An Example—Waveguide H-Bend

Introduction

This examples shows how to model a rectangular waveguide for microwaves. A single hollow waveguide can conduct two kinds of electromagnetic waves: transversal magnetic (TM) or transversal electric (TE) waves. This model examines a TE wave, one that has no electric field component in the direction of propagation. More specifically, for this model you select the frequency and waveguide dimension so that TE_{10} is the single propagating mode. In that mode the electric field has only one nonzero component—a sinusoidal with two nodes, one at each of the walls of the waveguide.

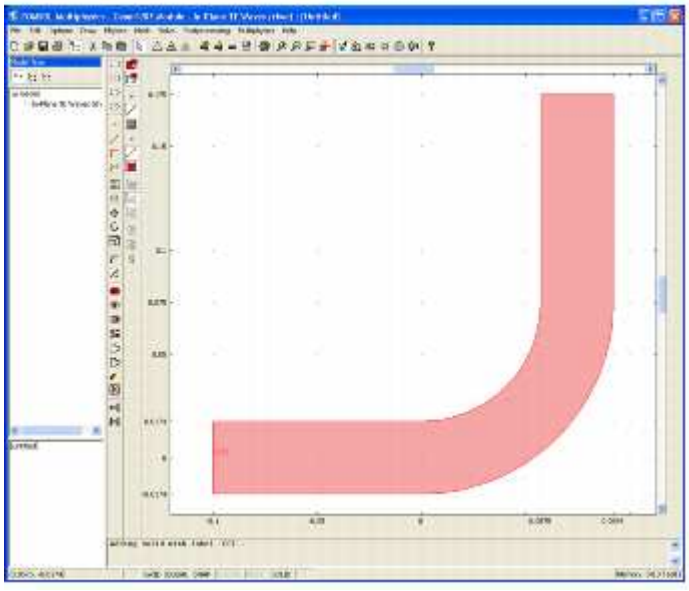
One important design aspect is how to shape a waveguide to go around a corner without incurring unnecessary losses in signal power. Unlike in wires, these losses usually do not result from ohmic resistance but instead arise from unwanted reflections. You can minimize these reflections by keeping the bend smooth with a large enough radius. In the range of operation the transmission characteristics (the ability of the waveguide to transmit the signal) must be reasonably uniform for avoiding signal distortions.

Model Definition

This example illustrates how to create a model that computes the electromagnetic fields and transmission characteristics of a 90-degree bend for a given radius. This type of waveguide bends changes the direction of the **H** field components and leaves the direction of the **E** field unchanged. The waveguide is therefore called an *H-bend*. The H-bend design used in this example is well-proven in real-world applications and you can buy similar waveguide bends online from a number of manufacturers. This particular bend performs optimally in the ideal case of perfectly conducting walls as is shown later on in this model by computing the (in this case optimal) transmission characteristics.

The waveguide walls are typically plated with a very good conductor, such as silver. In this example the walls are considered to be made of a perfect conductor, implying that $\mathbf{n} \times \mathbf{E} = 0$ on the boundaries. This boundary condition is referred to as a *perfect electric conductor* (PEC) boundary condition.

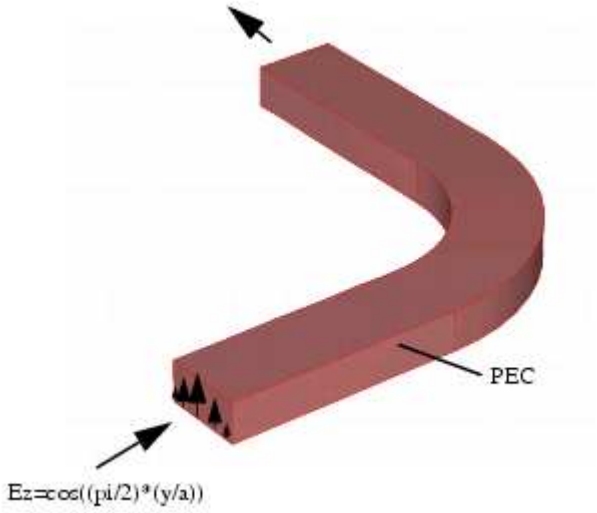
The geometry is as follows, as viewed in the *xy* plane.



DOMAIN EQUATIONS

The waveguide is considered to continue indefinitely before and after the bend. This means that the input wave needs to have the form of a wave that has been traveling through a straight waveguide. The shape of such a wave is determined by the boundary conditions of Maxwell's equations on the sides of the metallic boundaries, that is, the PEC boundary condition. If polarized according to a TE_{10} mode, the shape is known analytically to be $\mathbf{E} = (0, 0, \sin(\pi (a-y) / (2 a)))\cos(\omega t)$ given that the entrance boundary is centered around the $y = 0$ axis, and that the width of the waveguide, in the y direction, is $2a$.

In the RF Module you can model this waveguide in the 2D In-Plane TE Waves application mode or in the 3D Electromagnetic Waves application mode as a time-harmonic wave propagation model. This means that only the phasor component of the field is modeled. The incident field then has the form $\mathbf{E} = (0, 0, E_{0z}) = (0, 0, \sin(\pi (a-y) / (2 a)))$, and is considered as part of the expression $\mathbf{E}(t) = \text{Real}\{(0, 0, \sin(\pi(a-y)/(2a)))e^{jwt}\} = \text{Real}\{\mathbf{E}e^{jwt}\}$, where complex-valued arithmetic has been used (also referred to as the $j\omega$ method).



The width of the waveguide is chosen so that it has a cut-off frequency of 4.3 GHz. This makes the waveguide operational between 5.4 GHz up to 8.1 GHz. At higher frequencies other modes than the TE_{10} appear, causing a "dirty" signal. The input wave then splits into several modes that are hard to control without having large power losses. Below the cut-off frequency, no waves can propagate through the waveguide. This is an intrinsic property of microwave waveguides.

The cut-off frequency of different modes in a straight waveguide is given by the relation

$$(\nu_c)_{mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

where m and n are the mode numbers ($m = 1$, $n = 0$ for the TE_{10} mode), a and b are the lengths of the sides of the waveguide cross-section, and c is the speed of light.

For this waveguide, $a = 2b$ and $b = 0.0174245$.

The first few cut-off frequencies are $(\nu_c)_{10} = 4.3$ GHz, $(\nu_c)_{01} = 8.6$ GHz, $(\nu_c)_{11} = 9.6$ GHz and the operational range is chosen to be $1.25*(\nu_c)_{10} = 5.4$ GHz to $0.95*(\nu_c)_{10} = 8.1$ GHz. This is to have reasonable margins for manufacturing errors and to avoid the large reflections that occur at lower frequencies.

BOUNDARY CONDITIONS

This model makes use of the predefined port boundary condition. It is an automated version of the matched boundary condition described later in this section. An additional advantage is

that the port boundary condition automatically creates postprocessing variables for the S-parameters.

The input matched boundary condition consists of two parts: an incident planar wave and an absorbing boundary condition. The matched boundary condition is also used at the output boundaries to eliminate any reflections there. At the output boundaries there is no excitation. The walls of the waveguide are considered to be good conductors, so you can use the perfectly electric conductive (PEC) boundary condition.

For specifying the absorbing boundary condition you must know the propagation constant, β, of the wave. You can find the propagation constant from an eigenmode analysis of the waveguide cross section.

In this simple case, however, you can also compute the propagation constant by hand using the relation

$$k_0^2 = k_x^2 + k_y^2 + k_z^2$$

for the wavenumbers in the x, y and z directions, respectively, at the waveguide entrance port. Here x is the direction of propagation, and y and z are the transversal directions, with z as the out-of-plane direction. In an infinitely extended straight waveguide, the following equations define the free-space and the x, y, and z direction wavelengths:

$$k_0 = \frac{2\pi}{\lambda_0}$$

$$k_x = \frac{2\pi}{\lambda_x} = \beta$$

$$k_y = \frac{2\pi}{\lambda_y}$$

$$k_z = \frac{2\pi}{\lambda_z}$$

In this case, the y direction wavelength is known to be $2a$ because the model describes the lowest propagating mode, which is a half wavelength across the transversal direction. This means for the wave number in the y direction

$$k_y = \frac{2\pi}{2a}$$

Also, for the TE_{10} mode,

$$k_z = 0$$

The free-space wave number is

$$k_0 = \frac{2\pi}{\left(\frac{c_0}{\nu}\right)} = \frac{2\pi}{\left(\frac{3 \cdot 10^8}{\nu}\right)}$$

The propagation constant evaluates to

$$\beta = k_x = \sqrt{k_0^2 - k_y^2} = \sqrt{\left(\frac{2\pi}{\left(\frac{3 \cdot 10^8}{\nu}\right)}\right)^2 - \left(\frac{2\pi}{2a}\right)^2},$$

which is a frequency-dependent expression.

Now, for the matched boundary condition, entering the correct propagation constant eliminates all waves with that wavenumber in the propagating direction. To make the boundary condition perfectly absorbing, the propagating direction needs to be the same as the normal direction of the output boundary.

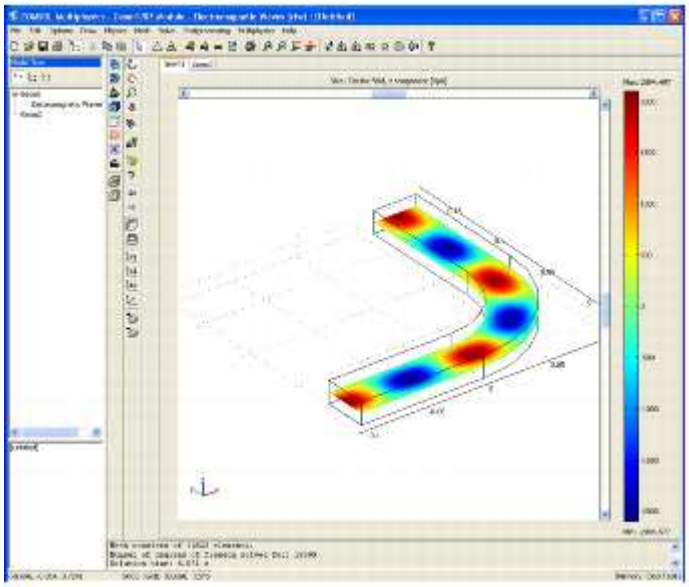
Note: The input amplitude can be replaced by any analytical expression. You can, for instance, create a wave corresponding to a truncated Gaussian beam or simply a planar wave, for which the expression would be 1.

The first part of the analysis is made for a frequency that is 20% above the cut-off

frequency. This is to show a generic propagating wave within the frequency range of operation.

Results and Discussion

The wave is found to propagate through the bend with very small reflection.



To verify that there is almost no reflection you can compare the power flow of the outgoing wave to that of the incoming wave. The entrance port is excited using the port boundary condition, which automatically normalizes the excitation to unity power (1 W). Comparing this to the power flow over the output port, 0.985795 W, shows that the power loss is less than 2%.

Model Library Path: `RF_Module/RF_and_Microwave_Engineering/hbend_3d`

3D Modeling Using the Graphical User Interface

MODEL NAVIGATOR

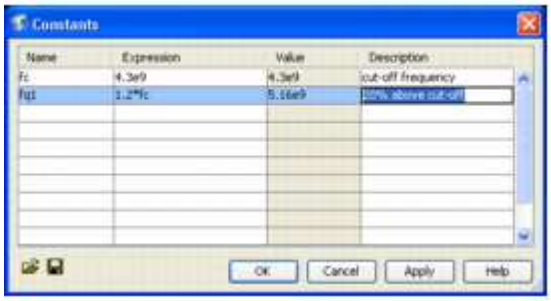
1 In the **Model Navigator**, select **3D** in the **Space dimension** list.

2 In the list of application modes, select **RF Module>Electromagnetic Waves>Harmonic propagation**.

3 Click **OK**.

OPTIONS AND SETTINGS

Define the following constants in the **Constants** dialog box.



NAME	EXPRESSION	DESCRIPTION
fc	4.3e9	cut-off frequency
fq1	1.2*fc	20% above cut-off

GEOMETRY MODELING

1 From the **Draw** menu, open the **Work Plane Settings** dialog box. Click **OK** to obtain the default workplane in the *x*-*y* plane.

2 From the **Options** menu, choose **Axes/Grid Settings**.

3 In the **Axes/Grid Settings** dialog box, specify the following axis and grid settings, and then click **OK**:

AXIS		GRID	
x min	-0.175	x spacing	0.05
x max	0.175	Extra x	0.075-0.0174245 0.075+0.0174245
y min	-0.04	y spacing	0.05
y max	0.2	Extra y	-0.0174245 0.0174245 0.075 0.175

To define the grid spacings, first click the **Grid** tab and clear the **Auto** check box.

The waveguide width 2*0.0174245 corresponds to a cut-off frequency of 4.3 GHz.

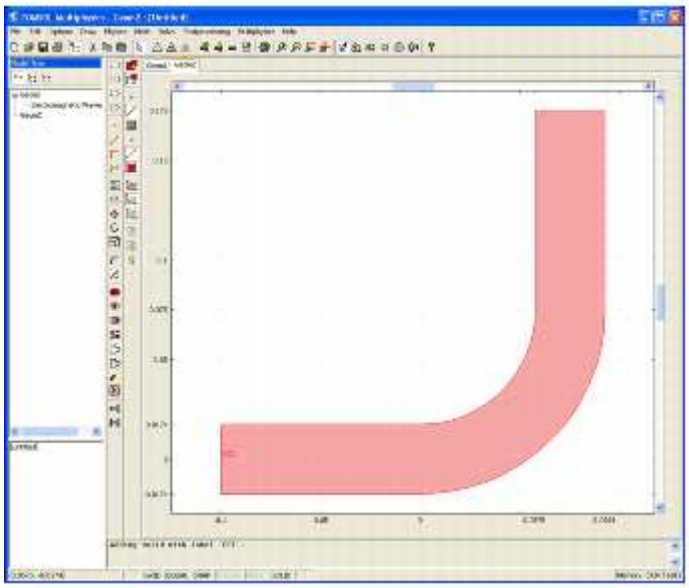
4 Start by drawing three lines. After selecting the **Line** button on the Draw toolbar, click at (0, −0.0174), (−0.1, −0.0174), (−0.1, +0.0174), and (0, +0.0174).

5 Click the **2nd Degree Bezier Curve** button, and click at (0.0576, 0.0174) and (0.0576, 0.075).

6 Click the **Line** button, and click at (0.0576, 0.175), (0.0924, 0.175), and (0.0924, 0.075).

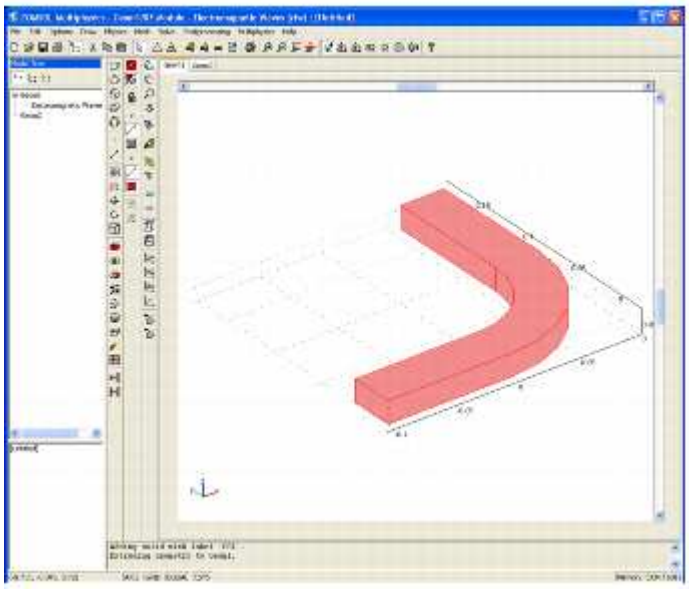
7 Click the **2nd Degree Bezier Curve** button, and click at (0.0924, −0.0174).

8 Finish by clicking the right mouse button to close the boundary curve and create a solid object.



9 Select **Extrude** from the **Draw** menu. Extrude the object using a distance of 0.0174.

10 Click the **Zoom Extents** toolbar button.



PHYSICS SETTINGS

Scalar Variables

1 From the **Physics** menu, choose **Scalar Variables**.

2 In the **Application Scalar Variables** dialog box, set the frequency `nu_rfw` to `fq1`, and then click **OK**.

Boundary Conditions

1 From the **Physics** menu, choose **Boundary Settings**.

2 Select boundaries 2–8 and 10.

3 In the **Boundary condition** list, select **Perfect electric conductor** as the boundary condition. These boundaries represent the inside of the walls of the waveguide which is plated with a metal, such as silver, and considered to be a perfect conductor.

4 On boundaries number 1 and 9, specify the **Port** boundary condition. On the **Port** tab set the values according to the following table; when done, click **OK**:

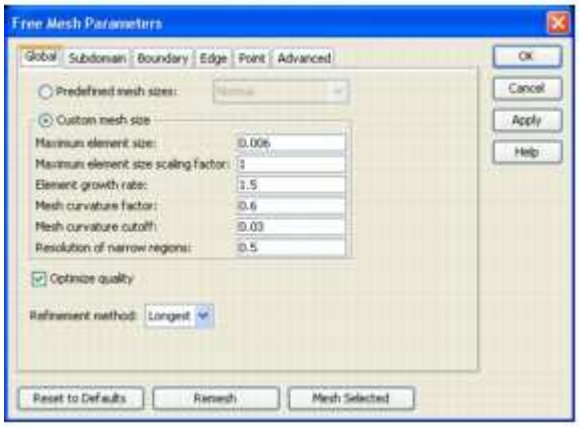
BOUNDARY	**1**	**9**
Port number	`1`	`2`
Wave excitation at this port	`Selected`	`Cleared`
Mode specification	`Rectangular`	`Rectangular`
Mode type	`Transverse electric (TE)`	`Transverse electric (TE)`
Mode number	`10`	`10`

Subdomain Settings

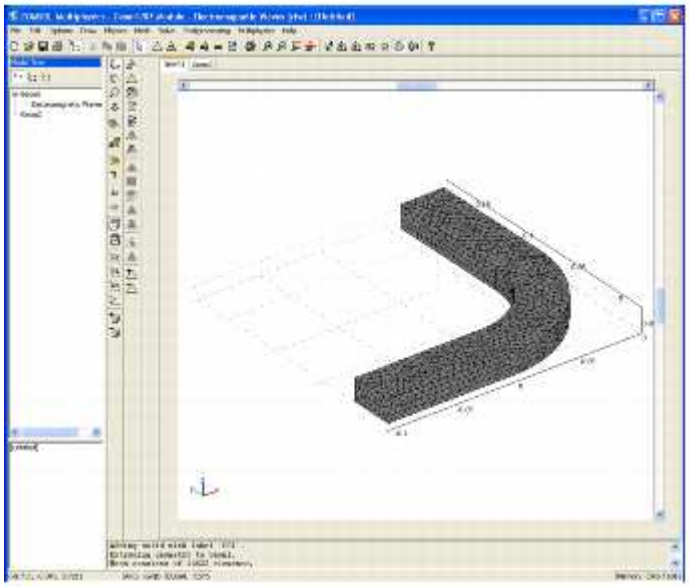
Use the default values for ε_r, μ_r, and σ, because the waveguide is filled with air.

MESH GENERATION

1 In the **Free Mesh Parameters** dialog box, click the **Custom mesh size** button, and then type `0.006` in the **Maximum element size** edit field.



2 Click **Remesh** to generate the mesh; then click **OK**.



COMPUTING THE SOLUTION

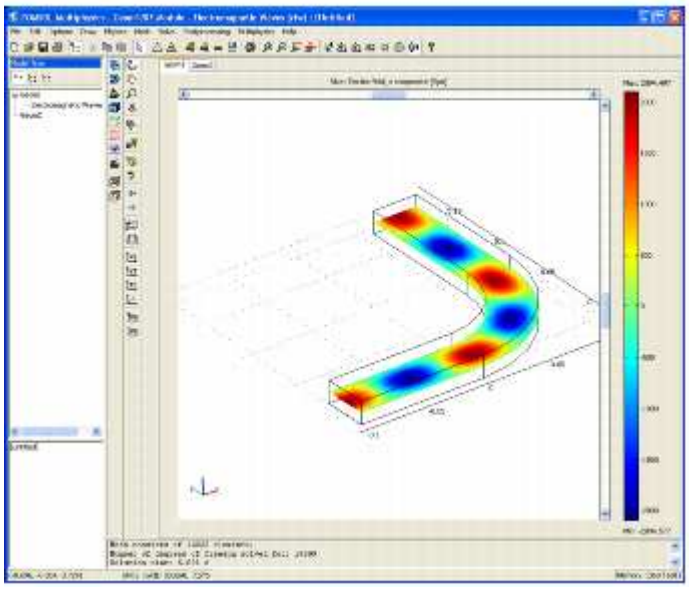
Click the **Solve** button on the Main toolbar to solve the problem.

POSTPROCESSING AND VISUALIZATION

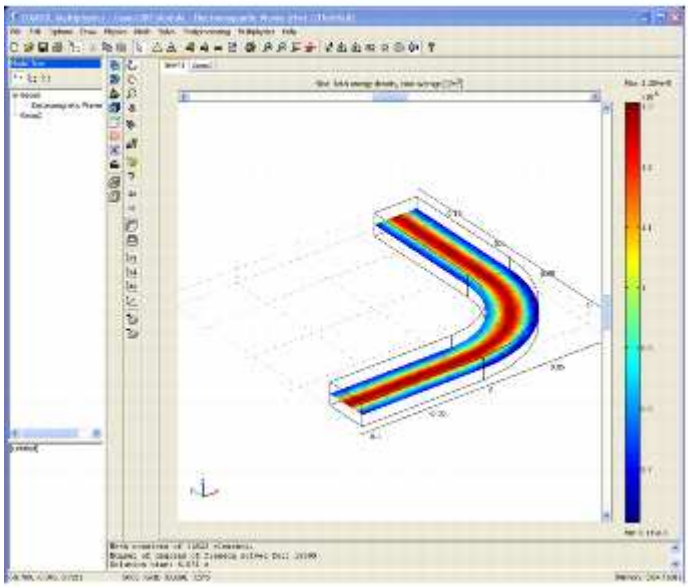
The default plot is a slice plot of the *z* component of the electric field. To better see the propagating wave change the position of the slices.

1 From the **Postprocessing** menu, choose **Plot Parameters**.

2 On the **Slice** tab in the **Plot Parameters** dialog box set the **Slice positioning x levels** to `0`, the **y levels** to `0`, and the **z levels** to `1`.



3 A convenient way to visualize the good transmission is to plot the total energy density. Reflections give rise to a wave pattern in the energy distribution. Select **Total energy density, time average** from the **Predefined quantities** list in the **Slice data** area.



4 To compare the power flow of the incident wave to the power flow of the outgoing wave, perform a boundary integration. The entrance port is excited using the port boundary condition, which provides an excitation with a power level of 1 W. To obtain the power outflow from the exit port, open the **Boundary Integration** dialog box from the **Postprocessing** menu, and integrate **Power outflow, time average** over boundary 9. The result is about 0.985795 W and appears in the message log at the bottom of the main window.

Model Library Path: `RF_Module/RF_and_Microwave_Engineering/hbend_2d`

2D Modeling Using the Graphical User Interface

The results obtained in the 3D calculation are independent of the height b of the waveguide as the TE_{10} wave does not vary in the z direction. This means that the model can just as well be made in 2D.

You find an extended version of this model with an S-parameter study in the section "Waveguide H-Bend with S-parameters" on page 60 in the *RF Module Model Library*. See also

the section "S-Parameters" on page 68 for more information about S-parameter calculations.

MODEL NAVIGATOR

1 Select **2D** in the **Space dimension** list.

2 Select the **RF Module>In-Plane Waves>TE Waves>Harmonic propagation** application mode.

3 Click **OK**.

OPTIONS AND SETTINGS

Define the same constants as was done in the 3D model on page 26.

GEOMETRY MODELING

Use the same axes/grid settings and draw the same geometry as was done in the 2D workplane in the 3D model.

PHYSICS SETTINGS

Scalar Variables

1 From the **Physics** menu, choose **Scalar Variables**.

2 In the **Application Scalar Variables** dialog box, set the frequency `nu_rfwe` to `fq1`, and then click **OK**.

Boundary Conditions

Use the same boundary conditions as in the 3D model:

1 On boundaries 2–4 and 6–8, select the **Perfect electric conductor** boundary condition.

2 On boundaries number 1 and 5, specify the **Port** boundary condition. On the **Port** tab set the values according to the table below:

BOUNDARY	**1**	**5**
Port number	1	2
Wave excitation at this port	Selected	Cleared
Mode specification	Analytic	Analytic
Mode number	1	1

MESH GENERATION

1 Initialize the mesh.

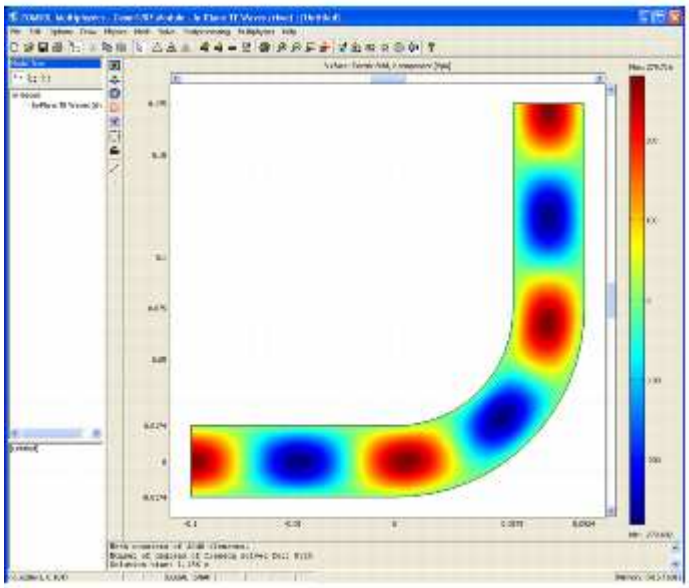
2 Refine the mesh twice.

COMPUTING THE SOLUTION

Click the **Solve** button on the Main toolbar.

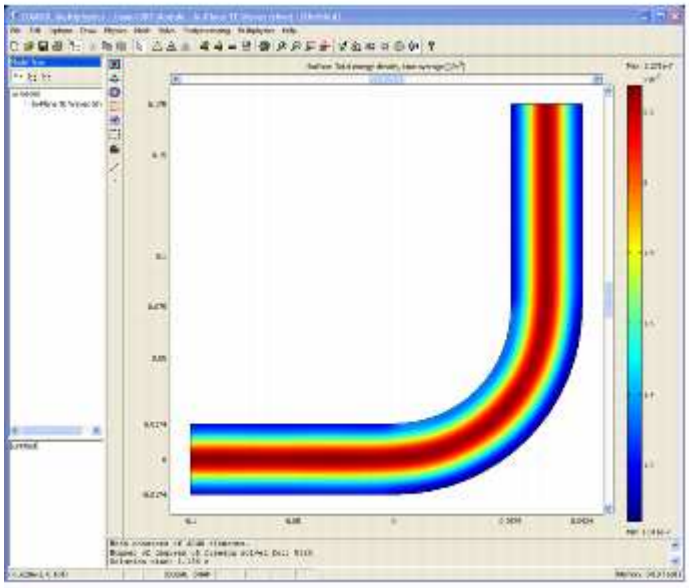
POSTPROCESSING AND VISUALIZATION

The default plot shows the *z* component of the electric field. Qualitatively, the result coincides with the result in the 3D model. The difference in amplitude is because the input power in the 2D port boundary condition is 1 W per unit depth (1m).



1 To plot the energy density, open the **Plot Parameters** dialog box.

2 Click the **Surface** tab, and select **Total energy density, time average** from the **Predefined quantities** list in the **Surface data** area. Click **OK**.



This again shows that the reflections are very small. To further verify this compare the power flow of the incoming wave to the outgoing wave. To compare the power flow of the incident wave to the power flow of the outgoing wave, perform a boundary integration. The entrance port is excited using the port boundary condition, which provides an excitation with a power level of 1 W (per unit depth). To obtain the power outflow from the exit port, open the **Boundary Integration** dialog box from the **Postprocessing** menu, and integrate **Power outflow, time average** over boundary 5. The result, 0.99993 W, which appears in the message log, indicates that any reflected power is too small to be detected. Note that the 2D model is more accurate than the 3D model, because it uses a finer mesh that better resolves the waves.