SATE1110 Sähkömagneettinen kenttäteoria / tenttikaavasto

Luonnon vakioita:

Suure	Tunnus	Arvo	Yksikkö
Valonnopeus	c	$299792458 \approx 3 \cdot 10^{-8}$	m/s
Tyhjön permeabiliteetti	μ_0	$4\pi \cdot 10^{-7}$	H/m
Tyhjön permittiivisyys	ε_0	$\frac{1}{\mu_0 c^2} \approx 8{,}8542 \cdot 10^{-12} \approx \frac{10^{-9}}{36\pi}$	F/m

Sähkömagnetiikan kaavoja:

Liikkuvaan varaukseen kohdistuva voima	$\boldsymbol{F} = q\left(\boldsymbol{U} \times \boldsymbol{B}\right)$
Aineen sisäinen impedanssi	$\eta = \sqrt{\frac{\mathrm{j}\omega\mu}{\sigma + \mathrm{j}\omega\varepsilon}}$
Hyvä johde	$\sigma >> \omega\varepsilon$
Hyvä eriste	$\sigma << \omega\varepsilon$

$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)}$	$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)}$
$\Gamma_\perp = \frac{\eta_2 \cos\theta_\mathrm{s} - \eta_1 \cos\theta_\text{läp}}{\eta_2 \cos\theta_\mathrm{s} + \eta_1 \cos\theta_\text{läp}}$	$\Gamma_\parallel = \frac{\eta_2 \cos\theta_\text{läp} - \eta_1 \cos\theta_\mathrm{s}}{\eta_2 \cos\theta_\text{läp} + \eta_1 \cos\theta_\mathrm{s}}$
$\alpha_\text{Nep/m} \cdot 20\log\left(\mathrm{e}^1\right) = \alpha_\text{dB/m}$ dB/m	$\varepsilon = \varepsilon_0\left(\varepsilon_\mathrm{r} + \frac{\sigma}{\mathrm{j}\omega\varepsilon_0}\right)$
$\delta = \alpha^{-1}$	

Trigonometrian kaavoja:

$\sin^2 x + \cos^2 x = 1$	$\tan\left(x \pm y\right) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
$\sin(x \pm y) = \sin x \cos x \pm \cos x \sin y$	$\cos(x \pm y) = \cos x \cos x \mp \sin x \sin y$
$\sin 2x = 2\sin x \cos x$	$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
$\sin x + \sin y = 2\sin\left(\frac{1}{2}\cdot\left(x+y\right)\right)\cdot\cos\left(\frac{1}{2}\cdot\left(x-y\right)\right)$ $\sin x - \sin y = 2\cos\left(\frac{1}{2}\cdot\left(x+y\right)\right)\cdot\sin\left(\frac{1}{2}\cdot\left(x-y\right)\right)$	$\cos x + \cos y = 2\cos\left(\frac{1}{2}\cdot\left(x+y\right)\right)\cdot\cos\left(\frac{1}{2}\cdot\left(x-y\right)\right)$ $\cos x - \cos y = -2\sin\left(\frac{1}{2}\cdot\left(x+y\right)\right)\cdot\sin\left(\frac{1}{2}\cdot\left(x-y\right)\right)$

Vektorimatematiikan kaavoja:

$$\boldsymbol{A}\cdot\boldsymbol{B}\times\boldsymbol{C}=\boldsymbol{B}\cdot\boldsymbol{C}\times\boldsymbol{A}=\boldsymbol{C}\cdot\boldsymbol{A}\times\boldsymbol{B}$$

$$\boldsymbol{A}\times\left(\boldsymbol{B}\times\boldsymbol{C}\right)=\boldsymbol{B}\left(\boldsymbol{A}\cdot\boldsymbol{C}\right)-\boldsymbol{C}\left(\boldsymbol{A}\cdot\boldsymbol{B}\right)$$

$$\vec{\nabla}\left(U+V\right)=\vec{\nabla}U+\vec{\nabla}V$$

$$\vec{\nabla}\cdot\left(\boldsymbol{A}+\boldsymbol{B}\right)=\vec{\nabla}\cdot\boldsymbol{A}+\vec{\nabla}\cdot\boldsymbol{B}$$

$$\vec{\nabla}\times\left(\boldsymbol{A}+\boldsymbol{B}\right)=\vec{\nabla}\times\boldsymbol{A}+\vec{\nabla}\times\boldsymbol{B}$$

$$\vec{\nabla}\cdot\left(V\boldsymbol{A}\right)=\left(\vec{\nabla}V\right)\cdot\boldsymbol{A}+V\left(\vec{\nabla}\cdot\boldsymbol{A}\right)$$

$$\vec{\nabla}\times\left(V\boldsymbol{A}\right)=\left(\vec{\nabla}V\right)\times\boldsymbol{A}+V\left(\vec{\nabla}\times\boldsymbol{A}\right)$$

$$\vec{\nabla}\cdot\left(\boldsymbol{A}\times\boldsymbol{B}\right)=\boldsymbol{B}\cdot\left(\vec{\nabla}\times\boldsymbol{A}\right)-\boldsymbol{A}\cdot\left(\vec{\nabla}\times\boldsymbol{B}\right)$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}V\right)=\vec{\nabla}^2V$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\boldsymbol{A}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\boldsymbol{A}\right)-\vec{\nabla}^2\boldsymbol{A}$$

$$\vec{\nabla}\times\left(\vec{\nabla}V\right)=0$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\boldsymbol{A}\right)=0$$

$$\int_V\vec{\nabla}\cdot\boldsymbol{A}\,\mathrm{d}V=\oint_S\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{S}\quad\left(\text{Divergenssiteoreema}\right)$$

$$\int_S\vec{\nabla}\times\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{S}=\oint_C\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{l}\quad\left(\text{Stokesin teoreema}\right)$$

Gradientti, divergenssi, roottori ja Laplacen operaattorit

Karteesisessa koordinaatistossa (x, y, z)

$$\vec{\nabla} V = \frac{\partial V}{\partial x}\boldsymbol{e}_x + \frac{\partial V}{\partial y}\boldsymbol{e}_y + \frac{\partial V}{\partial z}\boldsymbol{e}_z$$

$$\vec{\nabla} \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \boldsymbol{A} = \begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right)\boldsymbol{e}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right)\boldsymbol{e}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)\boldsymbol{e}_z$$

$$\vec{\nabla}^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Sylinterikoordinaatistossa (ρ, φ, z)

$$\vec{\nabla} V = \frac{\partial V}{\partial \rho}\boldsymbol{e}_\rho + \frac{\partial V}{\rho \partial \varphi}\boldsymbol{e}_\varphi + \frac{\partial V}{\partial z}\boldsymbol{e}_z$$

$$\vec{\nabla} \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho A_\rho\right) + \frac{\partial A_\varphi}{\rho \partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \boldsymbol{A} = \frac{1}{\rho}\begin{vmatrix} \boldsymbol{e}_\rho & \rho \boldsymbol{e}_\varphi & \boldsymbol{e}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\varphi & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\rho \partial \varphi} - \frac{\partial A_\varphi}{\partial z} \right)\boldsymbol{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right)\boldsymbol{e}_\varphi + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}\left(\rho A_\varphi\right) - \frac{\partial A_\rho}{\partial \varphi} \right)\boldsymbol{e}_z$$

$$\vec{\nabla}^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$$

Pallokoordinaatistossa (r, θ, φ)

$$\vec{\nabla} V = \frac{\partial V}{\partial r}\boldsymbol{e}_r + \frac{\partial V}{r \partial \theta}\boldsymbol{e}_\theta + \frac{1}{r \sin\theta}\frac{\partial V}{\partial \varphi}\boldsymbol{e}_\varphi$$

$$\vec{\nabla} \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}\left(A_\theta \sin\theta\right) + \frac{1}{r \sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

$$\vec{\nabla} \times \boldsymbol{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \boldsymbol{e}_r & r\boldsymbol{e}_\theta & \left(r \sin\theta\right)\boldsymbol{e}_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & rA_\theta & \left(r \sin\theta\right)A_\varphi \end{vmatrix}$$

$$= \frac{1}{r \sin\theta}\left(\frac{\partial}{\partial \theta}\left(A_\varphi \sin\theta\right) - \frac{\partial A_\theta}{\partial \varphi} \right)\boldsymbol{e}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}\left(rA_\varphi\right) \right)\boldsymbol{e}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial \theta} \right)\boldsymbol{e}_\varphi$$

$$\vec{\nabla}^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2}$$