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GENERALIZED AVAILABILITY CHARACTERISTICS FOR SYSTEMS WITH STATES OF REDUCED EFFICIENCY

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Abstract

The paper deals with the concepts and characteristics of system reliability, especially in such a case when the system has several different levels of performance. This kind of a situation for example arises when the failure of a component or subsystem only reduces the efficiency of the system instead of making the system completely inoperable. The traditional concepts of reliability are shown to be too narrow in extent in order to cover these systems with many possible levels of performance. In order to remove this deficiency, a new definition with an extension in content is given for the concepts of reliability. The conceptual extension is carried out in detail for the characteristic 'availability'. As a result of this analysis two generalized availability characteristics are obtained. Also properties and empirical interpretations of these characteristics are discussed.

1. INTRODUCTION

Numerous problems concerning the improvement of the efficiency of a production system have acquired special importance in the last few decades. Central among these efficiency problems is that of the ability of the system to function in the way intended by the user in advance. This general property of the system is commonly called the 'reliability of the system'.

This paper deals with some problems of definition and interpretation that appear in connection with the content of the concept 'reliability', especially with the quantitative characteristics of the reliability of certain kinds of systems. New definitions for the concepts of reliability, to extend their intension and extension, turn out to be essential in order to be able to make a valid reliability analysis for systems with states of reduced efficiency, i.e. for systems which after a failure, instead of becoming totally inoperable, may be able to operate, at a level of decreased performance however.

In section 2 we consider the definition and characteristics of traditional reliability, i.e. reliability that has been defined for two-stage operable or inoperable systems (the components of the system and the whole system are assumed to be either failure-free and thus capable of full performance or failed and thus totally inoperable). The section also contains a general mathematical definition of reliability which covers all the quantitative characteristics of reliability: the characteristics can be obtained as particular cases of this mathematical definition.

In section 3 the traditional concepts of reliability are shown to be too narrow in both intension and extension: the reliability of those systems, which as a consequence of failures have several possible levels of performance, remains on the basis of these definitions unresolved or gets a value that contradicts empirical observation. The concepts of reliability are now extended so that these deficiencies are removed. Explicitly the extension is carried out for the characteristic 'availability'. The extension is supported on the empirical interpretation of availability and so that the new characteristics preserve in ordinary operable or inoperable systems their previous content and meaning. Further, all the time within the framework of the general mathematical definion of reliability will be stayed.

2. ON THE TRADITIONAL CONCEPTS OF RELIABILITY

2.1 THE QUALITATIVE DEFINITION OF RELIABILITY

Definitions of reliability are in general based on the occurrence or non-occurrence of a failure at a given time or during a given time interval. According to their main features the definitions are divided into two groups, qualitative and quantitative definitions. The following is a typical representative of the group of qualitative definitions of reliability: "Reliability is the ability of the equipment to preserve its output characteristics (parameters) within established limits in given conditions of operation" ([5], p. 1). Similar definitions, only slightly differently worded, are given e.g. in [1], p. 6, [2], p. 6, [3], p. 70, and [4], p. 20.

In the qualitative definition the measurement and rendering of reliability still remain to some extent unresolved. The final specification of reliability is made in the form of quantitative definitions or quantitative characteristics.

2.2 QUANTITATIVE CHARACTERISTICS OF RELIABILITY

The number of quantitative characteristics of reliability is quite large, different indices playing the determining role when different systems or their different uses are considered. In the following, the three most general and important quantitative characteristics of reliability are presented.

The (pointwise) <u>availability</u> of the system at time t, A(t), is the probability (see e.g. [2], p. 7 and [6], p. 239):

$$A(t) = Pr\left\{\text{the system is operable at time } t\right\}. \qquad (1)$$

The availability of a system thus depends, not only on the ability of the system to operate without failures, but also on the efficiency with which the repair of the system has been arranged. Availability A(t) is thus typically a characteristic of repairable systems.

The <u>reliability function</u> or in short the <u>reliability</u> of a system at time t, R(t), is the probability (see e.g. [2], p. 7 and [3], p. 79):

$$R(t) = Pr\left\{\text{the system does not fail in } [0,t]\right\}. \qquad (2)$$

Reliability R(t) is typically a characteristic of systems that are non-repairable or are considered as non-repairable. We are only interested in the first failure of the system and the time of its occurrence.

<u>Mean time to system failure</u>, T, is, as its name shows, the expected value of the time that the system operates uninterruptedly without failing (see e.g. [3], p. 77 and [6], p. 240). As far as the characteristic T is concerned, we can consider the time before the first system failure as well as the time from the completion of the repair of a failure to the occurrence of the next failure. Thus T is a characteristic of reliability both for a repairable and for a non-repairable system.

2.3 THE GENERAL MATHEMATICAL DEFINITION OF RELIABILITY

In this section we present the general mathematical definition of reliability. The presentation follows the formulation of GNEDENKO (see [3], pp. 74 - 78).

Generally, let $S = \{s\}$ be the set of all the different possible states of the system, called the phase space of the system. With the passage of time, various changes may take place in the constituent parts of the system; the state of the system changes. Let x(t) denote the state of the system at the instant t. Then $x(t) \in S$ for all values of t ($t \geq 0$). The stochastic state at time t or the state distribution at time t is described by the random variable X(t). Then each x(t) observed or possible at time t is a single (sample) value of X(t). The ordered set $X = (X(t)|t \geq 0)$, indexed to the time variable t, is then a stochastic process describing the course of the states in time. A time series $\hat{X} = (x(t)|t \geq 0)$ is a realization or trajectory of this stochastic process. The set of these trajectories we denote by $\hat{X}$.

After definition of the phase space S and the stochastic process X we can formulate the general mathematical definition of reliability. Let Φ be a functional defined on the trajectories of the process X (on the set $\hat{X}$), whereupon to every trajectory $\hat{x}$ there corresponds a unique real number $\Phi(\hat{x})$. The reliability φ is now defined as the expected value of that functional

$$\varphi = E\left\{\Phi(\hat{X})\right\}. \tag{3}$$

The final specification of reliability, i.e. the choice of the quantitative characteristic to be used, thus remains dependent on the definition of the functional Φ.

Let S^O be a subset of the phase space S such that the system is inoperable, when its state $x(t)$ belongs to S^O and operable otherwise. Let the functional Φ be defined as follows

$$\Phi(\hat{x}) = \begin{cases} 0, & \text{if } \hat{x}(t) \in S^O \\ 1, & \text{if } \hat{x}(t) \in S - S^O. \end{cases} \tag{4}$$

Then we have

$$\varphi = E\left\{\Phi(\hat{X})\right\} = Pr\left\{\Phi(\hat{X})=1\right\} = Pr\left\{X(t) \in S-S^O\right\} \tag{5}$$
$$= Pr\left\{\text{the system is operable at time } t\right\} = A(t),$$

and we have achieved the characteristic availability as a special case of the definition (3). It can be shown (see [7], pp. 28-30) that also the other characteristics can be obtained using the definition (3).

3. THE EXTENDED CONCEPTS OF RELIABILITY

3.1 PROBLEMS CONCERNING THE CONCEPT OF SYSTEM RELIABILITY WHEN THE SYSTEM HAS LEVELS OF REDUCED PERFORMANCE

In section 2 we considered reliability from the point of view of the traditional approach: the system is at every moment regarded either as operable or inoperable. At the occurrence of any phenomenon differing from the system´s normal operation, for example, this means, that the phenomenon must be classified either as a failure or a factor having no influence on the operability of the system. Among production systems there exists, however, several examples which show that the ordinary concepts of reliability are too narrow to cover all kinds of systems. The problematic situation comes from the fact that the system is able after failure to operate with reduced efficiency or at a level of reduced performance, instead of becoming wholly inoperable due that failure. An example is given by an electric power system, where a power block can be in operation at a lower level of performance than the maximum: a failure of a smoke ventilator leads to a reduction in available power, but the block is still operating. Generally, the reduction in the level of performance may be a consequence of the property of an individual component (the component operates, but is e.g. retarded) or of the type of the composition of the components (there are parallel branches in the system, some of the branches having failed).

It is evident that the efficiency of a system which, as a consequence of failure, has operated at a level of reduced performance cannot be as high as the efficiency of a similar system which has all the time operated normally. On the other hand, its efficiency is higher than that of a similar system which, during the time in question, has been totally inoperable. Circumstances like this ought also to be included in the reliability of the system: the reliability of the system should decrease due to the failure, but not, however, as much as in the case of total system failure. But when the traditional concepts of reliability are used, the situation either remains unresolved (there exist situations when the system is neither fully operable nor fully inoperable, so that reliability cannot be determined at all), or it is incorrectly treated (if operation with reduced efficiency is regarded as normal operation, too high reliability is obtained; if a reduction in efficiency is regarded as total inoperability, too low reliability is obtained).

In this paper we extend the concepts of reliability in order to make it possible to determine also the reliability of systems with states of reduced efficiency. In making the extension we take care that it is done in a theoretically wellfounded and empirically adequate way. Furthermore it is not done any violation of the traditional concepts. These conditions will be fulfilled by setting for the new concepts the following requirements:

(1) Failures having a limiting effect on the efficiency of the system are referred to factors which decrease the reliability of the system, and do this to an extent that is dependent on seriousness of the consequences of the failure.

(2) When the new extended concepts are applied to general systems with many levels of performance, we get empirical interpretations analogical to those which result when the traditional concepts of reliability are applied to ordinary operable or inoperable systems.

(3) When a two-stage operable or inoperable system is under consideration the new concepts result in the same characteristics as the traditional ones.

(4) The definition of the new concepts can be given in the form of the general mathematical definition of reliability, i.e. using (3).

(5) The numerical value of reliability can be determined directly from the behaviour of the system, e.g. from the state probabilities.

3.2 THE GENERALIZED AVAILABILITY CHARACTERISTICS

Explicitly we carry out the extension of the reliability concepts for the characteristic availability only. In the course of the derivation of the generalized "availabilities" the main principles of the extension procedure will be given, however. Following these principles analogical extensions for the other characteristics of traditional reliability can be accomplished (see [7], pp. 44 - 48, where analogical generalized concepts for the traditional reliability and mean time to system failure are presented).

3.2.1 Notation

Let us assume, that the possible levels of performance of the system are $c_0, c_1, \ldots, c_K$, where $c_0=0$ stands for total inoperability, $c_K=C$ indicates full operability (C is the capacity of the system), and c_1 to c_{K-1} are levels of reduced performance caused by one or more simultaneous failures. The respective proportional levels of performance are denoted by $w_0, w_1, \ldots, w_K$. Then we have $w_0=0$, $w_K=1$, and $0<w_k<1$, when $k=1,2,\ldots,K-1$.

Between the performance levels and the states of the system there is an obvious relation. For each state there is a uniquely determined level of performance; several states, on the other hand may lead to one and the same performance level. Let the possible states of the system be numbered as follows the states corresponding to the performance level c_i are $s_i^1, s_i^2, \ldots, s_i^{n_i}$. Let the set of these states be denoted by S_i. So we have $S_i=\{s_i^1, s_i^2, \ldots, s_i^{n_i}\}$. The state probabilities of the system we denote by the symbol $P_{ij}(t)$, where

$$P_{ij}(t) = \Pr\{\text{the state of the system at time } t \text{ is } s_i^j\},\quad i=0,1,\ldots,K,\ j=1,2,\ldots,n_i. \tag{6}$$

By means of this notation not only the current state of the system but also its level of performance is revealed. Further, let the proportional level of performance of the system at time t be denoted by the random variable W(t). The probability distribution of the random variable W(t) we denote by $P_i(t)$:

$$P_i(t) = \Pr\{W(t)=w_i\},\quad i=0,1,\ldots,K. \tag{7}$$

Then we have

$$P_i(t) = \sum_{j=1}^{n_i} P_{ij}(t),\quad i=0,1,\ldots,K. \tag{8}$$

3.2.2 Mean availability of the capacity

The first generalized availability characteristic to be presented is defined as the expected value of the proportional level of performance of the system at time t. This new characteristic, denoted by $A_C(t)$, is thus

$$A_C(t) = E\{W(t)\} = \sum_{i=0}^{K} w_i \Pr\{W(t)=w_i\} = \sum_{i=0}^{K} w_i P_i(t). \tag{9}$$

It is easy to see that the characteristic mean availability of the capacity is such a generalization of the traditional availability that takes all the five requirements set in section 3.1 fully into account.

First, all kinds of failures have a decreasing effect on the reliability of the system, but the decrease is dependent on the "degree" of the failure: the more serious the consequences of the failure, the more will the reliability decrease.

Secondly, the definition of the characteristic mean availability of the capacity is fully analogical to the definition of the traditional availability. For, when the availability of an ordinary operable or inoperable system is considered, we have only two possible values for the proportional level of performance of the system: $W(t) = w_0 = 0$ when the system is inoperable and $W(t) = w_1 = 1$ when the system is operable (and we have K=1 in the notation of section 3.2.1). Now we can write

$$\begin{aligned} A(t) &= \Pr\left\{W(t)=1\right\} \\ &= \Pr\left\{W(t)=1\right\}\times 1 + \Pr\left\{W(t)=0\right\}\times 0 \\ &= E\left\{W(t)\right\}, \end{aligned} \qquad (10)$$

so that we have expressed A(t) as the expected value of the proportional level of performance of the system. It can further be shown (see [7], pp. 40 - 43) that the generalized characteristic $A_C(t)$ has under the steady state a lot of interpretations and properties which are in direct analogy to the corresponding properties of the traditional A(t): A_C gives (1) the mean proportional level of performance of the system during some interval, (2) the ratio of true and potential outputs of the system during that interval, and (3) the portion of the interval during which the full capacity should be available in order that an equivalent system output would be achieved.

In the third place, for a two-stage operable or inoperable system the generalized characteristic A_C yields the same result as the traditional A [see definition (9) and equation (10)].

Fourth, by defining the functional Φ as follows

$$\Phi(\hat{X}) = w_i, \text{ if } \hat{X}(t) \in S_i,\ i=0,1,\ldots,K, \qquad (11)$$

we have

$$\begin{aligned} E\left\{\Phi(\hat{X})\right\} &= \sum_{i=0}^{K} w_i \Pr\left\{\Phi(\hat{X})=w_i\right\} = \sum_{i=0}^{K} w_i \Pr\left\{X(t)\in S_i\right\} \\ &= \sum_{i=0}^{K} w_i P_i(t) = A_C(t), \end{aligned} \qquad (12)$$

so that we can obtain A_C as a particular case of definition (3).

Fifth and last, we can calculate the characteristic A_C from the state probabilities of the system:

$$A_C(t) = \sum_{i=0}^{K} w_i P_i(t) = \sum_{i=0}^{K} w_i \sum_{j=1}^{n_i} P_{ij}(t), \qquad (13)$$

as was presumed in requirement (5) in section 3.1.

3.2.3 Availability of levels of performance

The second way to take the many levels of performance into account is to define the "availability" as a function of the level of performance (when the traditional availability at a fixed time is simply a number). We define another generalized availability characteristic, availability of levels of performance, as the probability

$$A_0(c,t) = \Pr\left\{\text{the level of performance of the system at time } t \text{ is at least } c\right\}. \qquad (14)$$

We can immediately see that A_0 comes within the framework of the general definition (3). Defining the functional Φ as follows

$$\Phi(\hat{X}) = \begin{cases} 0, & \text{if } \hat{X}(t) \in \bigcup_{c_i<c} S_i \\ 1, & \text{if } \hat{X}(t) \in \bigcup_{c_i\geq c} S_i, \end{cases} \qquad (15)$$

we get $A_0(c,t)$ as the expected value of this functional

$$\begin{aligned} E\left\{\Phi(\hat{X})\right\} &= \Pr\left\{X(t) \in \bigcup_{c_i\geq c} S_i\right\} \\ &= \Pr\left\{\text{the level of performance of the system at time } t \text{ is at least } c\right\} \\ &= A_0(c,t). \end{aligned} \qquad (16)$$

Also the other requirements (1) to (5) in section 3.1 can be shown to be fulfilled (fore more detailed description and analysis of the characteristic A_0 see [7], pp. 34 - 37).

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