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Optimal investment and lot-sizing policies for improved productivity and quality

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Productivity and quality are an integrated component of the operational strategy of any firm. An increase in productivity implicitly assumes an improvement in quality. The concept of dynamic process quality control and smaller lot-size production have been employed to eliminate defective items, to reduce the cycle time of a product and to improve quality and productivity. We present a mathematical model to establish the relationship between various parameters of productivity and quality. In addition, the proposed model is used to determine the optimal levels of productivity and quality parameters such as batch sizes, and investment in set-up and process control operations. The basic criterion considered for optimizing the level of such parameters is the minimization of total system cost. The proposed model relates productivity and quality to set-up reduction, queueing of batches, batch sizes, and drift rate reduction. We conclude with an example problem to illustrate the behaviour and application of the model.

1. Introduction

Productivity and quality improvements are a matter of growing concern almost everywhere. Productivity improvement has been identified as one of the major ways of improving national economies under stable political situations. The Japanese productivity and quality improvements and their competitiveness in terms of price and quality have prompted other countries to seriously think about their productivity and quality problems. Productivity can be seen at the manufacturing level as the ratio between input and output and this indicates the effectiveness in utilizing the available capital and resources in producing quality goods and services. A number of technological changes such as flexible manufacturing systems (FMS), optimized production technology (OPT), computer-integrated manufacturing systems (CIM), and new concepts such as just-in-time (JIT), total quality management (TQM), and total preventive maintenance (TPM) in manufacturing systems can be traced to the on-going concern with improving productivity and quality. However, these efforts are concerned primarily with objectives such as set-up cost reduction, low scrappage level, and reduction of manufacturing cycle time. A related issue of concern is achieving smaller lot-size production. This is possible only when set-up cost is reduced by investing in engineering developments such as automatic tool changing or parts loading and unloading, jigs, fixtures, clamps, etc. However, this needs certain investment in these set-up cost reduction activities over a time period and this has to be considered while studying the set-up cost reduction alternatives. Also, there exists a strong relationship

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between batch size and inventory cost due to dynamic process control. For instance, set-up cost reduction facilitates a reduction in inventory cost due to dynamic process quality control using smaller batch sizes for processing the items.

In the case of dynamic process quality control, in-process inventory and scrappage levels are lower. For example, by placing responsibility directly on workers and supervisors, the Japanese prevent defects. This is based on the 'dynamic process quality control' concept. A number of authors such as Fetter (1967), Juran (1974), Pierskalla and Voelker (1976), Crosby (1979), Juran and Gryna (1980) and Feigenbaum (1983) deals with many aspects of quality control such as inspection, process control charts, machine maintenance, repair and replacement, quality improvement and designing for quality. Porteus (1985) explained the economic trade-offs associated with very small lot-size production systems. In particular, Porteus (1985) considered the option of investing in set-up cost reduction parameters in the classical undiscounted EOQ model and determined an optimal set-up cost level. For the same problem situation, Porteus (1986) dealt with discounting. Spence and Porteus (1987) determined the optimal set-up time reduction as well as optimal overtime. Rosenblatt and Lee (1986) considered the influence of imperfect production processes on the economic production cycles. These authors address some of the advantages associated with reducing the set-up cost, but do not take into account the additional benefits of improved quality control and increased capacity.

A number of research articles have been published in the areas of multi-stage production-inventory systems (for example, Maxwell and Muckstat 1985). Szendrovits (1975) presented a mathematical model for determining the manufacturing cycle time as a function of lot-size in a multistage production-inventory system. Goyal (1978) presented the extension of the economic production quantity model described in Szendrovits (1975), optimizing simultaneously both the lot size and the number of sub-batches. Imo and Das (1983) investigated the effects of scheduling according to the optimality of the various production stages. The results of their study conclude that scheduling according to the batch size optimality of various production states can be of real advantage. However, they do not account for the inventory carrying cost due to queueing of batches in their model. Karmarkar *et al.* (1983) presented a model for the multi-item production facility, at which jobs for work at a machine behave as an $M/G/1$ queue. A model of queueing behaviour of a multi-item multi-machine job-shop in which item lot sizes appear as explicit parameters, has been developed in Karmarkar *et al.* (1985). Karmarkar (1987) explained the relationship between lot-sizes, lead times, and in-process inventories through appropriate modelling and analysis. However, most of them appear not to be considering productivity and quality improvements activities such as set-up reduction, dynamic process control and their impact on lot-sizing policies in multistage production-inventory systems. Goyal and Gunasekaran (1990) presented a literature review on multistage production-inventory systems.

Recently, Porteus (1986) demonstrated that lower set-up costs can benefit production systems by improving quality control. He developed a simple model that captures a significant relationship between quality and lot size. The process has been assumed to be in control before beginning production of the lot. Once the process is out of control, it continues to produce defective units until the entire lot is produced. However, there is no model available to explain the relationship between quality control investment and set-up cost reduction in a multistage production system considering the dynamic process quality control. Realizing the importance of this particular problem in practice, an attempt has been made here to develop a mathematical model for studying different batching policies, set-up cost reduction investment and their impact on the performance of a multistage production system under the conditions of dynamic process control. The proposed model will determine the EPQs and optimal investments in set-up cost and process drift rate reduction programs by minimizing the total system cost.

The organization of this paper is as follows: § 2 presents the production system considered for modelling productivity and quality control problems. A mathematical model is developed in § 3. Section 4 contains the details of the solution methodology used. An example problem is solved in § 5 to explain the behaviour and application of the model. Section 6 presents the results and discussions. Finally, the conclusions of this research work are presented in § 7.

2. The production system

The production system considered for modelling is a multistage system which manufactures multiple products. The products are processed based on the priority assigned to them by the manufacturer (presented by 'x_{ij}' in the model). This factor is assumed to be known in advance. As we noted earlier, the system is assumed to be implementing the Japanese quality control approach, viz. dynamic process quality control. That is, whenever a process goes out of control (called 'process drift'), the machine automatically stops until the normal operating conditions of 'no defects' are restored. This particular approach completely avoids scrappage altogether. The process drift rate is a function of the investment in a quality improvement program. Generally, the process goes out of control at random intervals. Since the causes for the process drift is random, the time required to bring the process to normal working condition is also random. The products are processed at each stage and transported between stages in whole batches only. Products are processed with predetermined batch sizes with respect to each stage. According to the size of batches, the batches are either split into smaller batch sizes or added to form larger batches.

3. The mathematical model

The basic objective in the production system considered is to employ smaller lot-size production using set-up cost reduction and dynamic process quality control which would simultaneously improve both productivity and quality. The aim of the model is to estimate the total system cost. The development of the model including notations used, assumptions made, and the basic structure of the model are presented hereunder.

3.1. *Notations*

i = product index ($i = 1, 2, \ldots, M$)
j = stage index ($j = 1, 2, \ldots, N$)
S_j = number of machines at stage j
D_i = demand for product i per unit time (units per year)
A_{ij} = set-up cost per set-up for product i at stage j ($)
Q_{ij} = batch size for product i at stage j (decision variable)
x_{ij} = priority assigned in processing (capacity allocation) product i at stage j
α_{ij} = mean process drift rate while processing product i at stage j
β_{ij} = mean service rate for bringing the process to normal operating condition for product i at stage j
t_{ij} = processing time per unit of product i at stage j (years per unit)

U_{ij} = discounted cost of investment in set-up reduction program per unit time ($ year^{-1})
e_{ij}^{u} = set-up cost reduction elasticity
K_{ij} = a positive constant (defined as the set-up cost per set-up when the elasticity of the investment in the set-up cost reduction programme is zero)
e_{ij}^{v} = process drift rate reduction elasticity
V_{ij} = discounted cost of process control improvement methods ($ per year)
L_{ij} = a positive constant (defined as the process drift rate when the elasticity of investment in process drift rate reduction program is zero)
y = cost per unit time of controlling the process activities ($ year^{-1})
C_{ij} = cost per unit product i after processing at stage j ($ unit^{-1})
C_{i0} = raw material cost per unit product i ($ unit^{-1})
H = inventory cost per unit investment per unit time period ($ $$^{-1}$ year^{-1})
R_{ij} = number of production cycles for the given demand for product i at stage j
T_{ij} = processing time for a batch of product i at stage j (years)
F_{ij} = mean completion time for a batch of product i at stage j (years)
G_{ij} = mean cost per unit of product i between stages j and $j+1$ ($ unit^{-1})
λ_{ij} = production rate for product i at stage j
d_{ij} = lower bound on the value of batch size for product i at stage j
u_{ij} = upper bound on the value of batch size for product i at stage j
Z = total system cost.

3.2. *Assumptions*

The following assumptions are made in developing the model:

(1) Demand per unit time of a product is deterministic and known.
(2) Set-up cost per set-up is constant, independent of set-up sequence and batch sizes.
(3) Process drifts follow a Poisson distribution and service time required for each drift follows exponential distribution with a mean drift and service rates, respectively.
(4) Machines at each stage have identical capacities.
(5) There is no finished product inventory as the products will be dispatched once processing is completed at the final stage.
(6) There are no machine breakdowns.
(7) The cost per unit service time required to bring the process to normal condition is the same for all products and at all stages is known.
(8) The investment in process drift rate and set-up cost reduction programs leads to favourable results.
(9) The elasticities of the cost of investment in set-up cost and process drift rate reduction programs per unit of time are constant and known.

3.3. *The basic model*

The total system cost per unit time consists of: (*a*) set-up cost which is a function of investment per unit time in the set-up cost reduction programme and batch sizes, (*b*) discounted cost of investment in the set-up cost reduction programme, (*c*) discounted cost of investment in the process drift rate reduction programme, (*d*) inventory cost due to process control, (*e*) cost due to process control operations, and (*f*) inventory cost due to queueing of batches. These costs are derived hereunder.

3.3.1. *Discounted cost of investment in set-up cost reduction programme*

Generally, set-up cost per set-up is inversely proportional to the investment in set-up cost reduction programme per unit time. This relationship can be modelled as

$$A_{ij} \propto \frac{1}{U_{ij}^{e_{ij}^{u}}}, \tag{1}$$

where e_{ij}^{u} is the percentage change in set-up cost per set-up divided by the percentage change in investment in set-up cost reduction programme.

The proportionality relationship (1) can be written as

$$A_{ij} = \frac{K_{ij}}{U_{ij}^{e_{ij}^{u}}}, \tag{2}$$

where K_{ij} is the set-up cost per set-up when the elasticity of investment in the set-up cost reduction programme is zero.

The total discounted cost of investment in the set-up cost reduction programme per unit time considering all products and stages is given by

$$\sum_{i=1}^{M} \sum_{j=1}^{N} U_{ij}. \tag{3}$$

3.3.2. *Set-up cost*

The total set-up cost considering all products and stages is given by

$$\sum_{i=1}^{M} \sum_{j=1}^{N} \left\{ \frac{D_i}{Q_{ij}} \right\} A_{ij}. \tag{4}$$

3.3.3. *Discounted cost of investment in process drift rate reduction programme*

The drift rate is assumed to be inversely proportional to the investment in drift rate reduction programme per unit time. This relationship can be represented as

$$\alpha_{ij} \propto \frac{1}{V_{ij}^{e_{ij}^{v}}}, \tag{5}$$

where e_{ij}^{v} is the elasticity of investment in the process drift reduction programme. This can be defined as the percentage change in process drift rate divided by the percentage change in investment in the process drift reduction programme.

The above proportionality relationship (5) can be rewritten as:

$$\alpha_{ij} = \frac{L_{ij}}{V_{ij}^{e_{ij}^{v}}}, \tag{6}$$

where L_{ij} is the drift rate when the elasticity of investment in the process drift rate reduction programme is zero.

The total discounted cost of investment in the process drift rate reduction programme per unit time is given by

$$\sum_{i=1}^{M} \sum_{j=1}^{N} V_{ij}. \tag{7}$$

3.3.4. *Cost due to process control*

This cost arises due to the time spent by the operator to bring the drifted process to normal working condition. This cost is a function of the process drift rate and the service time required for each drift. However, this cost has been assumed to be independent of the type of service performed.

The cost due to process control can be obtained as

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\left\{\frac{D_i t_{ij}\alpha_{ij}}{\beta_{ij}}\right\} y. \tag{8}$$

3.3.5. *Inventory cost due to process control operations*

This cost arises from the waiting of batches due to stopping and restarting the machines for controlling the process. This process control supports the 'dynamic process quality control' which will improve quality control by not producing any defective items. In most occasions, the start-up and shut down of the machines for controlling the process lead to an in-process inventory carrying cost due to the waiting of batches while the process is brought to the normal operating condition.

The processing time for a batch is given by

$$T_{ij}=Q_{ij}\times t_{ij}. \tag{9}$$

It has been assumed here that the time between two successive drifts of the process at a particular machine follows an exponential distribution. Hence, the number of drifts per unit time follows the Poisson process with mean rate of drifts. The drift rate is a function of machine age, motivation of the workers in performing the process, and nature of the process. Depending upon the nature of process control required and the skill of the workers, the service time required to bring the process to a normal working condition varies. Therefore, it is assumed that the service time required for each process control task follows exponential distribution with mean service rate.

Suppose that the operator is an *M/M/1* server (or a server where there are S_j machines at stage j). Every time a machine drifts it has to be serviced by the operator. From *M/M/1* queueing theory, the total time spent (waiting time plus service time per drift) by a batch per drift can be estimated using the *M/M/1* (infinite source) queueing formula (Panico 1968) as

$$\frac{1}{(\beta_{ij}-\alpha_{ij})}. \tag{10}$$

The mean time spent by the batch due to process drifts while processing that batch can be obtained as

$$T_{ij}\left\{\frac{\alpha_{ij}}{(\beta_{ij}-\alpha_{ij})}\right\}. \tag{11}$$

The mean total time required (processing time for batch + mean time spent by the batch due to process drifts) for product i at stage j to complete the processing of a batch is given by

$$F_{ij}=T_{ij}\left\{\frac{\beta_{ij}}{(\beta_{ij}-\alpha_{ij})}\right\}. \tag{12}$$

The number of production cycles per unit time (year^{-1}) for product i at stage j is represented by

$$R_{ij}=\left\{\frac{D_i}{Q_{ij}}\right\}. \tag{13}$$

Since a process drift may occur at any time during the processing of a batch of a product, the value of per unit product at which the drift occurs may be difficult to obtain. Hence, the mean cost per unit product has been accounted to compute the cost due to process control. This cost can be calculated as

$$G_{ij}=\left\{\frac{C_{ij-1}+C_{ij}}{2}\right\}. \tag{14}$$

The total inventory cost due to process drifts is estimated as

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\left[\{R_{ij}Q_{ij}T_{ij}G_{ij}\}\left\{\frac{\alpha_{ij}}{(\beta_{ij}-\alpha_{ij})}\right\}\right]H. \tag{15}$$

3.3.6. *Inventory cost due to queueing of batches*

The production rate for a particular product depends upon the number of machines actually used for a product. If more than one machine is used for a product, then the production rate will be much higher than that with only one machine. The production rate for a product at a stage is a function of total process completion time of a batch, and the number of machines used for processing the product at the stage.

The production rate for product i from stage j can be calculated using equation (8) as

$$\lambda_{ij}=\left[\frac{\{x_{ij}\times S_j\}}{F_{ij}}\right], \tag{16}$$

where

$$\sum_{i=1}^{M} x_{ij}=1, \qquad \text{for } j=1,2,\ldots,N.$$

The parameter x_{ij} indicates the number of machines to be assigned for a particular product at a stage. In practice, the value of x_{ij} is being estimated based on marketing and financial factors.

The material flow for a product through a buffer with two adjacent production stages is modelled as an *M/M/1* production-inventory queueing system with an objective to calculate the waiting time for each batch and hence the inventory carrying cost of product i between stages j and $j+1$. In this system, the waiting bay constitutes the buffer storage. The arrival process of batches of a product at the buffer storage depends on the output rate at stage j, while the discharge process from the buffer depends on the output rate of the stage $j+1$. In case these two rates for a product are not equal, there is an imbalance in arrival and departure of batches and hence random forming and depletion of in-process inventory at the buffer from time to time.

Using the standard *M/M/1* queueing formulae (Panico 1986), the mean waiting time for a batch is given by:

$$W_{ij}=\left\{\frac{\lambda_{ij}}{[\lambda_{ij+1}(\lambda_{ij+1}-\lambda_{ij})]}\right\}. \tag{17}$$

In order to maintain the stability of the material flow and hence the system, the condition $\lambda_{ij+1} > \lambda_{ij}$ has to be satisfied. In case the restriction on batch sizes (due to technological and operational constraints) does not satisfy the condition $\lambda_{ij+1} > \lambda_{ij}$, then the capacities at each stage should be arranged to satisfy this condition.

Hence, the in-process inventory carrying cost due to queueing of batches between stages/at buffer, considering all products and buffers, is obtained using equation (17) and it can be obtained as:

$$\sum_{i=1}^{M} \sum_{j=1}^{N-1} \{R_{ij} W_{ij} Q_{ij} C_{ij}\} H. \tag{18}$$

3.3.7. *Problem formulation*

The total system cost per unit time can be obtained by the summation of cost equations (3), (4), (7), (15) and (18). The formulation of the batching problem can be given as:

$$\min Z = \sum_{i=1}^{M} \sum_{j=1}^{N} U_{ij} + \sum_{i=1}^{M} \sum_{j=1}^{N} \left\{\frac{D_i}{Q_{ij}}\right\} A_{ij} + \sum_{i=1}^{M} \sum_{j=1}^{N} V_{ij} + \sum_{i=1}^{M} \sum_{j=1}^{N} \left\{\frac{D_i t_{ij} \alpha_{ij}}{\beta_{ij}}\right\} y$$

$$+ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[\{R_{ij} Q_{ij} T_{ij} G_{ij}\} \left\{\frac{\alpha_{ij}}{(\beta_{ij} - \alpha_{ij})}\right\} \right] H + \sum_{i=1}^{M} \sum_{j=1}^{N-1} \{R_{ij} W_{ij} Q_{ij} C_{ij}\} H, \tag{19}$$

subject to the following constraints:

$$\beta_{ij} > \alpha_{ij} \quad \text{for all } i \text{ and } j \tag{20}$$

$$d_{ij} \leqslant Q_{ij} \leqslant u_i \quad \text{for all } i \text{ and } j. \tag{21}$$

Constraint (20) indicates that the service rate for the process control must be greater than the drift rates for a product at any stage. The lower and upper bounds on batch sizes are represented by the constraint (21). The mathematical model proposed here is capable of evaluating the the total system cost (Z) for different combinations of batch sizes, investment in the set-up cost reduction programme and investment in the process drift rate reduction programme.

4. Example

A three-stage production system manufacturing three products is considered to illustrate the model. The objective here is to determine optimal batch sizes, and the optimal discounted cost of investment in set-up and process drift rate reduction programs per unit time by minimizing the total system cost. The input to the example is presented in Table 1.

5. Solution methodology

In the literature a number of optimization methods has been reported to solve different types of problems or objective functions. The selection of a particular optimization method for solving a model depends upon the structure of the objective function. Here a brief description of the different optimization methods available in literature and the reasons for selecting the direct pattern search method (DPSM) to solve the proposed model here, DPSM and computational experience are reported hereunder.

Parameter/ variable	$M=3$ $N=3$								
	Product 1			Product 2			Product 3		
	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3
D		3000			2000			1500	
S	2	4	6	2	4	6	2	4	6
C_0		30·00			25·00			40·00	
C	50·00	70·00	85·00	40·00	55·00	75·00	65·00	80·00	95·00
t	0·0001	0·00006	0·00003	0·00008	0·00004	0·00002	0·0001	0·00005	0·00003
x	0·2	0·4	0·2	0·4	0·4	0·4	0·4	0·2	0·4
K	16000·00	18000·00	25000·00	22000·00	25000·00	40000·00	30000·00	40000·00	45000·00
L	20000·00	16000·00	10000·00	15000·00	10000·00	25000·00	70000·00	10000·00	10000·00
U	120·00	100·00	100·00	100·00	90·00	100·00	150·00	200·00	150·00
V	100·00	120·00	150·00	50·00	100·00	200·00	75·00	100·00	150·00
e^u	1·2	1·4	1·2	1·4	1·2	1·5	1·2	1·4	1·3
e^v	1·2	1·6	1·4	1·4	1·2	1·5	1·6	1·4	1·2
β	120	100	50	150	110	70	100	60	40

$H=0{\cdot}20$; $y=7500{\cdot}00$; $d_{ij}=100$; and $u_{ij}=600$ for all i and j.

Table 1. Input to the model.

5.1. *Available optimization methods*

The available optimization methods are usually classified into the classical or indirect methods and the mathematical programming and search methods. The classical methods are amenable to analytical solutions. They are based on the condition that the first derivatives of the objective function with respect to the independent variables must vanish at the optimum and are restricted to very few real world problems. The mathematical programming and search methods generally require a digital computer for finding a numerical solution to most realistic problems. Keuster and Maiz (1973) classified these methods into two broad categories. The first category is called special purpose methods which include several linear programming algorithms, quadratic programming, geometric programming, and dynamic programming and is expected to handle large problems. Each of these techniques is designed to be used with a specific problem structure. The second category consists of a variety of search methods such as multivariable unconstrained/constrained and used for much smaller problems. Decomposition techniques, such as dynamic programming, may be appropriate for restructuring large problems into a sequence of smaller optimization problems.

The unconstrained/constrained methods are normally divided into two categories: (1) derivative free methods, and (2) gradient methods. The gradient methods (e.g. SUMT algorithm) require function and derivative evaluations while the derivative free methods (e.g. the HOOKE algorithm) require function evaluations only. In general, one would expect the gradient methods to be more effective, due to the added information provided. However, if analytical derivatives are not available, the question of whether a search technique can be used should be answered. If numerical derivative approximations are utilized, the efficiency of the gradient methods should be approximately the same as that of the derivative-free methods. Gradient methods incorporating numerical derivatives would be expected to present some numerical problems in the vicinity of the optimum, i.e. the approximations would become very small. For further details on the comparison of different optimization methods (Kuester and Maiz 1973). Based on observations of the characteristics of different optimization methods available and nature of the objective function and constraints, DPSM has been selected as a suitable method for the solving the model developed here. The following are some of the reasons for selecting the DPSM to solve the model developed in § 4.

The structure of the objective function (Z) as represented by equation (19) does not satisfy the special structure required to use the spacial purpose methods such as linear and dynamic programming techniques. In addition, the objective function (Z) is not easily amenable to the classical optimization procedure, wherein the objective function should be differentiable at least once. The selection of a gradient search method (for example, SUMT) may require numerical derivative approximations and additional functional evaluations. These perhaps might increase the error in the objective function and the total number of functional evaluations required. Therefore, the direct pattern search method (DPSM) of Hooke and Jeeves (1966) is used for the present problem. This method has originally been proposed for unconstrained multivariable objective functions. However, DPSM can be converted into a constrained multivariable method using: (1) a feasibility check and (2) a modified objective function. The feasibility check approach is similar to the unconstrained methods except that a check section is added to verify if a constraint is violated. If this happens, the current point is relocated inside the feasible region in a prescribed manner. The modified objective function method incorporates the constraints into the objective function thus producing an unconstrained problem. Penalty functions are used which apply a penalty to the objective functions at non-feasible points thus forcing the search back into the feasible region. We have incorporated the feasibility check method in the DPSM (Kuester and Maiz 1973).

5.2. *Direct pattern search method*

This method is iterative in nature and the search moves in the minimum/maximum direction of the objective function. The search is performed either by increasing or decreasing the value of the decision variables Q_{ij}, U_{ij} and V_{ij}, by a constant factor called step sizes. It is based on the heuristic that if a move from one point to another in an n-dimensional space is successful, then the next move should also be made in the same direction. This move is called a 'pattern move' and utilizes the information concerning the local characteristics of the response surface supplied by an 'exploratory search' conducted around each base point to a new temporary base point by simulating the combined effect of moves from the previous base point. This feature enables the routine to reduce the computational effort. The failure of a pattern move returns the search to the last successful base point and begins an exploratory search to establish a new pattern. If this fails, the step sizes are reduced and another exploratory search is performed. This process continues until a new pattern or the step sizes fall below a preset minimum and the search terminates (Hooke and Jeeves 1966).

The search method is not unidirectional in nature as it moves in both directions of the values of decision variables. Hence, if more than one minimum exists or the shape of the surface is unknown, several sets of initial batch size values are recommended. Different starting values for decision variables may produce different optimal batch size values (Q), investment in set-up cost reduction program (U) and investment in process drift rate reduction program (V) (for more details about this search method see Hooke and Jeeves 1966). In the considered multi-product situation, the schedule of one product depends upon the scheduling of the other product. Therefore, appropriately sized (by batch splitting and forming) batches are processed at each stage. For instance, the waiting time of the larger batches can be reduced by splitting the batches into smaller batches. However, this is appropriate when the set-up cost is less than the inventory cost. Similar arguments hold for batch forming when the batches are smaller and set-up cost is significant compared to inventory cost. Simultaneously, the search routine considers the set-up cost and the speed of the machines while deciding about the batch splitting or forming. Furthermore, the search routine looks for machines wherein the items should have a minimum waiting time in front of the stages while compared to the level of cost incurred by the set-ups. Hence, the batch size optimization with respect to each stage embodies the feature of batch splitting and forming, flexibility in sequencing and scheduling of items and machines which may result in better system performance by reducing the total manufacturing cost and simultaneously increasing the utilization level of facilities. Determining the optimal level of investments in U and V may lead to an effective utilization of available resources.

5.3. *Computational experience*

The model has been coded in Fortran IV together with the DPSM. The computational time required for each run is only a few seconds on the Vax 3400 computer system which has a memory capacity of 29MB. It should be noted that larger problems can also be solved using this DPSM. In the computer program developed for this, a separate module has been incorporated to check the conditions at the end of every iteration, thus extending the 'Hooke and Jeeves search procedure' to the optimization problem with constraints. Moreover, the DPSM appears to have the following advantages over gradient methods: (*a*) derivative of the objective function is not required, (*b*) a better optimal solution as it does not require any approximations in the objective function, and (*c*) it is an easier method. However, the DPSM appear to perform very well when there are constraints on the decision variables. Otherwise, the search for an optimal solution is computationally long. It should also be noted that DPSM is applicable to optimization problems where the number of decision variables are indeed limited to a reasonable value.

6. Results and analysis

The details of the results obtained for the example problem such as comparison of the results obtained using different batching policies, sensitivity analysis, and comparison of the results with and without the optimization of batch sizes, discounted cost of investment in set-up and process drift rate reduction programs, are presented here.

6.1. *Comparison of different batching policies*

The results obtained by the DPSM are presented in Table 2. This table compares the results obtained for the cases of uniform batch sizes for each product at all stages and batch size with respect to each stage. A savings of 12·75% in total system cost has been achieved using the lot-sizing policies with respect to each stage as compared to that of uniform batch sizes for each product at all stages. The results indicate that the set-up cost is higher in case of uniform batch sizes ($5312) as compared with that of batch sizes ($3797) with respect to each stage. In the example situation considered, the set-up cost appears to be dominant as compared to the sum of inventory costs due to process control and queueing of batches between stages. Therefore, the search method looks for larger batch sizes with an attempt to reduce the set-up cost and inventory cost due to queuing of batch sizes. The reduction in set-up cost is due to batch splitting and forming with respect to each stage which are based on the characteristics of the facilities at each stage in terms of set-up and inventory costs for each of the alternative batch sizes. However, the inventory cost due to process control has increased ($2611–2776) because the search method selects larger batch sizes. In addition, the inventory cost due to queueing of batches has come down owing to the batch splitting and forming processes. The set-up cost appears to be highly significant when compared to inventory cost due to process control and waiting of batches. This indicates that there is an opportunity for further improvement by means of a suitable set-up cost reduction program. The reduction in set-up cost does not only reduce the total set-up cost, but also leads to smaller lot-size production and in turn reduces the inventory cost due to process control and waiting of batches. Furthermore, there is a need to investigate the balancing of production rates between successive stages from the capacity planning point of view.

As noted above, the batch splitting and forming reduce the queueing time of batches between stages and hence help to increase the speed of material flow within the production system. However, there are no changes in other costs such as discounted cost of investment in drift rate reduction, and process control cost, as they have been treated as constant. Furthermore, there are a number of operational and technological constraints (for example, material handling capacity, process constraints, inspection

Lot-sizing policies	Optimal batch sizes ($Q_{ij}, j=1, N; i=1, M$)	Set-up cost ($\times 10^2$)($)	Inventory cost due to process control ($\times 10^2$)($)	Inventory cost due to queueing of batches ($\times 10^2$)($)	Discounted cost of investment in set-up reduction ($\times 10^2$)($)	Discounted cost of investment in drift reduction ($\times 10^2$)($)	Process control cost ($\times 10^2$)($)	Total cost (Z) ($\times 10^2$)($)
Uniform batch sizes	300, 300, 300, 200, 200, 200, 150, 150, 150	53·12	26·11	13·67	11·10	10·45	36·15	150·60
Batch sizes with respect to each stage	199, 600, 330, 432, 511, 560, 192, 212, 205	37·97	27·76	7·970	11·10	10·45	36·15	131·40

Percentage decrease in total system cost = (150·60 − 131·40)*100/150·60 = 12·75%.

Table 2. Comparison of the results obtained for the cases of uniform batch sizes and batch size with respect to each stage.

Parameters/variables	Optimal batch sizes ($Q_{ij}, j=1,N; i=1,M$)	Set-up cost ($\times 10^2$)($)	Inventory cost due to process control ($\times 10^2$)($)	Inventory cost due to queueing of batches ($\times 10^2$)($)	Discounted cost of investment in set-up reduction ($\times 10^2$)($)	Discounted cost of investment in drift reduction ($\times 10^2$)($)	Process control cost ($\times 10^2$)($)	Total cost (Z) ($\times 10^2$)($)
$H=0{\cdot}10$	267, 600, 390 555, 600, 600, 271, 299, 289	29·93	18·39	5·080	11·10	10·45	36·15	111·10
0·20	199, 600, 330, 432, 511, 560, 192, 212, 205	37·97	27·76	7·970	11·10	10·45	36·15	131·40
0·30	169, 578, 291, 353, 417, 457, 157, 173, 167	45·05	34·80	10·15	11·10	10·45	36·15	147·70
$U=0{\cdot}5U$	285, 600, 403, 600, 600, 600, 294, 333, 321	67·77	39·38	10·80	5·550	10·45	36·15	170·10
$1{\cdot}0U$	199, 600, 330, 432, 511, 560, 192, 212, 205	37·97	27·76	7·970	11·10	10·45	36·15	131·40
$2{\cdot}0U$	137, 464, 235, 274, 331, 343, 126, 134, 130	23·82	18·52	5·360	22·20	10·45	36·15	116·50
$V=3{\cdot}0V$	600, 600, 368, 600, 600, 600, 600, 276, 408	23·95	6·774	6·988	11·10	31·35	8·394	88·56
$1{\cdot}0V$	199, 600, 330, 432, 511, 560, 192, 212, 205	37·97	27·76	7·961	11·10	10·45	36·15	131·40
$2{\cdot}0V$	470, 600, 360, 600, 600, 600, 600, 274, 361	25·17	11·65	7·076	11·10	20·90	14·33	90·23

Table 3. Results obtained by the sensitivity analysis.

Situation		$(Q_{ij}, j=1, N; i-1, M)$ $(U_{ij}, j=1, N; i=1, M)$ $(V_{ij}, j=1, N; i=1, M)$	Set-up cost $(\times 10^2)$($)	Inventory cost due to process control $(\times 10^2)$($)	Inventory cost due to queueing of batches $(\times 10^2)$($)	Discounted cost of investment in set-up reduction $(\times 10^2)$($)	Discounted cost of investment in drift reduction $(\times 10^2)$($)	Process control cost $(\times 10^2)$($)	Total cost (Z) $(\times 10^2)$($)
Without optimization	Q	300, 300, 300 200, 200, 200 150, 150, 150							
	U	120·00, 100·00, 100·00 100·00, 90·000, 100·00 150·00, 200·00, 150·00	53·12	26·11	13·67	11·10	10·45	36·15	150·60
	V	100·00, 120·00, 150·00 50·000, 100·00, 200·00 75·000, 100·00, 150·00							
With optimization	Q	388, 582, 277 600, 482, 369 600, 247, 243							
	U	200·00, 135·30, 200·00 122·40, 200·00, 161·90 178·80, 200·00, 200·00	17·30	9·825	5·471	15·98	16·96	13·93	79·470
	V	200·00, 177·40, 200·00 179·10, 200·00, 164·80 200·00, 174·90, 200·00							

Percentage decrease in total system cost = (150·60 − 79·47)*100/150·60 = 47·23%.

Table 4. Comparison of the results obtained without and with the optimization of batches, investment in set-up and process control reduction programs.

time, etc.), which in practice should be considered while optimizing the batch sizes with respect to each stage. A disadvantage of varying batch sizes is that the operational control of the production becomes more difficult. However, this disadvantage can be compensated by the advantage of lowered cost as this could be seen from the level of reduction in the total cost, as compared to the uniform batch sizes for all products. Besides, they can be overcome by standardizing the route of the products in the system and a computerized material flow information system.

6.2. *Results of the sensitivity analysis*

To study the behaviour of the model, a sensitivity analysis has been carried out. The details of the results obtained for different levels of inventory holding rate (H), discounted cost of investment in the set-up reduction program per unit time (U), discounted cost of investment in the drift reduction program per unit time (V), and process-service rate (β) are reported in Table 3.

The variation in inventory holding rate leads to significant changes in cost due to process control and the set-up cost. For example, lowering the inventory holding rate (H) from 0·20 to 0·10 results in a reduction in set-up cost ($3797–2993). This indicates that the model selects larger batch sizes when $H=0{\cdot}10$ as compared to those of when $H=0{\cdot}20$ in order to save some set-up cost. Because of the larger batch sizes, the cost due to the waiting of batches has also increased. This also explains how important it is to reduce the inventory level due to various disturbances in the system. Hence, one can achieve an increased productivity by reducing the cycle time of the product using smaller batch sizes.

A decrease in discounted cost of investment in the set-up reduction program per unit time ($1{\cdot}0U$–$0{\cdot}5U$) leads to a considerable increase in total set-up cost (from $3797–6777). Because of this increase, the model selects larger batch sizes and in turn this increases the inventory cost due to process control and queueing of batches. However, the discounted cost of investment in the set-up reduction program has come down from $1110–550. This implies that the model looks for higher savings in set-up cost when U is equal to $0{\cdot}5U$ by adjusting the size of batches. Therefore, the model prefers larger batches to achieve a reasonable reduction in set-up cost. An increase in discounted cost of investment in set-up reduction program per unit time reduces the inventory cost due to process control and queueing of batches. But the total discounted cost of investment in set-up reduction program has increased. This set-up cost reduction obviously helps to use smaller batch sizes and hence a reduced throughput time and an increased productivity. However, the optimal investment in the set-up cost reduction program should also be evaluated.

An increase in the discounted cost of investment in the drift reduction program per unit time ($1{\cdot}0V$–$2{\cdot}0V$) will allow the model to select larger batch sizes. This perhaps results in a lower total set-up cost ($3797–2517). Obviously, the increase in discounted cost of investment in drift rate reduction program per unit time leads to a corresponding reduction in inventory cost due to process control ($2776–1165) and queueing of batches ($7961–7076). This also leads to a decrease in process control cost ($3615–1433). However, one has to optimize the cost of investment in the set-up cost and drift rate reduction programs per unit time with a view to utilizing the available resources effectively. The results obtained for various parameter levels offer interesting scope for planning production and investment activities under different situations.

Furthermore, the results obtained by the sensitivity analysis indicate the potential for achieving significant savings in total system cost by employing the proposed batch sizing policies as compared with the uniform batch sizes. Also, the investments in the set-up reduction program (U) and the drift rate reduction program (V) influence the total system cost. Therefore, optimal values of the U and V may greatly improve the performance of the system in terms of productivity and quality.

6.3. *Comparison of the results with and without the optimization of Q, U and V*

The results obtained for the cases of with and without the optimization of Q, U and V are presented in Table 4 with a view to explaining the importance of investing in the set-up reduction program and dynamic process quality control to reduce the inventory cost and at the same time to increase the quality of products.

The simultaneous optimization of Q, U and V results in a savings of 47·23% in total system cost as compared with 12·75% when only the batch sizes are optimized by minimizing the total system cost. Simultaneous optimization of batch sizes and investment in set-up cost and process drift rate reduction programs decrease the set-up cost, inventory cost due to process control and queueing of batches and process control costs. At the same time, the investments in the set-up and drift rate reduction programs have increased. However, the increase in the investments is less than the reductions in other costs. Usually, a reduction in set-up cost leads to smaller batch production. However, the model selects larger batches due to the fact that the set-up cost is dominating among all other costs. Furthermore, investment in set-up cost and drift rate reduction programs at appropriate stages lead to an overall decrease in all other costs. However, in practice caution should be exercised while calculating the values of U and V taking into account the return of long-term investments. It should also be noted that there is a potential for achieving considerable savings in process control cost by reducing the drift rate by suitable investments. Hence, the proposed model optimizes the level of investments in set-up reduction and process drift reduction programs in order to improve productivity and quality with a minimum total system cost.

7. Concluding remarks

A mathematical model has been developed in this paper to study the implications of lot-sizing with respect to each stage, set-up cost reduction and process control in a multi-stage production system. A direct pattern search method has been used to determine the optimal values for lot-sizes, investment in a set-up cost reduction program, and investment in a process control program, by minimizing the total system cost. It must be acknowledged however, that the model is based on a number of assumptions and approximations. These, in turn, suggest directions for further research to enhance the accuracy of modelling the problem of multi-stage multi-product production environments.

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