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A model for integrated production-packaging systems

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Abstract. The problem of determining the economic packaging frequency of jointly replenished items has received a lot of attention from researchers. This problem is usually encountered while packaging the products after completing manufacture. Nevertheless, production batch quantities certainly influence the manufacturing cycle time of products and hence the economic packaging quantities for jointly replenished items. Therefore, realizing the importance of the relationship between production batch quantities and packaging quantities

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of jointly replenished items, a mathematical model is developed in this paper for determining the economic production and packaging quantities in a multi-stage production system. The basic criterion considered for optimizing the production and packaging quantities of jointly replenished items is the minimization of total system cost per unit time (TSC). The TSC consists of setup cost for production, packaging setup cost, inventory cost of production and inventory cost of packaged items. An example problem is solved to illustrate the model.

1. Introduction

In batch processing industries, finished products are generally packaged into several container sizes. When a manufacturer sells a number of products in different package sizes, there may not be sufficient bulk storage facilities for each of the manufactured products. In such a situation, finished products are packaged into more than a single-sized container. When a product is packaged into more than one type of container immediately after its manufacture, these items are said to be jointly replenished. The packaged product in a specific container has been designated an 'item'. The economic advantage of packaging such items jointly is that if items are replenished individually then each item is accountable for the full setup costs involved in undertaking the manufacture of the unpackaged product and for packaging the unpackaged product. In the case of joint replenishment, the economic lot size for each item cannot be determined without considering the rest of the items within the group. This is so because each time a batch of a product is made, there is a primary setup cost for the manufacture of the product and a secondary setup cost for each of the items to be packaged within the jointly replenished group. If the packaging setup costs are very high, it may be more economical to package certain 'slow moving' items less often than the 'fast moving items'. Specifically, one could package a 'slow moving' item once every other time or every third time a batch of product is made. However, joint packaging is not always economical for a product packaged into more than one container. It seems that a particular item with a very small demand could be made less frequently than the rest of the items in the same product group with resultant savings in total system cost. For a complete introduction to the problem see Shu (1971) and Goyal (1973a,b).

The packaging of products facilitates the process of transportation and other distribution activities. The problem of determining the economic packaging frequency of jointly replenished items has received considerable attention from researchers over the last 25 years (Shu 1971, Goyal 1973a,b, 1974, 1988, Nocturne 1973, Silver 1976). A common characteristic of all the suggested methods is that the production runs for producing unpackaged products are undertaken at equal intervals. Goyal (1974) developed an enumerative optimal solution procedure for determining the optimal policy when the production setups are undertaken at equal intervals. Silver (1976) suggested a simple heuristic procedure for determining a nearly optimal replenishment policy for jointly replenished items. Most of the models reported in the literature (for example, Shu 1971, Goyal 1973a,b, 1974) seem to be more suitable for only a single-stage production system.

In the past, a number of researchers have considered the problem of lot-sizing in multi-stage production-inventory systems. For example, Goyal (1978) presented an EPQ model for optimizing simultaneously both the lot size and number of sub-batches. Korgaonker (1979) extended the optimality condition for determining EPQs in a multi-stage assembly system as proposed by Crowston *et al.* (1973) to multiple items case. In addition, the optimality condition has been used for integrating the lot-sizing problems of production and packaging in a serial multi-stage production system. Goyal and Belton (1979) developed a simple method for determining order quantities in joint replenishment for deterministic demand.

However, most real-life manufacturing organizations have multi-stage production systems. In multi-stage production-inventory systems, in-process inventory tremendously influences the production batch quantity and in turn influences the packaging quantities for unpackaged items. But none of them has considered the problems of integrating production batch quantity and packaging quantities in a multi-stage production-inventory system. Realizing the importance of integrating production-packaging decisions in a multi-stage production system, an attempt has been made in this paper to develop a model for determining the optimal production and packaging policies in a multi-stage serial flow-shop batch production system. The basic criterion considered for determining the economic production and packaging quantities is the minimization of total system cost per unit time. The total system cost consists of setup cost for production, inventory cost of production, setup cost for packaging of items, and inventory cost of packaged items.

The organization of this paper is as follows: Section 2 presents a mathematical model to capture the relationship between production and packaging batch sizes in a multi-stage production system and the solution methodology. In Section 3, an example problem is solved to illustrate the model. The conclusions of this research are presented in Section 4.

2. The mathematical model

The production system considered for modelling is a flow-shop serial batch production system. The products are processed in batches and jointly replenished after their manufacture. The development of the model is presented hereunder.

Notation

- m total number of products
- n total number of production stages
- p_i total number of container-types (items) for product i
- i product index ($i = 1, 2, \ldots, m$)
- j stage index ($j = 1, 2, \ldots, n$)
- l item index ($l = 1, 2, \ldots, p_i$); it indicates the specific container type l.
- d_{il} demand for product i and item l per unit time or per year
- D_i demand for product i per unit time or per year (units per year)
- A_{ij} setup cost per setup for product i at stage j ($)
- Q_{ij} batch quantity for product i at stage j (units)
- PQ_{il} packaging quantity for product i and for item l
- C_{il} Container capacity for product i and for item l
- T production cycle time (in year) on the last processing stage n (decision variable)
- h_{ij} value added inventory cost per unit per year, for product i at stage j ($ per unit per year)
- H_{il} inventory cost of finished product per unit per year, for product i and item l
- k_{ij} a positive integer such that product i is processed at stage j once every $\prod_{q=j}^{n} k_{iq}$ cycles (decision variables)
- S_{il} cost of a packaging set-up for product i and item l ($)
- K_{il} a positive integer such that unpackaged product i is packaged into item l once every K_{il} cycles (decision variables)
- Z total system cost per unit time ($ per year).

2.1. *Assumptions*

The following assumptions are made in developing the model.

(a) Demand rate for each packaged item is uniform and deterministic.
(b) A flow-shop production system is considered where

all products are processed in batches at different stages in the same sequence. Setup cost per setup is constant.

(c) Products are packaged immediately after the manufacture in integer multiple production batch quantities.

(d) The production rates at different stages are assumed very high, so that instantaneous production results.

(e) The packaging setup cost is known and constant.

(f) Product shortages are not allowed.

(g) Planning horizon is infinite.

(h) Rate of packaging is infinite, that is, the packaging time is negligible.

2.2. *The basic model*

The total system cost consists of the following costs: (i) setup cost for production, (ii) inventory cost of production, (iii) setup cost for packaging of items, and (iv) inventory cost of packaged items. These costs are derived hereunder.

Crowston *et al.* (1973) have proved for multi-stage production systems, the economic production quantity at a stage is an integral multiple of economic batch size of its successor stage. This fact permits the consideration of only batch sizes that are integer multiples. Korgaonker (1979) extended the Crowston *et al.* (1973) optimal condition to the multi-product case in a serial multi-stage production system. With schedule feasibility guaranteed, the result clearly holds for each individual member of the multi-product family. We use the condition for the batch sizes to be optimal as suggested by Korgaonker (1979) for multi-item case for developing a model for integrated production-packaging systems.

The demand for product i is calculated as

$$D_i = \sum_{l=1}^{p_i} [d_{il} \times C_{il}] \tag{1}$$

The batch size for product i at the final stage (n) is given by

$$Q_{in} = k_{in} D_i T \tag{2}$$

The production batch size for product i at stage j is given by

$$Q_{ij} = k_{ij} k_{ij+1} \ldots k_{in} D_i T = \left(\prod_{q=j}^{n} k_{iq}\right) D_i T \tag{3}$$

2.2.1. *Setup cost for production*

The total setup cost for production considering all products and stages is given by

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{A_{ij}}{T\left(\prod_{q=j}^{n} k_{iq}\right)} \tag{4}$$

2.2.2. *Inventory cost of production*

Following Crowston *et al.* (1973), we shall use the concept of *echelon stock*. Echelon stock is the number of units in the system which have passed through the stage j but have not yet been sold. This concept allows consideration of value added inventory of a product at a stage and permits the inventory holding cost to be considered as a function of the stage concerned only. Also, the assumption of infinite production rate guarantees schedule feasibility and the production cycles will be repeatable in the long run.

The inventory holding cost for production is given by

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\left(\prod_{q=j}^{n} k_{iq}\right) D_i T h_{ij}}{2} \tag{5}$$

2.2.3. *Setup cost for packaging of items*

The total setup cost for packaging of items given by

$$\frac{1}{T} \sum_{i=1}^{m} \sum_{l=1}^{p_i} \left\{\frac{S_{il}}{K_{il}}\right\} \tag{6}$$

2.2.4. *Inventory cost of packaged items*

The packaging size for product i and item l is given by

$$PQ_{il} = TK_{il} d_{il} \tag{7}$$

The total inventory cost of packaged items is given by

$$\frac{T}{2} \sum_{i=1}^{m} \sum_{l=1}^{p_i} \{d_{il} K_{il} H_{il}\} \tag{8}$$

2.2.5. *Problem formulation*

The total system cost can be obtained by the summation of cost equations (5), (6) and (8). The formulation of the production and packaging problem can be given as

Minimize $Z =$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{A_{ij}}{T\left(\prod_{q=j}^{n} k_{iq}\right)} + \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\left(\prod_{q=j}^{n} k_{iq}\right) D_i T h_{ij}}{2} + \frac{1}{T} \sum_{i=1}^{m} \sum_{l=1}^{p_i} \left\{\frac{S_{il}}{K_{il}}\right\} + \frac{T}{2} \sum_{i=1}^{m} \sum_{l=1}^{p_i} \{d_{il} K_{il} H_{il}\} \tag{9}$$

A search method is proposed for determining the optimal values for T, k_{ij} and K_{il} by minimizing the total system cost (Z). The direct pattern search method (DPSM) of Hooke and Jeeves (1966) is used for the present problem. The search is performed either by increasing or decreasing the value of the decision variables by constant factor called step sizes. Different starting values may produce different optimal batch size values. For more details about this search method, see Hooke and Jeeves (1966). The model has been coded in Fortran IV together with the DPSM. The computational time required for each run is only a few seconds on a Vax 3400 computer system which has a memory capacity of 29Mb. It should also be noted that larger problems can also be solved using this DPSM.

3. Example

A three-stage production system processing three products is considered to illustrate the model. The objective here is to determine optimal batch sizes for production and packaging by minimizing the total system cost (Z). The input to the model is presented in Table 1. Table 2 contains the input to DPSM.

The results obtained by the DPSM are presented in

Table 1. Input to the model ($m = 3$; $n = 3$).

Parameter/ variable	Product 1			Product 2			Product 3		
	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3
p		3			3			3	
A	120·00	1500·00	50·00	100·00	1200·00	50·00	100·00	2000·00	50·00
h	14·20	15·00	17·60	15·00	17·00	19·40	16·60	18·80	20·4
Parameter/ variable	Product 1			Product 2			Product 3		
	Item 1	Item 2	Item 3	Item 1	Item 2	Item 3	Item 1	Item 2	Item 3
d	3000	2500	4000	1000	1800	1500	4000	3500	2000
S	3000·00	60·00	150·00	50·00	1500·00	100·00	30·00	1000·00	50·00
H	60·00	30·00	160·00	40·00	80·00	20·00	100·00	30·00	40·00
C	30	15	80	20	40	10	50	15	12

Table 2. Input to DPSM.

Parameters		
Notation	Description	Values
T, k_{ij}, K_{il}	Initial cycle time and values of integer multiples	0·05,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1
EPS	Initial step size values	0·01,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1
NSTAGE	Total number of stages	19
ITMAX	Maximum number of iterations allowed	2000
NKAT	Maximum number of times the initial step size reduced	30
EPSY	Error in objective function for convergence	0·00001
ALPA	Factor for extending initial step sizes	8·00
BTA	Factor for reducing initial step sizes	0·50

Table 3. Results obtained by DPSM.

Situation	Optimal cycle time (T) (year)	Optimal production batch sizes ($Q_{ij}, j=1, \ldots, n$; $i=1, \ldots, m$)	Optimal packaging sizes ($PQ_{il}, l=1, \ldots, p_i$; $i=1, \ldots, m$)	Setup cost ($\times 10^2$) ($)	Inventory cost of manufacturing ($\times 10^2$) ($)	Packaging setup cost ($\times 10^2$) ($)	Packaging inventory cost ($\times 10^2$) ($)	Total system cost ($\times 10^2$) ($)
Initial solutions	0·050	16 000, 16 000, 16 000	150,125,200					
		750,750,750	50,90,75					
		1200,1200,1200	200,175,100	1034·00	4271·55	1190·00	423·50	6919·05
(k_{ij}, $j=1,\ldots,n$; $i=1,\ldots,m$) and (K_{il}, $l=1, \ldots, p_i$; $i=1, \ldots, m$		1,1,1	1,1,1					
		1,1,1	1,1,1					
		1,1,1	1,1,1					
Optimal solutions	0·010	6406,6406,3203	330,50,80					
		1350,1350,150	20,198,135					
		2160,2160,240	40,385,40	1336·44	1855.20	700·41	353·30	4245·.35
(k_{ij}, $j=1,\ldots,n$; $i=1,\ldots,m$) and (K_{il}, $l=1,\ldots,p_i$; $i=1,\ldots,m$)		1,2,1	11,2,2					
		1,9,1	2,11,9					
		1,9,1	1,11,2					

Percentage decrease in total system cost = (6919·05 − 4245·35) × 100/6919·05 = 38·64 %.

Table 4. Results obtained by sensitivity analysis.

Situation	Optimal cycle time (T) (year)	Optimal production batch sizes ($Q_{ij}, j=1, \ldots, n$; $i=1, \ldots, m$)	Optimal packaging sizes ($PQ_{il}, l=1, \ldots, p_i$; $i=1, \ldots, m$)	Setup cost ($\times 10^2$) ($)	Inventory cost of manufacturing ($\times 10^2$) ($)	Packaging setup cost ($\times 10^2$) ($)	Packaging inventory cost ($\times 10^2$) ($)	Total system cost ($\times 10^2$) ($)
Basic solutions	0·010	6406,6406,3203	330,50,80					
		1350,1350,150	20,198,135					
		2160,2160,240	40,385,40	1336·44	1855·20	700·41	353·30	4245·35
($k_{ij}, j=1,\ldots,n$; $i=1,\ldots,m$) and ($k_{il}, l=1,\ldots,p_i$; $i=1,\ldots,m$)		1,2,1	11,2,2					
		1,9,1	2,11,9					
		1,9,1	1,11,2					
$h=0·5h$	0·050	16 000,16 000,16 000	330,125,200					
		1500,1500,750	50,180,150					
		2400,2400,1200	200,350,100	694·00	2301·98	630·00	538·25	4164·23
($k_{ij}, j=1,\ldots,n$; $i=1,\ldots,m$) and ($k_{il}, l=1,\ldots,p_i$; $i=1,\ldots,m$)		1,1,1	2,1,1					
		1,2,1	1,2,2					
		1,2,1	1,2,1					
$h=2h$	0·010	3203,3203,3203	330,50,80					
		600,300,150	90,198,165					
		960,480,240	80,385,40	3416·58	1968·23	1327·23	390·34	7103·11
($k_{ij}, j=1,\ldots,n$; $i=1,\ldots,m$) and ($k_{il}, l=1,\ldots,p_i$; $i=1,\ldots,m$)		1,1,1	11,2,2					
		2,2,1	9,11,11					
		2,2,1	2,11,2					

Table 4. *Continued.*

Situation	Optimal cycle time (T) (year)	Optimal production batch sizes ($Q_{ij}, j=1, \ldots, n; i=1, \ldots, m$)	Optimal packaging sizes ($PQ_{il}, l=1, \ldots, p_i; i=1, \ldots, m$)	Setup cost ($\times 10^2$) (\$)	Inventory cost of manufacturing ($\times 10^2$) (\$)	Packaging setup cost ($\times 10^2$) (\$)	Packaging inventory cost ($\times 10^2$) (\$)	Total system cost ($\times 10^2$) (\$)
$H=0\cdot5H$	0·010	6406,6406,3203 1350,1350,150 2160,2160,240	330,50,80 90,198,165 80,385,40	1336·44	1855·20	663·98	198·17	4050·79
($k_{ij}, j=1, \ldots, n; i=1, \ldots, m$) and ($k_{il}, l=1, \ldots, p_i; i=1, \ldots, m$)		1,2,1 1,9,1 1,9,1	11,2,2 9,11,11 2,11,2					
$H=2H$	0·010	6406,6406,3203 1350,1350,150 2260,2160,240	330,50,80 20,180,135 40,350,40	1336·44	1855·20	723·12	681·68	4596·44
($k_{ij}, j=1, \ldots, n; i=1, \ldots, m$) and ($k_{il}, l=1, \ldots, p_i; i=1, \ldots, m$)		1,2,1 1,9,1 1,9,1	11,2,2 2,10,9 1,10,2					
$A=0\cdot5A$	0·010	3203,3203,3203 600,300,150 960,480,240	330,50,80 20,198,135 40,385,40	1708·29	984·11	700·41	353·30	3746·12
($k_{ij}, j=1, \ldots, n; i=1, \ldots, m$) and ($k_{il}, l=1, \ldots, p_i; i=1, \ldots, m$)		1,1,1 2,2,1 2,2,1	11,2,2 2,11,9 1,11,2					
$A=2A$	0·040	12 800,12 800,12 800 1200,1200,600 1920,1920,960	240,100,160 40,144,120 160,280,80	1735·00	3683·16	787·50	430·60	6636·26
($k_{ij}, j=1, \ldots, n; i=1, \ldots, m$) and ($k_{il}, l=1, \ldots, p_i; i=1, \ldots, m$)		1,1,1 1,2,1 1,2,1	2,1,1 1,2,2 1,2,1					
$S=0\cdot5S$	0·010	6406,6406,3203 1350,1350,150 2160,2160,240	330,50,80 20,180,135 40,350,40	1336·44	1855·20	361·56	340·84	3894·04
($k_{ij}, j=1, \ldots, n; i=1, \ldots, m$) and ($k_{il}, l=1, \ldots, p_i; i=1, \ldots, m$)		1,2,1 1,9,1 1,9,1	11,2,2 2,10,9 1,10,2					
$S=2S$	0·010	6406,6406,3203 1350,1350,150 2160,2160,240	330,50,80 90,198,165 80,385,40	1336·44	1855·20	1327·97	390·34	4909·95
($k_{ij}, j=1, \ldots, n; i=1, \ldots, m$) and ($k_{il}, l=1, \ldots, p_i; i=1, \ldots, m$)		1,2,1 1,9,1 1,9,1	11,2,2 9,11,11 2,11,2					

Table 3. The comparison of results for the cases without and with optimization of production batch and packaging sizes as shown in Table 3 indicates a potential saving of 38·64 % in total system cost by optimizing the production batch and packaging sizes. Different starting values for the decision variables (T, k_{ij} and K_{il}) may lead to different optimal solutions. This procedure can be used to obtain global optimal solutions. After conducting a number of experiments with different starting values for the decision variables, it has been observed that there is only 2–3 % difference in the total system cost. But the probability of obtaining optimal solutions seems to be very high using this search method.

To study the behaviour of the model, a sensitivity analysis has been carried out. The details of the results obtained for different levels of inventory holding rates

(h, H), setup cost per setup (A), and packaging setup cost per setup (S) are reported in Table 4. One can easily observe that the changes in setup cost (A) and inventory holding rate (h) lead to significant changes in optimal packaging sizes and their respective costs for the given set of data. This explains the influence of production batch sizes on the optimal packaging policies in a multi-stage production system. The results obtained by the sensitivity analysis may generate further interest in integrating the production and packaging problems and the advantages of integrating the production-packaging decisions. Moreover, the model reported here can easily be extended to the case of finite production rate (see Goyal *et al.* (1990)).

4. Concluding remarks

In this paper a mathematical model is presented to integrate the problems of economic production and packaging quantities in a multi-stage production system. The model simultaneously determines the optimal economic production and packaging quantities. The model assumes instantaneous production and no capacity bottleneck. A direct pattern search method has been employed for determining the economic production and packaging quantities by minimizing the total system cost.

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