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Internal rate of return estimation methods vs. accountant's rate of return revisited, a simulation evaluation

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INTERNAL RATE OF RETURN ESTIMATION METHODS VS. ACCOUNTANT'S RATE OF RETURN REVISITED

A SIMULATION EVALUATION

Abstract

Four methods for estimating the firm's long-term profitability as the internal rate of return (IRR) of the firm's capital investments are revisited and evaluated using simulated financial statements. The methods of Kay, Ijiri-Salamon, Ruuhela and the averaged accountant's rate of return (ARR) are analysed. In the literature's long-standing dispute about the validity of ARR as a proxy for the IRR the simulation results strongly support the school of thought siding with the validity. The conclusion of the research is to recommend the average ARR method in financial analysis practice.

Key words: Long-term profitability, IRR, ARR, simulation.

JEL: M40, G31, C88, C15

1. INTRODUCTION

1.1. Background

The firm's ability to find and implement successful capital investment opportunities decides its long-run profitability and financial position. There is no doubt that the questions of profitability measurement and the valuation of the firm's financial assets are the most important questions in financial accounting research. The question of a theoretically sound and pragmatic profitability measurement is of crucial importance not only to the firm but also to an economy's overall welfare. The allocation of resources in an economy is directly affected by the validity and

reliability of the decision makers' measures of the firms' performance (profitability) and financial position. For example, in loan and credit decisions the creditors are not only interested in the applicant's short-term situation but in the firm's long-term ability to generate income.

The firm's profitability is crucially reflected in the financial statements of the firm. The stakeholders of the firm need the profitability information for their decision making both for the short and for the long run. In the economics literature the internal rate of return (IRR) is the widely used theoretical long-run profitability concept. A recent survey by Pike (1996) in the area capital budgeting confirms that IRR is a well-established measure also among practitioners. Furthermore, the investment theory of finance recognises IRR as a profitability measure, albeit under restrictive assumptions.

Strictly speaking the theory of finance states that, for example, under capital rationing only the net present value method is uniquely consistent with maximising the value of the stockholders' wealth. However, under ordinary practical conditions of investment opportunities in the same size categories and conventional cash-flow patterns the internal rate of return method can in most cases be expected to give conforming evaluation for the capital investment evaluation. In this paper we accept IRR as the valid long-run profitability measure for the firm.

The accountant traditionally measures profitability as the ratio between the firm's annual income and the book value of its assets. This ratio is often called the accountant's rate of return (ARR) in literature. Other common terms for it are the return on the capital invested (ROI) and the book yield. This measure looks at profitability after the fact. The economist has a different definition of income. It is based on the changes in the market value of the firm defined as its discounted future cash flows. The economist's definition is based on expectations about the future. The internal rate of return (IRR) is consistent with the economist's concept of income. The internal rate of return also is prominent in the capital investment theory.

One traditional way of looking at the firm is to regard it as a series of capital investments. The IRR of the capital investments making up the firm is the well-accepted, theoretically valid measure of the firm's profitability. The problem with this theoretical notion is, however, that the IRR of the firms is not readily measurable in actual business and financial analysis practice, while the annual values of the ARR are calculated routinely for business firms. See the Introduction in Fritsche and Dugan (1997) for the same view. There is a considerable body of literature that discusses the possibility of analytically deriving or empirically estimating the firm's IRR. Since the mid 1960's there is a long-standing controversy, both conceptual and technical, whether it is possible successfully to estimate the firm's IRR. The discussion is too extensive to review in the presentation at hand. For the references see Butler, Holland and Tippet (1994), Stark (1994) and the review article by Salmi and Martikainen (1994).

The approaches in literature to the IRR estimation can be classified into several, partly overlapping categories. The first approach is trying to establish a link from ARR to IRR. This approach is exemplified by Kay (1976) and later by Peasnell (1982a, 1982b). A second approach is to derive the IRR by utilising an auxiliary estimate such as CRR (the cash recovery rate). This approach has been suggested by Ijiri (1979 and 1980), extended and tested by Salamon (1982) and Gordon and Hamer (1988). A third approach seeks to establish the IRR directly from the published financial statements. This category is represented by Ruuhela (1972) and its mathematically streamlined rederivation in Salmi (1982). Furthermore, Kay (1976: 455) presented how his IRR estimate could be improved if the ratio of the accountant's valuation of the firms assets and the economist's valuation of the firms assets were available. Steele (1986) suggested the use of market values from the stock market to represent the economist's valuation of the firm's assets needed in Kay's correction. Lawson (1980) presented an approach based on cash flows and market values.

Which of the various methods put forward in literature should one select? For the business practitioner, as well as for an academic researcher, facing the number of the various long-run profitability estimation methods, and the theoretical controversy of their correctness, the question becomes the following. What methods are reliable and applicable for evaluating the long-run profitability of a business firm? In other words which method or methods work both in practice and in theory? In particular, might it be, after all, that the practice of calculating a straight-forward average of the annual ARRs would be at par with the more theoretical IRR estimation methods?

1.2. Research Approach

The discussion in literature on the possibility of a sound estimation of the firm's IRR has been inconclusive. The controversy has concerned both the generality of the theoretical derivations and the empirical applicability. Given this background, it is our view that the various methods are best evaluated in their empirical context. But even the empirical investigation has not been unproblematic. The following difficulty has arisen. The empirical estimates of the IRR given by the various methods have been compared only relative to each other in the earlier literature. Thus, the earlier empirical approach has not resolved the absolute reliability of the methods compared. The true IRRs of the firms under observation are needed as benchmarks for an objective reliability evaluation. Unfortunately, the true IRRs cannot be known when actual financial statement data are used. This dilemma can be solved by using a simulation approach. A simulation approach with a preset IRR facilitates an objective evaluation of the ability of the various methods to estimate the firm's true IRR. For a rationalisation of the use of a simulation approach in this context see Fritsche and Dugan (1997:782).

The IRR estimation methods of Kay, Ijiri-Salamon and Ruuhela all are mathematically non-trivial. They are not straight-forward to apply in practice on actual financial data. The practitioner's obvious alternative would be to use the averaged accountant's rate of return as a surrogate of the IRR estimate. However, in earlier literature there are reservations on using the average ARR as the estimate. The reservations can be traced as far back as to Vatter (1966). Later e.g. Fisher and McGowan (1983: 82) stated that "accounting rates of return provide almost no information about economic rates of return". On the other hand, as pointed out by Pike (1996: 83-84) in connection with capital budgeting, the technically simple methods such as the payback period and the average ARR has been condoned by several authors starting from Weingartner (1969).

In the literature on IRR estimation some general assumptions have become conventions. We use these same conventions. An important, established convention in the long-run profitability research is to consider the firm as a series of repetitive capital investments. Stating this research convention in Salamon's (1982: 294) words "... the firm is a collection of projects that have the same useful life, same cash-flow pattern, and same IRR". See, however, the critique of this standard assumption by Kelly and Tippet (1991). The assumption of the constant cash-flow pattern has usually been presented as a necessary, technical simplification of the business reality. However, this restriction is not an unrealistic, technical assumption. It can be posed that the assumption conforms with observing often long periods of stable business culture in individual firms. The business culture of the firm is above all created by its CEO-level management and their ability to generate and utilise capital investment opportunities.

Another strong convention is the firm's access to the financial markets freely to obtain the funding for the capital investments. In other words the implied capital markets in this area of research conventionally are perfect and complete. There is no capital rationing. Therefore, the question of financing of the simulated capital investments need not be considered in this paper.

1.3 Specific Research Questions

In the current paper we are interested, in general terms, in evaluating the accuracy of the selected long-term profitability estimation methods under different economic circumstances, under different capital investment payback profiles and under different accounting decisions on depreciation. More specifically, the following research questions will be considered.

In the earlier research a constant growth approach to the capital investments has been fairly common. This restriction has meant the absence of business cycles and noise. A priori one would expect that the cycles can have a drastic effect on the ability of the methods to estimate the correct IRR. Furthermore, the IRR estimation methods are largely based on the idea of regular development uninterrupted by structural changes or other major one-time events causing exceptional capital investment peaks. We relax the steady-state restrictions. Therefore, our first research question is:

1. Are the methods sensitive to business cycles in the capital investment activities?
   Are the methods sensitive to ordinary irregularities in the capital investments?
   Are the methods sensitive to major capital investment shocks?

Second, an outside stakeholder has to base the profitability estimates on the financial data provided by the firm. In the financial statement data the capital investments and their cash flows are totally mixed. It is not possible to know the contribution pattern of the capital investments based on the external data. The question of the effect of the different contribution patterns arises as in Salamon (1982) and Gordon and Hamer (1988). Hence, our second research question is:

2. Are the methods sensitive to the underlying, alternative cash contribution patterns and life-span of the firm's capital investments?

Third, it has been put forward in the earlier literature that there are some particular instances where the profitability estimates given by the accountant's rate of return theoretically become close or equivalent to the underlying, true profitability of the capital investments making up the firm. These include the case where growth equals profitability as presented by Solomon (1966: 115) and the case where the theoretical annuity method of depreciation is postulated as presented in e.g. Salmi and Luoma (1981: 28) and Peasnell (1982a: 364). The annuity depreciation is the economist's depreciation in defining the concept of economic income discussed e.g. in Bromwich (1992: 31-51). Hence, our third research question is:

3. Are the methods sensitive to disparities between the firms growth and profitability?

Fourth, in accounting practice the choice between the depreciation methods such as the prevalent straight-line and the declining-balance methods affects the reported annual income figure. Our fourth question is:

4. Are the methods sensitive to the depreciation choice that the firm has used in producing its financial statements?

An economic time series is made up by several constituents. These are the growth trend, the business cycle, the seasonal variation and the noise. Furthermore, there can be regular or irregular shocks. The growth trend and the business cycle are relevant in this paper. Seasonal variations are intra-year. Thus they do not arise in our research questions. It is true that the economic activities of the firm are continuous in nature. However, the financial data used for the profitability estimation in the methods under observation use discontinuous observations from the annual statements.

This paper contributes to the existing knowledge in the following, related ways. First, it lends new evidence in the long-standing theoretical controversy of the ability of the ARR to serve as a useful proxy of the IRR. Second, it improves on the simulation approach to evaluate various IRR estimation methods by closely emulating the flow-patterns of real-life business firms. Third, it comes up with a theoretical and empirical comparison of four major IRR estimation methods presented in earlier literature. Finally, it arrives at a well-founded practical recommendation (usage of averaged ARR) for the evaluation of the long-run profitability of business firms.

The rest of the paper is organised as follows. The simulation approach to evaluating the IRR estimation methods is presented in section two. Section three presents the IRR estimation methods under investigation. Section four first presents the simulation experiments design with data generation. The results of the evaluation are presented and analysed. The section ends with the comparison of the results. The summary concludes.

2. SIMULATION EVALUATION APPROACH

This section presents our simulation model. The central time-series in the simulation is the capital investments. They are generated by the following process with a trend, cyclical, random and a potential shock component

(1) $$g_t = g_0(1+k)^t\{1+A\sin[(2\pi t/C) + \phi]\}(1 + \sigma z)(1 + \delta_{t\tau}S)$$

where

g_t = capital investments in year t

g_0 = initial level of capital investments

k = growth rate (the trend component in the capital investments)

A = amplitude of the business cycle

C = length of the cycle

ϕ = a technical phase adjustment for the cycle

σ = standard deviation of the random fluctuation in the capital investments

z = random variable following the (0,1)-normal distribution

S = capital investment shock coefficient

τ = the year of the capital investment shock

δ = Kronecker's delta, $\delta_{t\tau} = 1$ when $t = \tau$, and 0 otherwise.

Using this capital investment generating process produces financial time series which emulate the time series profiles observed for actual business firms. For an illustration see Figure 2 in the next section.

The capital investments g_t induce (later) cash inflows which can be defined in terms of a contribution distribution b_i, i = t+1,...,t+N, where N is the (common) life-span of a capital investment project. This formulation is in line with the cash flow profiles in Ruuhela (1972), Ijiri (1979), Salamon (1982) and Gordon and Hamer (1988). The total cash inflow f_t in year t is cumulated from the annual contributions f_{ti} from the capital investments g_{t-i} made in the earlier years:

(2) $$f_t = \sum_{i=1}^{\min(N,t)} f_{ti} = \sum_{i=1}^{\min(N,t)} b_i\, g_{t-i}\,.$$

As is the custom in growth models, constant returns of scale on the capital investments are assumed. In other words, when the firm grows, there are no economics of scale. See e.g. the standard reference Levhari and Shrinivasan (1969: 153). The question of financing and its costs do not arise in our simulations as long as it can be safely assumed that the firm remains sufficiently profitable to be able to obtain new capital as the need arises. Hence chronically declining activities (divestments) or infeasible combinations of growth and profitability will not be considered in our research, since in actual business practice this would in the long-run cause restrictions or even a cessation of the availability of capital to the firm.

It readily follows from (2) and from the definition of the internal rate of return (r) that

$$\sum_{i=1}^{N} b_i \,(1+r)^{-i} = 1. \tag{3}$$

In the simulations a conventional cash-flow contribution pattern will be used. It is a well-known fact that under non-conventional cash flows (more than one sign alteration) there can be multiple or no real roots for the internal rate of return r in Equation (3). See e.g. Teichroew, Robichek and Montalbano (1965). This problem does not arise in the simulations.

The true internal rate of return r determined by Formula (3) is a function of the contribution distribution b_i. The true form of the contribution distribution is not generally known for real-life business firms. In order to assess the effect the different, potential contribution patterns of the firm's capital investments we will perform our simulations with three different contribution patterns from the capital investments. Figure 1 illustrates a neutral, a typical growth-maturity-decline life-cycle pattern and a steadily declining distribution.

Figure 1 about here

We choose the uniform, the negative binomial and a linearly declining contribution distribution (Anton distribution) to represent the three cases. The uniform contribution distribution that leads to the profitability of r for the capital investments is given by the annuity factor

$$b_i = \left[\sum_{t=1}^{N} (1+r)^{-t}\right]^{-1}, \quad i = 1,\ldots,N. \tag{4}$$

Several negative binomial contribution distributions with different profiles lead to the profitability r. For the definition and the properties of the negative binomial distribution see Fisz (1967: 167). We choose the following alternative with the shape parameter q fixed at 0.15

$$b_i = \left\{\sum_{t=1}^{N} (t+1)\left(\frac{1-q}{1+r}\right)^{t}\right\}^{-1} (i+1)(1-q)^{i}, \quad i = 1,\ldots,N. \tag{5}$$

The Anton distribution presented in Anton (1956) is defined as

$$b_i = \frac{1 + (N-i+1)\,r}{N}, \quad i = 1,\ldots,N. \tag{6}$$

To complete the simulation model we need the formulas for alternative depreciation methods in order to have the annual profit and book value figures. First consider the accounting relationships between these concepts.

The profit p_t is defined by the cash inflow f_t less depreciation d_t as

$$p_t = f_t - d_t. \tag{7}$$

The book value v_t of the firm at the end of year t is defined by book value at the beginning of the year plus the capital investments g_t less the depreciation d_t. For simplicity cash, inventories and other assets are not modeled separately. We have

$$v_t = v_{t-1} + g_t - d_t. \tag{8}$$

The firm's choice of the depreciation method is central to profit measurement and asset valuation both in accounting theory and practice. We build into our simulation model the possibility of three alternative depreciation methods to be employed by the simulated firm in its financial statements. The alternatives are the straight-line depreciation method, the double declining-balance method and the theoretical annuity depreciation method.

Straight-line depreciation is calculated as

$$d_t^S = \sum_{i=1}^{\min(N,t)} (1/N)\, g_{t-i} \; . \tag{9}$$

Double-declining-balance depreciation is a decreasing depreciation used in the U.S. practice. See Davidson and Weil (1977). Double-declining-balance depreciation method formula is

$$d_t^D = \sum_{i=1}^{\min(N,t)} (2/N)[1 - (2/N)]^{i-1} g_{t-i} \; . \tag{10}$$

All the remaining book value of the relevant capital investment is depreciated in the last year of the life-span.

The well-accepted definition for the theoretical annuity depreciation is that the profit (before interest and taxes) p_t is assessed as the interest on the initial capital stock v_{t-1} in year t. It readily follows that

$$d_t^A = f_t - r\, v_{t-1} \, . \tag{11}$$

3. EXPOSITION OF THE PROFITABILITY ESTIMATION METHODS

The current section presents four IRR estimation methods from earlier literature to be analysed and evaluated with our simulation approach. Before any IRR estimation method can be applied on the simulated (or actual financial) statements, the IRR estimation method must be made operational for the financial data available.

Kay (1976) presented a model for deriving IRR from ARR. For Kay's model we have from Kay (1976: 451), Salmi and Luoma (1981: 25) and Peasnell (1982a: 371)

$$\hat{r} = \frac{\sum_{t=2}^{n} p_t (1+\hat{r})^{-t}}{\sum_{t=2}^{n} v_{t-1} (1+\hat{r})^{-t}} \tag{12}$$

where n stands for the number of observations for the subsequent years which have been renumbered from 1 to n. The internal rate of return estimate $\hat{r}$ is solved from the above formula by numerical iteration. For the conditions of convergence see Steele (1986: 2-4). Our solution procedures for all the methods are Turbo Pascal computer programs for MS-DOS PC (available on request on the Internet from the authors).

Ijiri (1979) presented what Salamon (1982) interpreted and expanded as an IRR estimation method based on the concept of the cash recovery rate, CRR. Ijiri (1979: 259) derived the following relationship between CRR and IRR

$$CRR = \frac{r}{1 - (1+r)^{-N}} \, . \tag{13}$$

Ijiri (1980: 55) gives the calculation of an annual CRR_t from published financial statements as Cash Recoveries / Gross Assets, which can be stated as

$$CRR_t = \frac{f_t}{V_{t-1}} , \tag{14}$$

where V_t is calculated from

$$V_t = v_t + \sum_{i=0}^{\frac{N}{2}-1} d_{t-i} \, . \tag{15}$$

The calculated CRR_t values are averaged and the average is substituted as CRR into Formula (13) to obtain the IRR estimate.

The third method to be included in our analysis is the IRR estimation component of Ruuhela's "Growth, Profitability and Financing" model. As we have seen in the above, Kay's method is based on a relationship between the ARR and the IRR and Ijiri-Salamon method on the relationship between the CRR and IRR. Ruuhela's method can be considered to fall into a category of direct estimation of the IRR from the financial statements without the intermediate ARR or CRR concepts.

The method was first presented in Ruuhela (1972) and mathematically streamlined by Salmi (1982). The method was restructured in Ruuhela et al. (1982). Ruuhela's IRR estimate is given by

$$\hat{r} = k \frac{N a_{N,k} - F}{(N a_{N,k} - 1) F}, \tag{16}$$

where k is the growth-rate trend of the capital expenditures, $a_{N,k}$ is the well-known annuity factor and F is defined as the capital investment ratio g_t/f_t assumed a constant by Ruuhela.

In applying Ruuhela's method an estimate of the common growth rate of the firm's time series is needed. The OLS estimate of the growth-trend of the firm's cash inflows f_t is used as the estimate. Given the OLS estimate $\hat{k}$ of the growth trend the capital investment ratio is estimated from

$$\hat{F} = \frac{\sum_{t=1}^{n} g_t (1+\hat{k})^{-t}}{\sum_{t=1}^{n} f_t (1+\hat{k})^{-t}}. \tag{17}$$

The fourth and last method included into our analysis is based on straight-forward accounting practice. Much of the discussion, ever since Vatter (1966), in the ARR vs. IRR debate has centred around the question whether or not the ARR is a good approximation of the IRR. Instead of re-entering the deductive debate we seek a resolution to this question by including the averaged ARR in our simulation and comparison. The inclusion of the average ARR method is prompted by the fact that accounting practitioners routinely use and are comfortable with the concept of annual profits and return on investment. Employing the averaged ARR as the IRR estimate can be considered a direct extension of this business practice.

The average ARR is calculated from

$$\overline{ARR} = \frac{1}{n-1} \sum_{t=2}^{n} \frac{p_t}{v_{t-1}}. \tag{18}$$

Our advance hypothesis is that the average ARR method will not be empirically inferior to the other IRR estimation methods. Our hypothesis is based on the concept of economic Darwinism. Quoting Watts and Zimmerman (1986: 195) "Competition among firms implies that operating procedures ... that are used systematically by surviving organisations are efficient."

4. EVALUATION OF THE ESTIMATION METHODS

4.1. Simulation Design and Data Illustration

To tackle the research questions posed financial data is generated for the parameter combinations for Formula (1) listed in Table 1.

Table 1 about here

Solomon (1966: 115) posed that when the growth rate and the true internal rate of return are equal, the accountant's rate of return also becomes the same. Consequently, it is interesting to observe the behaviour of methods under different growth-profitability combinations. To produce different combinations either parameter can be fixed without a loss of generality. We fix a growth rate of $k = 8\%$ in the simulation. The data is generated to produce true profitability figures of $r = 4\%$, 8%, 12% and 16% on both sides of the growth rate.

Our first research question concerns the effect of the business cycles on the robustness of the four IRR estimation methods. For our simulation it is realistic to assume that the long-run average length of a business cycle is six years ($C = 6$ in Formula (1)). In the simulation the length of the observation period is set at 13 years covering two full business cycles. Three alternative amplitudes of the cycles are used in our simulations. For no cycles we set $A = 0$, for medium cycles we set $A = 0.50$ and for strong cycles $A = 1.00$.

The noise term $1+\sigma z$ represents random fluctuations in the level of the capital investments. We choose a moderate noise level of $\sigma = 20\%$ to arrive at a realistic capital investment time series. Only one realisation (for each parameter combination) of the randomised time series is picked in our simulation. Stated in terms of statistics and operations research our approach is not a Monte Carlo simulation that would repeat the same parameter combinations with the random term varied.

The robustness of a profitability estimation method can be tested by including capital investment shocks in the model. In business terms such a shock is usually related to a major deviation from the level of capital investment pattern. Experiments are made with different magnitudes and timing of a one-time shock. The shock alternatives simulated are a five-fold shock and a seventeen-fold shock relative to the normal capital investment level in the third or in the ninth year.

The life-span of the capital investments affects the numerical values of the chosen contribution distribution and the annual depreciation figures. The life-span of the capital investments is known in the simulation (we have chosen a typical 20 years), but it cannot be accurately

known in applications on real-life business firms. The effect of the life-span will be considered in the analysis section by comparing the IRR estimates with a 20-year life-span to the results with a 16-year and a 24-year life-span.

The firm's capital investments can produce revenues in accordance with various contribution distributions. We use the three alternatives discussed and presented in Table 1. Three different depreciation alternatives chosen by the simulated firms are considered.

Figure 2 gives an example of one realisation of the time series from the simulated financial statements. The observation period is 13 years from the simulated year 22 to 34 (the years 1-21 are skipped to first reach the going-concern phase in the simulation). The realisation presented is for the case of the negative binomial contribution distribution with a true profitability of 12%, a growth trend of 8%, medium amplitude ($A = 0.50$) of business cycles, with noise ($s = 0.20$), no shock ($S = 0$), a life-span of 20 years of the capital investments, and a double-declining-balance depreciation of 10%.

Figure 2 about here

4.2. Evaluation of Kay's Method

We begin the evaluation by assessing the effect of business cycles on the performance of the IRR estimation methods (our first research question). The IRR estimates by Kay's method are presented in Table 2 for the three different levels of amplitudes in the business cycle. To see the pure effect of the cyclical component we first omit the random noise term. The results are presented for the four different growth-profitability combinations and the three different depreciation methods "Str" straight-line depreciation, "Decl" double-declining-balance depreciation and "Ann" annuity depreciation.

Table 2 about here

The effect of the business cycles is marginal for Kay's IRR estimation method. In the worst case with the strong cycles (A = 1.00) the difference between the IRR estimates 18.9% and 18.6% (16% true profitability and double-declining-balance depreciation) is only 0.3%. The presented result is for the negative binomial contribution distribution. The results for the other two contribution distributions, the uniform distribution and the Anton distribution, indicate a similar insensitivity. (The tables for these cases are not displayed for brevity. This goes for space reasons through the rest of this paper. Only select example tables are presented. All the results can, however, be replicated with the computer programs available from the authors.) We can safely conclude that Kay's IRR estimation method is not affected by regular business cycles. This being the case the rest of the analysis of Kay's method can be conducted without a loss of generality using the medium cycle strength (A = 0.50). A further analysis shows that while the noise term in the capital investment level seems to have more effect on the Kay's IRR estimation than the regular cycles the effect of noise is rather mild.

As to capital investment shocks, Kay's IRR estimation method is reasonably robust to the capital investment shocks even if there is some disruption in the estimates. The effect of the shock tends to decrease the IRR estimates, the more the bigger and later the shock appears. This behaviour is easy to explain. The one-time investment shock becomes dominating, and its effects are much outside the period under observation. (The numerical results for this part are not included for brevity. All the non-displayed material is available on request on the Internet from the authors).

Table 3 about here

We can now analyse the total error in Kay's IRR estimates. Table 3 presents the results for Kay's IRR estimation method under medium business cycles, with the noise component included and the three contribution distributions.

The general impression conveyed by Table 3 is that the level of Kay's IRR estimates is fairly well in line with the true profitability. In particular, when the firm's growth and profitability are near each other, Kay's method performs excellently. There are, however, parameter combinations where Kay's method performs poorly. The biggest absolute discrepancy in Table 3 in an estimate ($\hat{r}$ = 19.5% vs. r = 16%) takes place when the true internal rate of return deviates most from growth, the capital investments contribute according to the uniform distribution and the firm uses the double-declining-balance method. Kay's IRR estimate is off by 3.5% (by a fifth in relative terms). Likewise, at the true profitability of 4% the IRR estimate, with the uniform contribution distribution and the double-declining-balance depreciation, is off by a third (2.6% vs. 4.0%). These are marked deviations.

Our second research question concerns the effect of the alternative contribution patterns of capital investments available to the firm. As noted, the shape of the contribution distribution of the capital investments is not readily known for real-life firms. Therefore it is of interest to test whether the IRR estimation results are sensitive to this factor. It is seen from Table 3 that under capital investment opportunities that contribute in accordance with the negative binomial distribution, or the Anton distribution, the results are more accurate than under the non-declining uniform contribution distribution.

Our third research question concerns the effect of the discrepancy between growth (k) and profitability (r). It is obvious from the results that a discrepancy between growth and profitability levels is the crucial source of error in the Kay's IRR estimates. It is also noted that when $r > k$ Kay's IRR systematically overestimates the true profitability. Thus it appears that Kay's method gives even too optimistic IRR estimates to firms with good profitability. For $r < k$ the direction on the estimation error depends on the contribution distribution, the depreciation combination and the irregularities in the capital investments (the noise). Thus it would seem that it is not possible to make any predictions whether Kay's estimates for firms with low profitabilities are optimistic or pessimistic.

Our fourth research question concerns the effect of the depreciation method choice that the firm makes. The effect of the firm's accounting choice appears highly important to the accuracy of the IRR estimates. The error in the estimates in Table 3 is about half or less when the firm applies the straight-line depreciation method instead of the double-declining-balance method. This observation raises interesting accounting issues about the depreciation method choice.

There are several theoretical assertions about the relationship between the internal rate of return and the accountants rate of return under the specific growth rates, depreciation methods and contribution distributions presented in earlier literature. Next we consider these assertions, under the more general conditions of business cycles and noise, utilising our simulation results.

Solomon (1966: 115) posed that when the growth rate and the true internal rate of return are equal, the accountant's rate of return also becomes the same. The expected equality is supported by our simulation results not only in the case of constant, exponential growth but also in the case with the business cycles added. However, when irregularities are introduced in terms of the noise, the expected theoretical result no more fully holds. The deviation is not marked numerically, but theoretically the assertion breaks. As is natural, the disruptive effect of the one-time capital investments shocks is more marked than that of the noise.

Analytically, the accountant's rate of return and the internal rate of return are equal when the annuity method of depreciation is used (see e.g. Salmi and Luoma, 1981: 28 and Peasnell, 1982a: 364). The simulation results for Kay's method are in agreement with this contention for all the observed combinations of growth vs. profitability and for all contribution distributions even with the irregularities introduced upon the growth-trend and the business cycles in terms of the noise and the capital investments shocks. The theoretical results about the annuity depreciation are very strong. They are in line with discussions and results in literature about accountant's and the economist's concepts of income.

To summarise, the main findings about Kay's IRR estimation method are the following. Under ordinary circumstances Kay's method performs quite well. However, the deviation of Kay's IRR estimates from the true internal rate of return can be considerable if the growth rate of the

firm and its profitability are not near each other. This is the main source of error in Kay's method. Kay's method seems to lead to systematically overoptimistic profitability estimates when the firm's true profitability exceeds the firm's growth considerably. If the true profitability is below the firm's growth the nature of Kay's IRR estimate is ambiguous. The magnitude of the error caused by a growth-profitability gap is jointly dependent on the contribution pattern of the capital investments, the firm's depreciation choice and the noise in the capital investment time series. Kay's method is not affected by regular business-cycle fluctuations in the capital investment time series, but it is mildly affected by noise. Kay's method is reasonably robust to major capital investments shocks. The irrelevance of business cycles and the mild effect of noise on the accuracy of the estimates are important advantages in Kay's method.

Kay's method has a firm theoretical background in the theory of accountant's and economist's profit concepts. This fact is reflected in always getting exactly the expected IRR estimates under the theoretical annuity depreciation and getting fairly accurate IRR estimates under the equality of growth and profitability. Furthermore, if the capital investments contribute in accordance to the Anton distribution, the estimates under the firm applying a straight-line depreciation are accurate.

4.3. Evaluation of Ijiri-Salamon Method

The cash-recovery-rate-based Ijiri-Salamon IRR estimation method differs from Kay's method in two respects in the data that it needs. An estimate of the life-span of the firm's capital investments and an estimate of the gross book value of the firm's assets are needed. This fact introduces two additional, potential sources of error into the method: a misestimation of the life-span of the capital investments and a misestimation of the gross book value. However, in a simulation approach for evaluating the Ijiri-Salamon method one can utilise the fact that the life-span (N = 20) and the true accumulated depreciation, and hence the gross book value of the firm's assets are known precisely.

Table 4 presents an example of IRR estimates with Ijiri-Salamon method. The table lists the results for three alternative estimates of the capital investments' life-span E(N). The IRR estimates are presented assuming a correctly estimated life-span of 20 years, an underestimate 16 years, and an overestimate 24 years.

Table 4 about here

The IRR estimation results are presented assuming that the firm either employs the straight-line depreciation ("Str") or the double-declining-balance depreciation ("Decl"). The accumulated depreciation must be estimated from the financial statements. In accounting practice, the accumulated depreciation figure usually is an approximation based on a time series of recent financial statements. We use the estimate given by Formula (15). The accumulated depreciation can also be calculated accurately in the simulation approach. Ijiri-Salamon's IRR estimates with accurate accumulated depreciation is presented in the "Accu" column of the tables. This particular information facilitates a decomposition analysis of the error sources in the IRR estimates.

As for Kay's method our simulations for Ijiri-Salamon method indicate that the method is not sensitive to cycles. Likewise, the effect of noise is rather mild. Furthermore, like Kay's method the Ijiri-Salamon method is reasonably robust to capital investment shocks. Overall, Ijiri-Salamon method fares on the average in the simulations comparably to Kay's method. The worst cases in the representative Table 4 appear when the profitability is low compared to the growth. However, in the Ijiri-Salamon method there is no clear pattern to the errors. Unlike in Kay's method there are no cases where the error would disappear. Furthermore, there is no clear pattern to the direction and the magnitude of the error.

The realisation of the theoretical assertions concerning the growth-profitability equality conditions and the annuity depreciation could be checked also for Ijiri-Salamon results. However, these assertions do not cover the relationship between the cash recovery rate and the internal rate of return. This state of matters also is clearly reflected in the simulation results as a lack of similar theoretical regularities as were observed in the results for Kay's IRR estimation method. This can be considered a disadvantage.

As noted, the simulation results for Ijiri-Salamon method seem at rough par with Kay's method. However, a decomposition of the sources of the overall error exposes a more critical picture of the potential quality of the IRR estimates by Ijiri-Salamon method. The total error in Ijiri-Salamon's IRR estimates is made up by several components, which individually can be larger in absolute terms than the total error, but the components of the error compensate each other in the presented simulations. Table 5 gives the decomposition of the total error in Table 4 into three components in the case of the double-declining-balance depreciation.

Table 5 about here

4.4. Evaluation of Ruuhela's Method

Also Ruuhela's IRR estimation method differs from Kay's in the financial statement data that it uses. Like Ijiri-Salamon method an estimate of the life-span of the capital investments is needed. Furthermore, Ruuhela's method needs the estimate of the growth rate of the firm. On the other hand, and very importantly, Ruuhela's method does not need the time series of depreciation. Ruuhela's IRR estimation method is independent of the depreciation method that the firm chooses.

Table 6 presents an example of IRR estimates with Ruuhela's method. The table lists the results for the three alternative cyclical intensities and the three alternative estimates of the capital investments' life-span. Ruuhela's method needs an estimate of the firm's growth rate. This growth rate is estimated in Ruuhela's method by OLS regression from the time series of the cash inflows f_t. The OLS-estimated growth rates are given within the parentheses.

Table 6 about here

It is readily seen that unlike in Kay's and Ijiri-Salamon methods Ruuhela's method is highly sensitive to the business cycles. It is obvious both from the presentation of Ruuhela's method and the simulation results that Ruuhela's constant-growth assumption is crucial for his method. The business cycles cause a deviation under even perfect growth estimates and correctly estimated life-spans of the capital investments. The sensitivity of Ruuhela's method to noise alone is mild, but noise aggravates the effect of the cyclical fluctuations. With the introduction of major capital investment shocks the OLS growth estimation procedure is derailed. If there are major capital investment shocks, Ruuhela's method should not be applied on the time period including such a structure-changing shock. This is in line with the reservations in the original derivation of the method.

In the special case when there are no cycles (A = 0.00), when the life-span estimate is equal to the true life-span (20 years) of the capital investments and when the capital investments contribute according to the Anton distribution Ruuhela's method produces exactly the correct IRR estimates. This is to be expected from the method's basic premises. Like Kay's method Ruuhela's method has under its own assumptions a direct linkage to the income (and depreciation) theory.

The contribution pattern of the capital investments has an effect, but the effect is a joint effect with the other parameters of the IRR estimation situation. In the case of Ruuhela's method the Anton distribution has a special role since it is assumed in the derivation of the method. Not surprisingly, the best IRR estimates are gained under the Anton contribution distribution.

The comparison of the simulation results for the case when growth equals profitability will produce near-correct but not perfect estimates under no business cycles. A discrepancy between growth and profitability has a considerable effect on the quality of Ruuhela's IRR estimates. The effect of the fluctuations in the capital investments caused by business cycles is overriding in Ruuhela's method. With the increase of the cyclical fluctuations the growth vs. profitability equality loses its effect in Ruuhela's method.

A decomposition of the estimation error for Ruuhela's method is presented in Table 7.

Table 7 about here

4.5. Averaged Accountant's Rate of Return Method

The last method to be analysed in this paper is the method of using the average ARR as the IRR estimate. As for Kay's method the effect of cycles is negligible for the average ARR method. Table 8 gives the IRR estimates using the average ARR method in the case of medium level of business cycles (A = 0.50).

Table 8 about here

The IRR estimates produced by the average ARR method are strikingly similar to the simulation results with Kay's method. This closeness is not an unexpected result, since Kay's method in Formula (12) can be interpreted as an iterative weighted-average ARR method. Under ordinary conditions the average ARR method and Kay's method give virtually equivalent results. In practical long-run profit evaluation terms of the accountant there is no numerical difference between the two methods. Only if major investment shocks are introduced the average ARR method gives estimates that are markedly different from Kay's estimates.

4.6. Comparison of the Results

In comparing the different methods for estimating the internal rate of return of the firm's capital investments the following aspects are relevant: numerical performance, theoretical foundations and practical applicability. In this section we summarise the results in general terms.

First, consider numerical performance. In our simulations the relevant parameters are given such values as should put them in a realistic range with regard to actual business firms. Within the observed range none of the methods unequivocally outperforms the others in the simulation. The deviations in Kay's and the average ARR method are more regular and predictable than the deviations in Ijiri-Salamon and Ruuhela's methods. The number of potential sources of errors in Ijiri-Salamon and Ruuhela's method is greater than the other two methods. Since the errors of these methods partly compensate for each other, the resulting total error, while less predictable, is no worse for Ijiri-Salamon method than for the other methods. Ruuhela's method is the most dependent of the methods on its internal assumptions. Under its restrictive assumptions it works perfectly, but in a general situation it also produces the worst of the overestimation errors if there are strong business cycles and if the firm's profitability exceeds its growth considerably.

No common, general pattern of errors emerged for the observed, different parameter combinations, with one tentative exception. Kay's method, Ruuhela's method and the average ARR method all have a tendency to overestimate rather than underestimate the true profitability when the firm's profitability exceeds its profitability considerably.

Second, consider the methods' theoretical foundations in the light of the simulation results. Kay's method came out as the theoretically most generic, with the average ARR method very close by. Ruuhela's method is theoretically as sound, but its constant-growth and Anton contribution distribution assumptions make it empirically more vulnerable than Kay's and the average ARR method. Ijiri-Salamon method does not conform empirically to any of the expected theoretical propositions. Ijiri-Salamon method can be regarded as an elaborate, good rule of thumb. The other methods have deep roots within income theories of accounting and economics.

Last, consider practical applicability. In this area the average ARR method has the outstanding merit of being directly based on established accounting practice of performance measurement. It would be trivial to use computers to calculate Kay's elaborate weighted-average IRR estimates in business practice. However, the marginal improvement compared to the average ARR method does not compensate the obvious disadvantages of having to "sell" an iterative method to the users of financial information over the suggestion of using an average return on investment for long-term profitability measurement. Ijiri-Salamon and Ruuhela's method are at a considerable disadvantage compared to the average ARR method since they require a fairly involved estimation process. In this light, for the practitioner it is our recommendation to choose for long-term profitability estimation the average ARR method over the more sophisticated IRR estimation methods. Knowing and understanding the analysed, more sophisticated methods is not wasted, however. On the contrary, the practitioner should be aware of and familiar with the foundations of the methods s/he applies in order to make sound decisions.

5. SUMMARY

This research analyses four internal rate of return (IRR) estimation methods from literature for assessing the long-term profitability of a business firm from its published financial statements. The IRR estimation methods considered are Kay's, the Ijiri-Salamon, Ruuhela's and the average ARR methods. A realistic simulation approach is developed to evaluate and compare the methods. A simulation approach with a known internal rate of return makes it possible to study the ability of the various methods to estimate the firm's true IRR. The research contributes by evaluating the performance of selected IRR estimation methods under more general conditions than the earlier literature. This is facilitated by including cyclical fluctuations, noise and the possibility of major capital investment shocks into the simulated financial data. Most importantly the research contributes in literature's long-standing dispute about the validity of accountant's rate of return ARR as a proxy for the IRR.

Four research questions are posed concerning four IRR estimation methods. First, the effect of business cycles and ordinary noise around the growth-trend of the firm's capital investments and capital investment shocks is of interest. It is observed that three of the four methods are insensitive to cyclical fluctuations. The exception is Ruuhela's method which relies heavily on its constant-growth assumption. In the case of Kay's, Ijiri-Salamon and the average ARR method the insensitivity to business cycles is an important result because it confirms the applicability of the methods beyond the common steady-state assumptions. It is observed that ordinary noise in the capital investment time-series does not have a marked effect on the IRR estimates. The simulations mostly indicate an unexpectedly good tolerance of the analysed IRR estimation methods to major capital investment shocks. Ruuhela's method is the exception in this respect since its growth estimation is disrupted by such shocks.

Second, the sensitivity of the methods to the capital investments' payback patterns is of interest. It is observed that all the methods can be sensitive to the contribution distribution. The effect of the shape of the contribution distribution on the IRR estimates is interactively dependent on the depreciation methods applied by the firm and the relationship between growth and profitability. The conclusion is that contribution distribution of the firm's capital investments can have an effect of the quality of the IRR estimates given by the analysed IRR estimation methods. Furthermore, contrary to the other two IRR estimation methods, Ijiri-Salamon and Ruuhela's methods require an estimate of the life-span of the firm's capital investments. The reliability of the IRR estimates by Ijiri-Salamon and Ruuhela's method depends on the quality of the life-span estimate.

Third, it is to expected from theory that a disparity between the firm's growth rate and its long-term profitability affects the quality of the IRR estimates. It is observed that the reliability of the IRR estimates of all the methods is very sensitive to the relationship between the underlying true profitability and the firm's growth rate. In accordance to the simulation results the discrepancy between the true growth and profitability is the dominating source of the error in the IRR estimates in all the methods analysed.

Fourth, the depreciation method applied by the firm in its financial statements can affect the IRR estimation result in concert with the contribution distribution of the capital investments.

Also this effect is strongly related to the growth-profitability discrepancy. In this respect Ruuhela's method has an advantage over the other methods since it is unaffected by the firm's depreciation choice.

To conclude, the simulation comparison of the selected IRR estimation methods shows that none of the analyzed sophisticated methods performs consistently better than the average ARR method. Thus, considering the various facets discussed in this paper, the accounting-practice-based average ARR method can be recommended as the best choice for the long-term profitability estimation. However, none of the methods, including the average ARR, is an unbiased estimator of the firm's IRR. For fast growing firms with low profitability and for slow-growth firms with good profitability the long-term profitability estimates should be interpreted with much caution. On the other hand, the average ARR method can be safely used when a firm has comparable growth and profitability even when there are ordinary fluctuations and noise in the capital investment intensity.

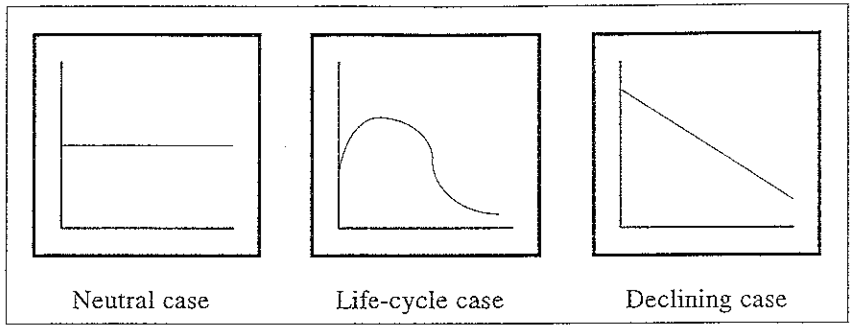

Figure 1. Three contribution pattern alternatives.

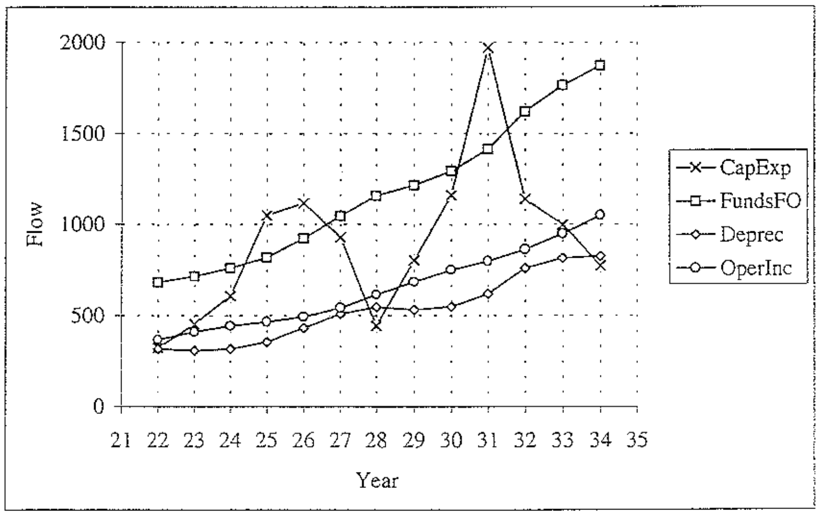

Figure 2. Visualisation of simulated observations; the case of negative binomial contribution distribution, declining balance depreciation, growth 8%, IRR 12%, amplitude 50%, noise 20%.

Table 1. The variation of the parameters in the simulation runs.

Parameter	Symbol	Values
First initial investment	g_0	100.00
Growth rate	k	0.08
True internal rate of return	r	0.04, 0.08, 0.12, 0.16
Amplitude	A	0.00 0.50 1.00
Cycle length	C	6 years
Technical phase adjustment	ϕ	$\pi/6$
Noise	σ	0, 0.20
Shock timing	τ	None, early, late
Shock coefficient	S	0, 5.309, 17.924
Life-span of investments	N	16, 20, 24
Length of observation period	n	13 (years 22-34)
Contribution distribution		Uniform, negative binomial, Anton
Depreciation method		Straight-line, declining, annuity

Table 2. Estimation of IRR with Kay's method, negative binomial contribution distribution, growth rate k = 8%, no noise.

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	4.1	3.5	4.0	4.1	3.4	4.0	4.1	3.4	4.0
	8%	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0
	12%	12.3	13.1	12.0	12.3	13.2	12.0	12.4	13.2	12.0
	16%	17.0	18.6	16.0	17.1	18.8	16.0	17.1	18.9	16.0

Table 3. Estimation of IRR with Kay's method, growth rate k = 8%, amplitude A = 50%, noise = 20%.

Contr. distribution		Uniform			Neg. binomial			Anton		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	3.5	2.6	4.0	4.2	3.3	4.0	4.0	3.1	4.0
	8%	7.8	7.6	8.0	8.0	7.8	8.0	8.0	7.8	8.0
	12%	12.6	13.3	12.0	12.3	12.9	12.0	12.0	12.5	12.0
	16%	17.9	19.5	16.0	17.0	18.4	16.0	16.0	17.3	16.0

Table 4. Estimation of IRR with Ijiri-Salamon method, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 50%, noise = 20%.

Estimated life-span		16 years			20 years			24 years		
Depreciation		Str	Decl	Accu	Str	Decl	Accu	Str	Decl	Accu
True r	4%	3.5	4.2	2.7	4.8	5.3	4.6	5.6	6.0	5.8
	8%	7.6	8.4	6.6	8.4	9.0	8.1	8.9	9.3	9.1
	12%	11.6	12.5	10.5	12.0	12.6	11.6	12.2	12.7	12.5
	16%	15.6	16.7	14.4	15.6	16.4	15.2	15.5	16.1	15.9

Table 5. Decomposition of the estimation error in Ijiri-Salamon method. An example with negative binomial distribution, declining balance depreciation, growth k = 8%, amplitude A = 50%, noise = 20%.

Est. life-span		16 years				20 years				24 years			
Source of error		For-mula	Life-span estim	Cumu depr calc	Total error	For-mula	Life-span estim	Cumu depr calc	Total error	For-mula	Life-span estim	Cumu depr calc	Total error
True r	4%	0.4	-1.9	1.7	0.2	0.5	0	0.8	1.3	0.6	1.3	0.1	2.0
	8%	0.0	-1.5	1.9	0.4	0.1	0	0.9	1.0	0.1	1.1	0.1	1.3
	12%	-0.5	-1.2	2.2	0.5	-0.4	0	1.0	0.6	-0.3	0.9	0.1	0.7
	16%	-1.0	-0.9	2.6	0.7	-0.8	0	1.2	0.4	-0.9	0.8	0.2	0.1

Table 6. Estimation of IRR (and growth) with Ruuhela's method, negative binomial contribution distribution, true growth rate k = 8%, true life-span N = 20, noise = 20%.

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Estimated life-span		16	20	24	16	20	24	16	20	24
True r	4%	3.8	4.4	4.8	4.6	5.1	5.4	5.5	5.9	6.2
		(9.2)	(9.2)	(9.2)	(9.2)	(9.2)	(9.2)	(9.3)	(9.3)	(9.3)
	8%	8.1	8.3	8.3	9.2	9.2	9.2	10.4	10.3	10.2
		(9.2)	(9.2)	(9.2)	(9.2)	(9.2)	(9.2)	(9.3)	(9.3)	(9.3)
	12%	13.1	12.6	12.3	14.4	13.8	13.4	15.9	15.2	14.7
		(9.2)	(9.2)	(9.2)	(9.2)	(9.2)	(9.2)	(9.3)	(9.3)	(9.3)
	16%	18.4	17.3	16.6	20.0	18.8	18.0	21.9	20.5	19.5
		(9.2)	(9.2)	(9.2)	(9.2)	(9.2)	(9.2)	(9.3)	(9.3)	(9.3)

Table 7. Decomposition of the estimation error in Ruuhela's method. An example with negative binomial distribution, growth rate k = 8%, amplitude A = 50%, noise = 20%.

Est. life-span		16 years				20 years				24 years			
Source of error		For-mula	Grwt estim	Life-span estim	Total error	For-mula	Grwt estim	Life-span estim	Total error	For-mula	Grwt estim	Life-span estim	Total error
True r	4%	0.1	1.0	-0.5	0.6	0.1	1.0	0	1.1	0.1	1.0	0.3	1.4
	8%	0.0	1.2	0.0	1.2	0.0	1.2	0	1.2	0.0	1.2	0.0	1.2
	12%	0.3	1.5	0.6	2.4	0.3	1.5	0	1.8	0.3	1.5	-0.4	1.4
	16%	1.0	1.8	1.2	4.0	1.0	1.8	0	2.8	1.0	1.8	-0.8	2.0

Table 8. Estimation of IRR with the average ARR method, growth k = 8%, amplitude A = 50%, noise = 20%.

Contr. distribution		Uniform			Neg. binomial			Anton		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	3.6	2.6	4.0	4.2	3.4	4.0	4.0	3.2	4.0
	8%	7.8	7.6	8.0	8.0	7.9	8.0	8.0	7.9	8.0
	12%	12.6	13.3	12.0	12.3	12.9	12.0	12.0	12.6	12.0
	16%	17.9	19.5	16.0	17.0	18.4	16.0	16.0	17.2	16.0

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