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AUTO1030 Signaalien käsittely
Harjoitus 1, tehtävä 3

Complex numbers.

$$z_1 = (1+2\mathrm{i})$$

$$z_2 = (3-4\mathrm{i})$$

$$z_1 + z_2 = (1+2\mathrm{i}) + (3-4\mathrm{i}) = (1+3) + (2\mathrm{i}-4\mathrm{i}) = 4-2\mathrm{i}$$

$$z_1 z_2 = (1+2\mathrm{i})(3-4\mathrm{i}) = 1\cdot 3 + 1\cdot(-4\mathrm{i}) + 2\mathrm{i}\cdot 3 + 2\mathrm{i}\cdot(-4\mathrm{i}) = 3-4\mathrm{i}+6\mathrm{i}-8\mathrm{i}^2 = 3+2\mathrm{i}-8\cdot(-1) = 3+2\mathrm{i}+8 = 11+2\mathrm{i}$$

$$z_1 / z_2 = ^{3+4\mathrm{i})}\ \frac{1+2\mathrm{i}}{3-4\mathrm{i}} = \frac{(1+2\mathrm{i})(3+4\mathrm{i})}{(3-4\mathrm{i})(3+4\mathrm{i})} = \frac{(3-8)+(4\mathrm{i}+6\mathrm{i})}{9+16} = \frac{-5+(10)\mathrm{i}}{25} = \frac{-5}{25} + \frac{10}{25}\mathrm{i} = -0{,}2+0{,}4\mathrm{i}$$

$$2z_1 + z_2/3 = 2(1+2\mathrm{i}) + (3-4\mathrm{i})/3 = (2+4\mathrm{i}) + (1-\frac{4}{3}\mathrm{i}) = 3+2\frac{2}{3}\mathrm{i}$$

Seuraavaksi lainaus sivulta: http://mathworld.wolfram.com/ComplexExponentiation.html

A complex number may be taken to the power of another complex number. In particular, complex exponentiation satisfies

$$(a+bi)^{c+di} = (a^2+b^2)^{(c+id)/2} e^{i(c+id)\arg(a+ib)}, \qquad (1)$$

where $\arg(z)$ is the complex argument. Written explicitly in terms of real and imaginary parts,

$$(a+bi)^{c+di} = (a^2+b^2)^{c/2} e^{-d\arg(a+ib)} \times \{\cos[c\arg(a+ib) + \tfrac{1}{2} d\ln(a^2+b^2)] + i\sin[c\arg(a+ib) + \tfrac{1}{2} d\ln(a^2+b^2)]\}.$$

Eli nyt (Matlabilla laskettuna):

$$z_1^{z_2} = 932{,}14 + 95{,}947\mathrm{i}\ .$$

$$\mathrm{i}z_1/(1-z_2) = \frac{\mathrm{i}(1+2\mathrm{i})}{1-(3-4\mathrm{i})} = \frac{-2+\mathrm{i}}{-2+4\mathrm{i}} =$$

$$\left|\frac{-2+\mathrm{i}}{-2+4\mathrm{i}}\right| \mathrm{e}^{\arg(-2+\mathrm{i})-\arg(-2+4\mathrm{i})} = 0{,}4+0{,}3\mathrm{i}$$

$$\frac{(z_1-1)}{(z_1+1)(z_2-1)} = \frac{2\mathrm{i}}{(2+2\mathrm{i})(2-4\mathrm{i})} = \frac{2\mathrm{i}}{12-4\mathrm{i}} = \frac{2\mathrm{i}(12+4\mathrm{i})}{(12-4\mathrm{i})(12+4\mathrm{i})} =$$

$$\frac{-8+24\mathrm{i}}{160} = -0{,}05+0{,}15\mathrm{i}$$