperiod 2; Three-Factor Solution)

	Factor	Subperiod 2		
		1	2	3
Sub-	1	0.985	−0.145	0.097
period	2	0.166	0.947	−0.273
1	3	−0.052	0.285	0.957

Table 4. Transformation Matrix Between the Factor Loadings of Stock Returns (Subperiod 2 vs. Subperiod 3; Three-Factor Solution)

	Factor	Subperiod 3		
		1	2	3
Sub-period 2	1	0.220	0.975	0.009
	2	0.975	−0.220	0.001
	3	−0.003	−0.009	1.000

and 4 present the transformation matrices between three-factor solutions of subperiods 1 and 2, and of subperiods 2 and 3, respectively. The results show that the stability of factors in very high during different subperiods. This means that we have found at least three very stable factors. Table 4 also shows that subperiods 2 and 3 have produced the first and the second factor in different order. This means that the first and second factor have changed their places in the order of appearance in the third subperiod as compared to the first and second subperiods. Table 5 presents the residual matrix between subperiods 1 and 2 (the residual matrix between subperiods 2 and 3 is very similar to the matrix of Table 5; the other residual matricies are not presented in the paper). The residual matrices show that any remarkable abnormal transformation does not exist (there are no large non-zero elements in the matrices).

Tables 6 and 7 present, for four-factor solutions, the transformation matrices between subperiods 1 and 2 and between subperiods 2 and 3, respectively. The results show that the factor structure remains quite stable also in the case of four-factor solutions. Some minor unstable elements have appeared, however, in the factor structure between the first and second subperiod. Residual matrices for four-factor solutions showed also now only minor abnormal transformation.

The results for five- and six-factor solutions are not presented in the paper. These transformation matrices and the corresponding residual matrices showed considerable instability and abnormal transformation.

So, the analysis carried out shows that the stability of factor structure over time is best for three-factor solution (and also quite good for four-factor solutions). First, according to the transformation matrices the factor structure is very stable. Second, the Cattel's scree-tests and eigenvalues also supported the results obtained by this transformation analysis.

The following step involved examining the effect of factors on equilibrium returns. In cross-sections (one for each subperiod) the dependent variable is the mean return for the subperiod and the independent variables are factor loadings from factor analysis. The OLS regression coefficients will be the estimated risk premia. Because there is no meaning to the signs of the parameters and the scaling of the factors in factor analysis the magnitude of regression coefficients is arbitrary. Therefore, only the statistical significance of regression coefficients is relevant instead of their numerical values.

Table 5. Residual Matrix E_{12} and Abnormal Transformation for Subperiod 2 (Three-Factor Solution)

Firm	Factor 1	Factor 2	Factor 3	Abnormal transformation $t_i^2 = \Sigma_j e_{ij}^2$
Kop	−0.291	0.084	−0.110	0.104
Syp	0.089	−0.010	−0.374	0.148
Pohjola	0.521	−0.116	−0.464	0.500
Effoa	−0.324	0.108	−0.212	0.161
Kesko	0.065	−0.066	0.539	0.299
Stock.	0.264	−0.396	0.728	0.757
Tamro	0.245	−0.014	0.099	0.070
Enso	0.171	0.159	−0.179	0.086
Fisk.	−0.080	−0.213	0.028	0.053
Huhtam.	−0.056	0.060	0.071	0.012
Kajaani	−0.089	−0.015	0.202	0.049
Kemi	0.391	0.242	−0.104	0.222
Kone	−0.065	0.114	−0.256	0.083
Kymmene	0.088	−0.281	−0.020	0.087
Lassila	−0.569	−0.141	0.510	0.603
Lohja	0.274	−0.227	−0.068	0.131
Metsäl.	0.352	0.297	0.027	0.212
Nokia	0.050	−0.025	−0.028	0.004
Otava	0.099	−0.103	−0.243	0.079
Partek	0.084	−0.192	−0.121	0.058
Rauma-R.	0.084	−0.110	0.091	0.027
Rosenlew	−0.010	−0.374	0.362	0.271
Schauman	−0.116	−0.464	0.604	0.593
Serlachius	0.108	−0.212	0.093	0.065
Suomen S.	−0.066	0.539	0.180	0.327
Suomen Tr.	−0.396	0.728	−0.124	0.702
Tamfelt	−0.014	0.099	0.084	0.017
Tampella	0.159	−0.179	−0.061	0.061
Wärtsilä	−0.213	0.028	0.268	0.118
Yhtyneet	0.060	0.071	−0.272	0.083
Abnormal transformation $s_j^2 = \Sigma_i e_{ij}^2$	1.623	1.906	2.454	5.984

Table 6. Transformation Matrix Between the Factor Loadings of Stock Returns (Subperiod 1 vs. Subperiod 2; Four-Factor Solution)

	Factor	Subperiod 2			
		1	2	3	4
Sub-	1	0.813	0.533	−0.206	−0.111
period	2	−0.368	0.779	0.383	0.333
1	3	0.368	−0.298	0.220	0.853
	4	0.261	−0.141	0.873	−0.387

Table 7. Transformation Matrix Between the Factor Loadings of Stock Returns (Subperiod 2 vs. Subperiod 3; Four-Factor Solution)

	Factor	Subperiod 3			
		1	2	3	4
Sub-	1	0.944	−0.143	0.292	0.052
period	2	0.212	0.942	−0.243	0.097
2	3	−0.194	0.302	0.844	−0.399
	4	−0.161	0.041	0.379	0.910

Table 8. Regression Analysis Estimates.
Dependent variable: average monthly return for security
independent variables: factor loadings ($k = 3$)
(t-values in parantheses)

Subperiod	Constant	Coefficients of			R-square	F
		Fact 1	Fact 2	Fact 3		
1	0.0257 (5.354)	−0.0067 (−1.122)	−0.0164 (−2.766)	−0.0078 (−1.347)	0.235	2.664
2	0.096 (4.536)	−0.0237 (−4.160)	−0.0170 (−2.875)	−0.0038 (−0.574)	0.489	8.293
3	0.0185 (4.115)	0.0111 (1.712)	0.0097 (1.388)	0.0039 (0.540)	0.128	1.276

The results of the cross-sectional regressions for subperiods are presented in Tables 8 and 9. They show that only two factors are greater than zero at the 1% level of significance and the third and fourth factor have only a bit more explanatory power compared to the two-factor solution. In addition, the non-stationarity and non-normality in return series indicate that the regression results must be interpreted with relatively high caution. However, it is very important to remember that transformation analysis is necessary in testing if the contents of factors in different subperiods are the same. The transformation analysis showed that we can extract three or four factors with the same content in different subperiods. The seeming inconsistency of the results rises from the fact that transformation analysis gives the number of the factors which have the same content in different subperiods. Regression analysis on the other hand gives the number of significant common factors. So, the transformation analysis gives the maximum number of factors with the same content in different time periods. It is possible that some very stable factors extracted by factor analysis are so firm- or industry-specific that their t-statistic is so low in cross-sectional regression that they are not common.

Table 9. Regression Analysis Estimates.
Dependent variable: average monthly return for security
independent variables: factor loadings ($k = 4$)
(t-values in parantheses)

Subperiod	Constant	Coefficients of				R-square	F
		Fact 1	Fact 2	Fact 3	Fact 4		
1	0.0172 (2.486)	0.0017 (0.145)	−0.0107 (−1.574)	−0.0002 (−0.025)	0.0091 (1.285)	0.269	2.302
2	0.0199 (3.584)	−0.024 (−3.912)	−0.0150 (−2.241)	−0.0091 (−1.391)	0.0051 (0.740)	0.537	7.240
3	0.0165 (2.609)	0.0125 (1.649)	0.0116 (1.483)	0.0052 (0.580)	0.0036 (0.445)	0.133	0.959

Table 10. Regression Analysis Estimates.
Dependent variable: error term of the Equation (2.2); three factors
independent variables: variance of the returns, size of the firm
(t-values in parantheses)

Subperiod	Constant	Coefficients of		R-square	F
		variance	size		
1	−0.0069 (−2.501)	1.7606 (2.861)	0.0000 (0.242)	0.233	4.098
2	−0.0018 (0.718)	0.8774 (1.338)	−0.0000 (−0.377)	0.0845	1.245
3	0.0027 (0.638)	0.5873 (0.073)	−0.0001 (−2.102)	0.1513	2.406

Finally, we perform the test of APT against the "own variance" effect and the "firm size" effect (see Chen 1983:1405–1409). The apparently significant explanatory power of the own variance suggests that APT may be "false", because investors should be able to diversify away the nonsystematic part of variance, that is, it should not be priced. The small firm effect has attracted attention in research (see Reinganum 1981 and Chen 1983). Small firms seem to have higher risk-adjusted average returns than large firms. The dependent variable in Table 10 is the residual term of Equation (2.2) (three-factor case). Corresponding dependent variable in Table 11 is the residual term of Equation (2.2) in the four-factor case.

The results show that during the first subperiod the own variance of the stock returns seems to have a slight explanatory power. The explanatory power becomes weaker when we go from the three-factor solution to the four-factor solution. This is parallel to our earlier interpretation that there seem to be three or even four priced factors although they are not all common factors. Finally, during the third subperiod we can find a slight size effect.

Table 11. Regression Analysis Estimates.
Dependent variable: error term of the Equation (2.2); four factors
independent variables: variance of the returns, size of the firm
(t-values in parantheses)

Subperiod	Constant	Coefficients of		R-square	F
		variance	size		
1	−0.0036 (−1.273)	1.0235 (1.609)	−0.0000 (−0.286)	0.0950	1.416
2	−0.0021 (−0.863)	0.9874 (1.550)	−0.0000 (0.365)	0.1059	1.600
3	0.00295 (0.701)	−0.0237 (−0.029)	−0.0001 (−2.046)	0.1412	2.220

The results presented above have been extracted from the sample which includes all shares quoted at the Helsinki Stock Exchange during the whole period examined. At the end of the period the number of the firms was much larger. That is why we selected the second sample containing the returns of 30 most traded stocks for each subperiod. However, these two samples included mostly the same firms. Therefore also the results were very similar and the tables for the second sample have not been presented in the paper (during the third subperiod the values of t-statistics were slightly higher in the second sample).

5 Summary

The main purpose of this study was to test the APT using monthly time series data of Finnish firms quoted on the Helsinki Stock Exchange and, as a part of this, especially to test, using transformation analysis, the stability of the factor structure over time. That means: transformation analysis tells us if the content of the factors remains the same in different time periods (and in principle also in different samples during the same time period).

At the beginning of the empirical part of our research we saw that the mean and variance of the returns were not fully time invariant. In addition, hypotheses of normally distributed returns could be rejected among many firms examined. This indicates that the empirical results must be interpreted with moderate caution.

The empirical verification of the APT involved in the first stage the estimation of the systematic risks for each asset using factor analysis. The second stage involved testing by transformation analysis if the number and structure of factors remained unchanged or stable across different time periods. For stability analysis the whole period (from February 1970 to December 1986) was divided into three subperiods: 1970–1975, 1976–1980 and 1981–1986. The factor and transformation analysis showed that we had at least three very stable factors. Also the four-factor solution was quite stable.

The next step involved examining the effects of factors on equilibrium returns. The results of the cross-sectional regression showed that we had at least two different factors which were significantly greater than zero. The third and fourth factor had only a slight additional explanatory power as compared to the two-factor solution. As a summary we can state that transformation analysis gave us the maximum number of stable factors which preserved the same content across different time periods and could thus serve as the common factors. It is possible that some very stable factors extracted by factor analysis are so firm- or industry-specific that their t-statistic is so low in cross-sectional regression that they are not common.

Finally, we found that the "own variance" and "the firm size" had only a slight explanatory power on equilibrium returns after the two or three priced factors. The analysis was carried out using the residual term of cross-sectional returns as the dependent variable in regression analysis.

Appendix

Stocks Included in the Study

Sample 1: (Same for all periods.)

Kop	= Kansallis-Osake-Pankki
Syp	= Suomen Yhdyspankki Oy
Pohjola	= Vakuutusosakeyhtiö Pohjola
Effoa	= Effoa-Suomen Höyrylaiva Oy
Kesko	= Kesko Oy
Stock.	= Oy Stockmann Ab
Tamro	= Oy Tamro Ab
Enso	= Enso-Gutzeit Oy
Fisk.	= Fiskars Oy Ab
Huhtam.	= Huhtamäki Oy
Kajaani	= Kajaani Oy
Kemi	= Kemi Oy
Kone	= Kone Oy
Kymmene	= Kymmene Oy
Lassila	= Lassila & Tikanoja Oy

Lohja	= Oy Lohja Ab
Metsäl.	= Metsäliiton Teollisuus Oy
Nokia	= Oy Nokia Ab
Otava	= Kustannusosakeyhtiö Otava
Partek	= Oy Partek Ab
Rauma-R.	= Rauma-Repola Oy
Rosenlew	= Oy W. Rosenlew Ab
Schauman	= Oy Wilh. Schauman Ab
Serlachius	= G. A. Serlachius Oy
Suomen S.	= Suomen Sokeri Oy
Suomen Tr.	= Suomen Trikoo Oy Ab
Tamfelt	= Tamfelt Oy Ab
Tampella	= Oy Tampella Ab
Wärtsilä	= Oy Wärtsilä Ab
Yhtyneet	= Yhtyneet Paperitehtaat Oy

Sample 2: Subperiods used in the study and numbers of monthly returns (sample 2)

Subperiod 1: 1. 2. 1970 – 31. 12. 1975 N = 70
Subperiod 2: 1. 1. 1976 – 31. 12. 1980 N = 59
Subperiod 3: 1. 1. 1981 – 31. 12. 1986 N = 71

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