

# **A Simulation Assessment of Three Methods for Deriving the Long-run Profitability of the Firm as its Internal Rate of Return**

**Timo Salmi and Ilkka Virtanen**



**University of Vaasa  
P.O. Box 700  
FIN-65101 Vaasa  
Finland**

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# Deriving the Internal Rate of Return from the Accountant's Rate of Return

by

Timo Salmi and Ilkka Virtanen  
University of Vaasa, Finland

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A Simulation Assessment of Three Methods for Deriving the Long-Run Profitability of the Firm as its Internal Rate of Return

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# Introduction

- A central task in accounting theory and practice: to measure the firm's profitability both in the long-run and in the short-run
- In the economics literature the internal rate of return (IRR) is the widely used theoretical long-run profitability concept. IRR is a well-established measure also among practitioners (in capital budgeting decisions)
- Methods to estimate the IRR
  - methods based on the accountant's rate of return (ARR): Kay's model, the average ARR method
  - methods based on the cash recovery rate (CRR): Ijiri's and Salamon's models
  - methods based on market values (Lawson) etc.

# Research Problem and Methodology

- Research problem in general:
  - To develop an objective and operational methodology for assessing the various long-run profitability (IRR) estimation methods presented in literature
  - To use this methodology for finding out which of the methods works best both in practice and in theory
- Evaluation based on simulated financial statements
  - Evaluations using actual financial data from business enterprises suffer from missing an objective profitability benchmark (true IRR unknown)
  - Results based on an analytic deduction are valid only under strict assumptions and have arrived at conflicting conclusions
  - Using simulated data allows one to know the true IRR in advance for providing the objective benchmark needed

# Specific Research Questions

- 1** Are the methods sensitive to cyclical fluctuations in the capital investment activities?
  - earlier research typically based on a constant growth approach
- 2** Are the methods sensitive to the underlying, assumed cash contribution patterns and duration of the firm's capital investments?
  - sensitivity analysis of the contribution distribution and the life span
- 3** Are the methods sensitive to disparities between the firm's growth and profitability?
  - verification of analytic results under more general assumptions
- 4** Are the methods sensitive to the depreciation choice used to produce the financial statements?
  - sensitivity analysis of the firm's accounting practice

# Capital Investment Simulation Model

$$(1) \quad g_t = g_0 (1 + k)^t \left\{ 1 + A \sin \left[ \left( 2\pi t / C \right) + \phi \right] \right\} [1 + \delta_{t\tau} S],$$

where

$g_t$  = capital expenditures in year  $t$

$k$  = growth rate of the investments

$A$  = amplitude of the business cycle

$C$  = length of the business cycle

$\phi$  = phase adjustment for the business cycle

$\delta_{t\tau}$  = Kronecker's delta ( $\delta_{t\tau} = 1$ , when  $t = \tau$ , and 0 otherwise)

$S$  = capital investment shock coefficient

# Some Basic Formulas

The total contribution  $f_t$  in year  $t$  cumulated from the contributions of the earlier years' investments:

$$(2) \quad f_t = \sum_{i=1}^{\min(N,t)} f_{ti} = \sum_{i=1}^{\min(N,t)} b_i g_{t-i}$$

$f_t$  = cash inflow in year  $t$

$f_{ti}$  = contribution in year  $t$  from capital investment  $i$  years back

$b_i$  = relative contribution from capital investment  $i$  years back

$N$  = life-span of every capital investment project

The true internal rate of return  $r$ :

$$(3) \quad \sum_{i=1}^N b_i (1+r)^{-i} = 1$$

The accountant's profit  $p_t$  is defined as the cash inflow  $f_t$  less the depreciation  $d_t$ :

$$(4) \quad p_t = f_t - d_t$$

The book value  $v_t$  of the firm at the end of period  $t$  is defined as:

$$(5) \quad v_t = v_{t-1} + g_t - d_t$$

The accountant's rate of return  $ARR$  is defined as:

$$(6) \quad ARR = p_t / v_{t-1} = (f_t - d_t) / v_{t-1}$$

# Depreciation Methods

Annuity depreciation (theoretical):

$$(7) \quad d_t = f_t - p_t = f_t - r v_{t-1}$$

Straight-line depreciation:

$$(8) \quad d_t = \sum_{i=1}^{\min(N,t)} (1/N) g_{t-i}$$

Double declining balance method:

$$(9) \quad d_t = \sum_{i=1}^{\min(N,t)} (2/N) (1 - 2/N)^{i-1} g_{t-i}$$

# Contribution Distributions

The uniform distribution

$$(10) \quad b_i = 1 / N, \quad i = 1, 2, \dots, N$$

The negative binomial distribution

$$(11) \quad b_i = s(i + 1)p^2(1 - p)^i, \quad i = 1, 2, \dots, N$$

$s$  = a scaling parameter

$p$  = a shape parameter

The Anton distribution

$$(12) \quad b_i = 1 / N + (N - i + 1)r / N, \quad i = 1, 2, \dots, N$$

# Kay's Model

$$(13) \quad IRR = \frac{\sum_{t=2}^n p_t (1 + IRR)^{-t}}{\sum_{t=2}^n v_{t-1} (1 + IRR)^{-t}}$$

where

$n$  = the length of the observation period

## Average ARR Method

$$(14) \quad ARR = \left[ \frac{1}{n-1} \right] \sum_{t=2}^n p_t / v_{t-1}$$

# Ijiri's Model

Cash recovery rate  $CRR_t$ :

$$(15) \quad CRR_t = f_t / V_{t-1}$$

$$(16) \quad V_t = v_t + \sum_{i=0}^{N/2-1} d_{t-i}$$

where  $V_t$  denotes the gross assets at the end of year  $t$ .

IRR estimate (by iteration):

$$(17) \quad CRR = IRR / [1 - (1 + IRR)^{-N}]$$

## Results (Kay's Model)

- \* The estimates are insensitive to the cyclical fluctuations and their amplitude (Table 2)
- \* When the growth rate and the true IRR are equal, the profitability estimates contain no error (Table 3)
- \* When the annuity method of depreciation is used, the estimates are perfectly accurate
- \* The main source of error in the estimates is the disparity between the firm's growth and profitability
- \* The firm's capital investment opportunities (the contribution distribution) and accounting choice (the depreciation method used) also affect the estimates
- \* The three sources of error interact with each others

## Results (Ijiri's Model)

- \* Two additional sources of error:
  - the estimate of the cumulated depreciations for the GBV
  - the estimate of the life-span  $N$
- \* Ijiri's model fares numerically on the average at least as well as does Kay's model (Table 4). However
  - perfectly accurate estimates do not exist (cf. Kay's model in the case  $k = r$  and under annuity depreciation)
  - there is no clear pattern (direction/magnitude) in the errors
  - the individual error components can be much larger than the resulting total error due to compensating effect (Table 5)
  - + the method is insensitive to cyclical fluctuations

## Results (Average ARR Method)

- \* The results produced by the average ARR method are very similar to the results by Kay's model (Table 6)
  - maximum difference in the estimates of the two methods is 0.1 per cent
  - perfect accuracy in the case  $k = r$  does not hold (but holds in the case of annuity depreciation)
  - the method is insensitive to cyclical fluctuations

# Comparison of Results

## \* Numerical performance

- none of the methods clearly outperforms the others
- the errors in Kay's and the average ARR method are more regular and predictable than in Ijiri's method (Ijiri's method has more, although compensating, sources of error)

## \* Theoretical foundations

- Kay's model is theoretically best founded, with the average ARR method very close by
- Ijiri's model can be regarded more as a good rule of thumb

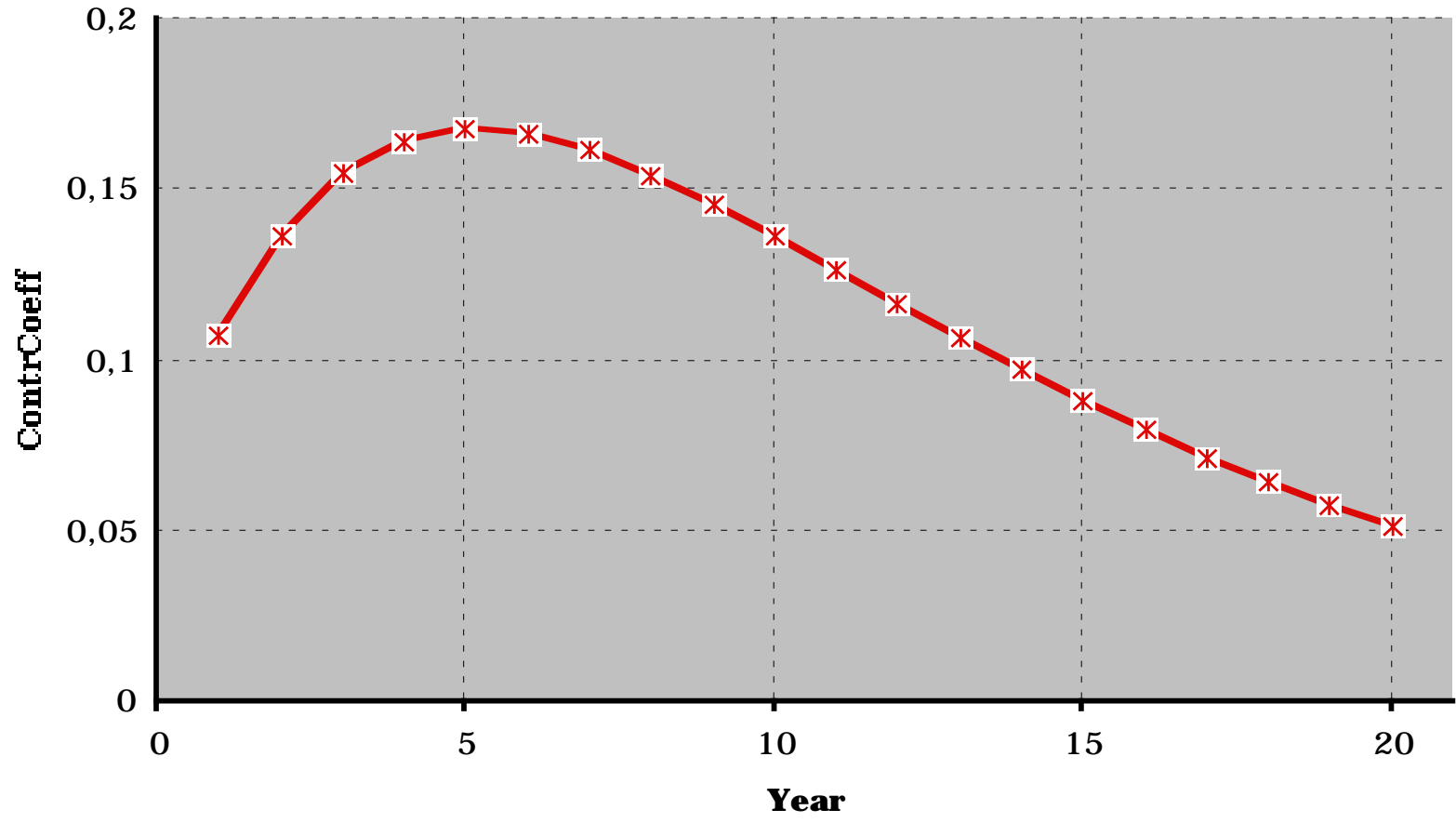
## \* Practical applicability

- the average ARR method has the outstanding merit of being directly based on an established accounting practice
- Kay's and Ijiri's methods difficult to "sell" to practitioners

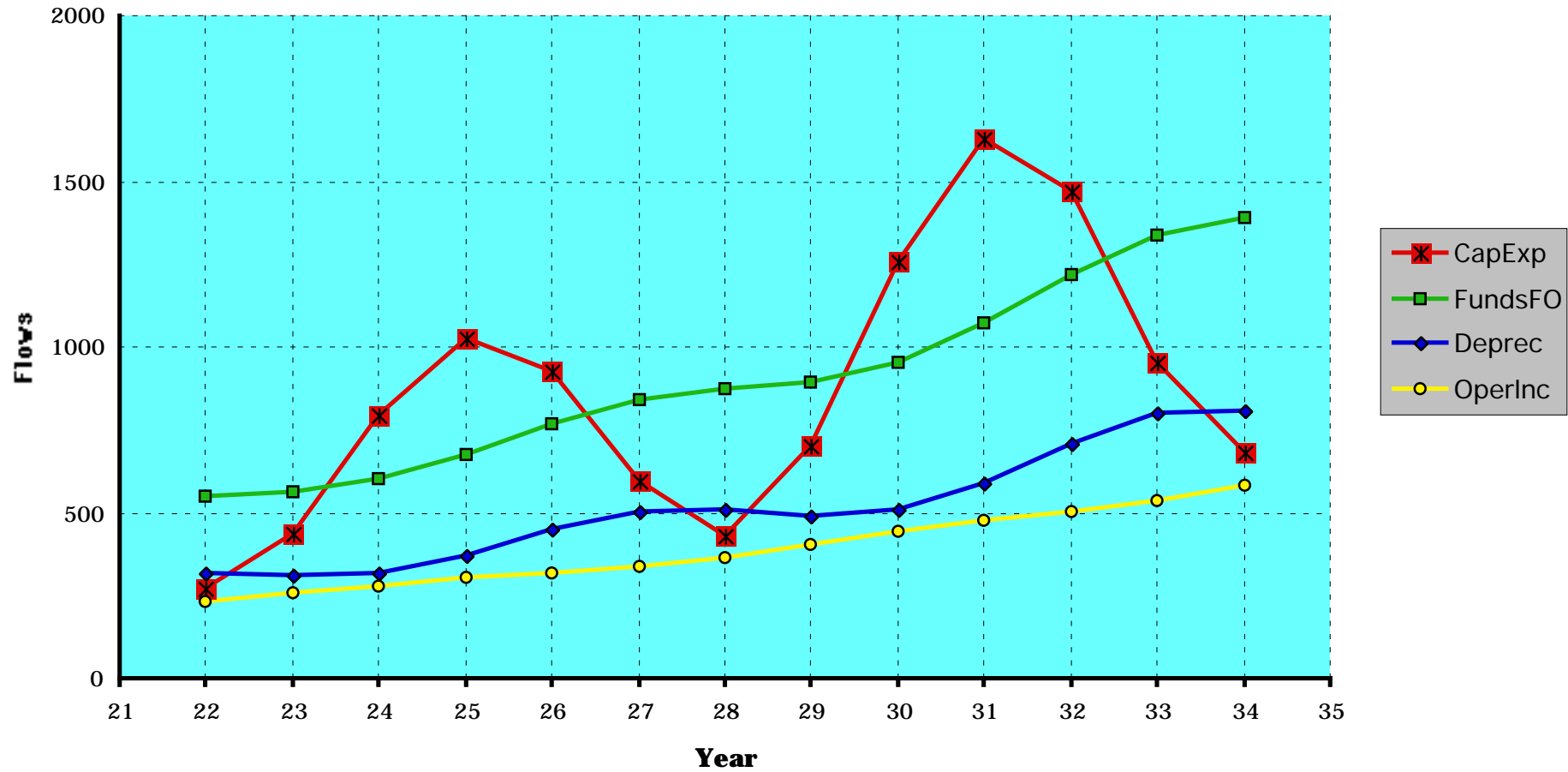
# Conclusion

- \* The simulation approach makes it possible to know the true IRR in advance and provides an objective benchmark for an assessment of the different methods
- \* Due to the inclusion of the investment cycles into the models the applicability of the models beyond the steady state assumptions is confirmed
- \* The discrepancy between the the growth and true profitability is the dominating source of error in all the three methods
- \* The numerical performance of the methods is roughly at par. Kay's method is theoretically best founded and the average ARR method most easy and straightforward to apply in practice

**Negative Binomial Contribution Coefficients for 12% Profitability**



**Visualization of Simulated Observations: Negative Binomial  
Contribution, Declining Balance Depreciation, No Shock, Growth 8%,  
IRR 12%, Amplitude 0.50**



**Visualization of Simulated Observations:  
Negative Binomial Contribution, Declining Balance Depreciation,  
Amplitude 0.50, Shock in Year 24, No Noise, Growth 8%, IRR 12%**

