

COMSOL MULTIPHYSICS™

Three-port Ferrite Circulator

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Introduction

A microwave circulator is a multiport device that has the property that a wave incident in port 1 is coupled into port 2 only, a wave incident in port 2 is coupled into port 3 only, and so on. Circulators are used to isolate microwave components to couple a transmitter and a receiver to a common antenna, for example. They typically rely on the use of anisotropic materials, most commonly ferrites. In this example, a three-port circulator is constructed from three rectangular waveguide sections joining at 120° where a ferrite post is inserted at the center of the joint. Figure 1 shows the geometry of the circulator.

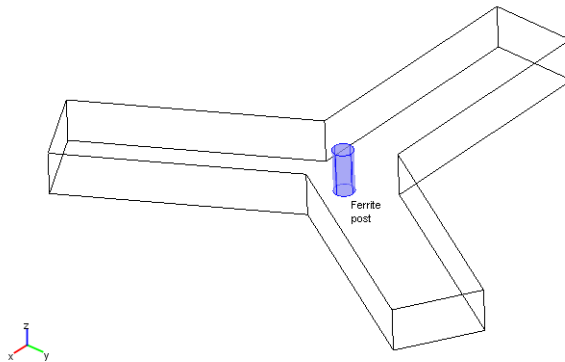


Figure 1: Geometry of the three-port microwave circulator.

To match the junction, identical dielectric tuning elements are inserted into each branch (not shown above). The ferrite post is magnetized by a static H_0 bias field along the axis. The bias field is usually supplied by external permanent magnets. Here, the focus is on the modeling of the ferrite and how to minimize reflections at the inport by matching the junction by the proper choice of tuning elements. For a general introduction to the modeling of rectangular waveguide structures, see the Waveguide H-bend model on page 20 in the *RF Module User's Guide* and the corresponding 2D

model found on page 60 in this document. Matching the circulator junction involves calculating how well a TE_{10} wave propagates between ports in the circulator for different materials in the tuning element. This is done by calculating the scattering parameters, or S-parameters, of the structure as a function of the permittivity of the tuning elements for the fundamental TE_{10} mode. The S-parameters are a measure of the transmittance and reflectance of the circulator. For a theoretical background on S-parameters, see the section “S-Parameters” on page 67 in the *RF Module User’s Guide*.

This model only includes the TE_{10} mode of the waveguide. Thus the model can be made in 2D as the fields of the TE_{10} mode have no variation in the transverse direction. Figure 2 shows the 2D geometry including the dielectric tuning elements.

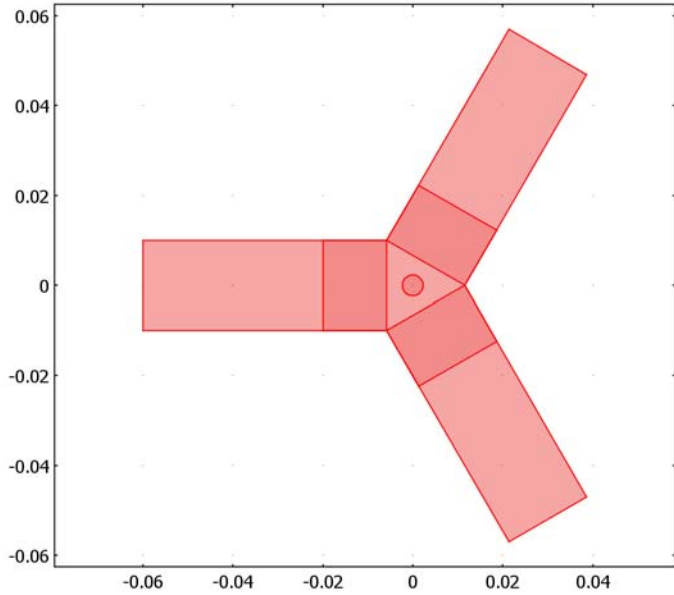


Figure 2: 2D geometry with dielectric tuning elements.

Model Definition

The dependent variable in this application mode is the z component of the electric field \mathbf{E} . It obeys the following relation:

$$\nabla \times (\mu_r^{-1} \nabla \times \mathbf{E}_z) - \left(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0} \right) k_0^2 \mathbf{E}_z = 0$$

where μ_r denotes the relative permeability, ω the angular frequency, σ the conductivity, ϵ_0 the permittivity of vacuum, ϵ_r the relative permittivity, and k_0 is the free space wave number. Losses are neglected so the conductivity is zero everywhere. The magnetic permeability is of key importance in this model as it is the anisotropy of this parameter that is responsible for the nonreciprocal behavior of the circulator. For the theory of the magnetic properties of ferrites, see Ref. 1 and Ref. 2. The model assumes that the static H_0 magnetic bias field is much stronger than the alternating magnetic field of the microwaves, so the quoted results are a linearization for a small signal analysis around this operating point. Further assume that the applied magnetic bias field is strong enough for the ferrite to be in magnetic saturation. Under those assumptions and neglecting losses, the anisotropic permeability of a ferrite magnetized in the positive z -direction is given by:

$$[\mu] = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$

where

$$\kappa = \mu_0 \left(\frac{\omega\omega_m}{\omega_0^2 - \omega^2} \right)$$

$$\mu = \mu_0 \left(1 + \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2} \right)$$

and

$$\omega_0 = \mu_0\gamma H_0$$

$$\omega_m = \mu_0\gamma M_s$$

Here μ_0 denotes the permeability of free space, ω is the angular frequency of the microwave field, ω_0 is the precession frequency or Larmor frequency of a spinning electron in the applied H_0 magnetic bias field, ω_m is the electron Larmor frequency at the saturation magnetization M_s of the ferrite, and finally γ is the gyromagnetic ratio of the electron. For a lossless ferrite, the permeability clearly becomes unbounded at $\omega = \omega_0$. In a real ferrite, this resonance becomes finite and is broadened due to losses. For complete expressions including losses, the interested reader is referred to Ref. 1 and Ref. 2. In this analysis the operating frequency is chosen sufficiently off from the

Larmor frequency to avoid the singularity. The material data, $M_s = 2.39e5$ A/m and $\epsilon_r = 12.9$, are taken for magnesium ferrite from Ref. 2. The applied bias field is set to $H_0 = 2.72e5$ A/m, which is well above saturation. The electron gyromagnetic ratio is set to $1.759e11$ C/kg. Finally, the model uses an operating frequency of 10 GHz, which is well above the cutoff for the TE_{10} mode which for a waveguide cross section of 2 cm by 1 cm is at about 7.5 GHz. At the ports, matched port boundary conditions make the boundaries transparent to the wave.

Results and Discussion

The S_{11} parameter as a function of the relative permittivity of the matching elements, ϵ_{r_m} , is shown in Figure 3. The S_{11} parameter corresponds to the reflection coefficient at port 1. Thus matching the junction is equivalent to minimizing the magnitude of S_{11} .

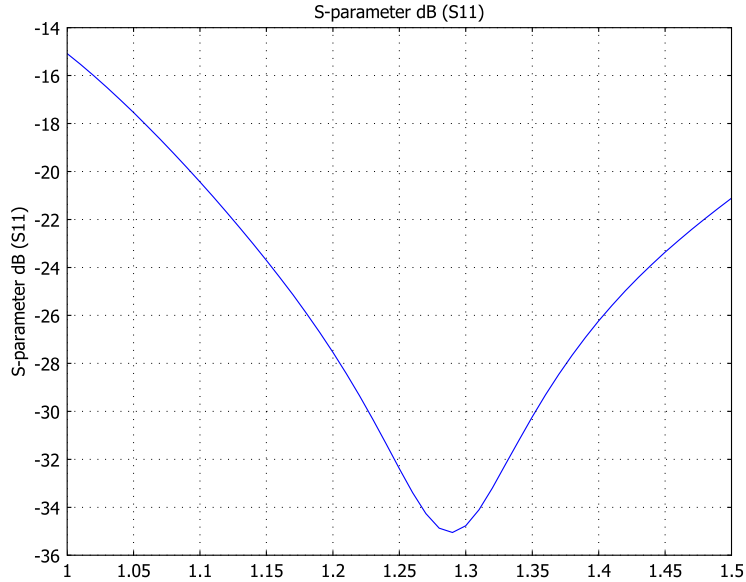
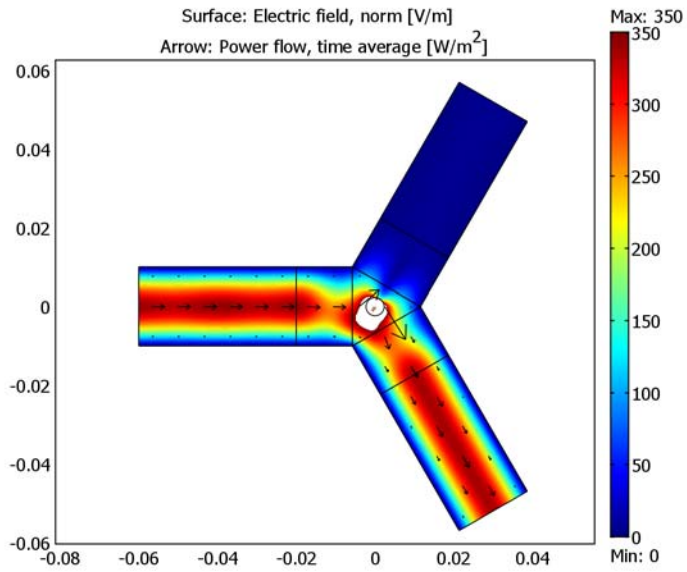


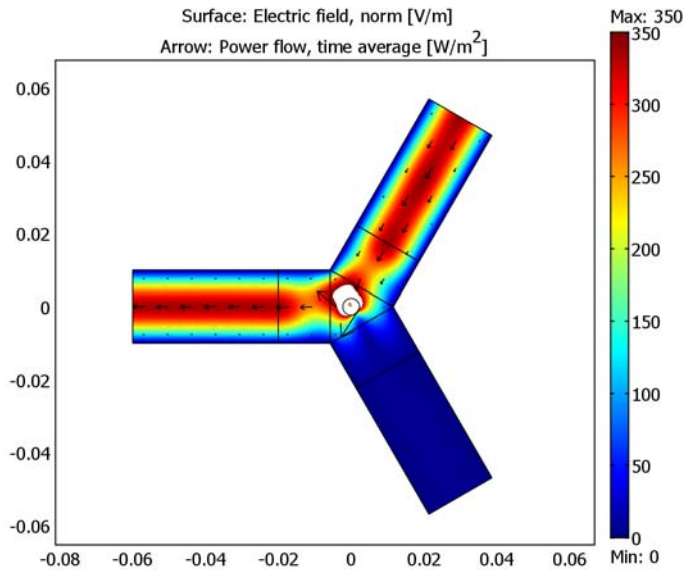
Figure 3: S_{11} parameter as a function of the relative permittivity.

By choosing ϵ_{r_m} to about 1.28, you obtain a reflection coefficient of about -35 dB, which is a good value for a circulator design. Judging from the absence of standing wave patterns in the magnitude plot of the electric field and by looking at the direction

of the microwave energy flow in the result plot below, it is clear that the circulator behaves as desired.



By feeding the circulator at a different port, the nonreciprocal behavior of the device becomes quite apparent.



As a final study, you can extract the full S-parameter matrix at the optimum value of ϵ_{ps_r} . The script `sparametermatrix.m` automatically use all ports, one at a time, as inputs and extracts the matrix. The matrix contain all the information about the relation between the ports.

$$S = \begin{bmatrix} -34,8 & -0,003 & -34,9 \\ -35,0 & -34,8 & -0,003 \\ -0,003 & -34,9 & -34,8 \end{bmatrix}$$

If you feed the signal through port 2 for example, check column 2, and you see that it is passed on to port 1 where you find the highest value.

References

1. Robert E. Collin: *Foundations for Microwave Engineering*, Second Edition, IEEE Press/Wiley-Interscience
2. David M. Pozar: *Microwave Engineering*, Third Edition, John Wiley & Sons Inc.

