Vaasan yliopisto, kevät 2019

Lineaarialgebra II, MATH.1240

3. harjoitus, (viikko 12, 18.3.–22.3.2019)

R01	ma	12-14	F425
R02	to	12-14	F425
R03	to	14-16	F425

1. Määritä käänteismatriisi matriisille $\boldsymbol{A} = \begin{pmatrix} 7 & 4 \\ 3 & 2 \end{pmatrix}$,

a) käyttäen rivioperaatioita;
b) käyttäen adjungaattia.
Ratkaisu: a)

$$
\begin{aligned}
&\left(\begin{array}{cc|cc} 7 & 4 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right)\begin{array}{|l} \leftarrow + \\ *(-2) \end{array} \\
\sim &\left(\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 3 & 2 & 0 & 1 \end{array}\right)\begin{array}{|l} *(-3) \\ \leftarrow + \end{array} \\
\sim &\left(\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 0 & 2 & -3 & 7 \end{array}\right)\begin{array}{|l} \\ *(0.5) \end{array} \\
\sim &\left(\begin{array}{cc|cc} 1 & 0 & 1.0 & -2.0 \\ 0 & 1 & -1.5 & 3.5 \end{array}\right) \qquad \longrightarrow \boldsymbol{A}^{-1} = \begin{pmatrix} 1.0 & -2.0 \\ -1.5 & 3.5 \end{pmatrix}
\end{aligned}
$$

b) Minorit

$$
\begin{aligned}
m_{11} &= 2, \quad m_{12} = 3 \\
m_{21} &= 4, \quad m_{22} = 7
\end{aligned}
$$

$$
\boldsymbol{A}^{-1} = \frac{1}{\det(\boldsymbol{A})}\mathrm{Adj}(\boldsymbol{A}) = \frac{1}{\det(\boldsymbol{A})}\begin{pmatrix} m_{11} & -m_{21} \\ -m_{12} & m_{22} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & -4 \\ -3 & 7 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -3/2 & 7/2 \end{pmatrix}
$$

Vastaus: $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ -1.5 & 3.5 \end{pmatrix}$

Tarkistus:

$$
\boldsymbol{A}^{-1}\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ -1.5 & 3.5 \end{pmatrix}\begin{pmatrix} 7 & 4 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{Ok}
$$

$$
\boldsymbol{A}^{-1}\boldsymbol{A} = \begin{pmatrix} 7 & 4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -1.5 & 3.5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{Ok}
$$

---

2. Ratkaise yhtälö:

$$
\begin{vmatrix} 1-x & 2 \\ 3 & 2-x \end{vmatrix} = 0.
$$

Ratkaisu:

$$\begin{aligned}
\begin{vmatrix} 1-x & 2 \\ 3 & 2-x \end{vmatrix} &= 0 \\
\Leftrightarrow (1-x)(2-x) - 2\cdot 3 &= 0 \\
\Leftrightarrow x^2 - 3x - 4 &= 0 \\
\Leftrightarrow x &= \frac{3 \pm \sqrt{3^2 - 4\cdot 1\cdot(-4)}}{2\cdot 1} = \frac{3\pm 5}{2} \\
\Leftrightarrow x &= 4 \quad \text{tai} \quad x = -1
\end{aligned}$$

Vastaus: $x = 4$ tai $x = -1$.
Tarkistus:

$$x = 4: \qquad \begin{vmatrix} 1-4 & 2 \\ 3 & 2-4 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ 3 & -2 \end{vmatrix} = 6 - 6 = 0 \qquad \text{Ok}$$

$$x = -1: \qquad \begin{vmatrix} 1-(-1) & 2 \\ 3 & 2-(-1) \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix} = 6 - 6 = 0 \qquad \text{Ok}$$

---

3. Ratkaise yhtälö:

$$\begin{vmatrix} 3 & 4 & 5 \\ 7 & 7 & 7 \\ x & x+1 & 2x \end{vmatrix} = 0.$$

Ratkaisu: Kehitetään determinantti kolmannen rivin suhteen

$$\begin{aligned}
\begin{vmatrix} 3 & 4 & 5 \\ 7 & 7 & 7 \\ x & x+1 & 2x \end{vmatrix} &= 0 \\
\Leftrightarrow +x\cdot \begin{vmatrix} 4 & 5 \\ 7 & 7 \end{vmatrix} - (x+1)\cdot \begin{vmatrix} 3 & 5 \\ 7 & 7 \end{vmatrix} + 2x \cdot \begin{vmatrix} 3 & 4 \\ 7 & 7 \end{vmatrix} &= 0 \\
\Leftrightarrow +x\cdot(-7) - (x+1)\cdot(-14) + 2x\cdot(-7) &= 0 \\
\Leftrightarrow -7x + 14x + 14 - 14x &= 0 \\
\Leftrightarrow x &= 2
\end{aligned}$$

Vastaus: $x = 2$
Tarkistus:

$$\begin{aligned}
\begin{vmatrix} 3 & 4 & 5 \\ 7 & 7 & 7 \\ 2 & 3 & 4 \end{vmatrix} &= +3\cdot \begin{vmatrix} 7 & 7 \\ 3 & 4 \end{vmatrix} - 4\cdot \begin{vmatrix} 7 & 7 \\ 2 & 4 \end{vmatrix} + 5\cdot \begin{vmatrix} 7 & 7 \\ 2 & 3 \end{vmatrix} \\
&= 3\cdot(28-21) - 4\cdot(28-14) + 5\cdot(21-14) \\
&= 21 - 56 + 35 = 0 \quad \text{Ok}
\end{aligned}$$

---

4. Määritä determinantti ja käänteismatriisi matriisille $A = \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 6 & -2 & -3 \end{pmatrix}$.

Ratkaisu:
Determinantti:

$$
\begin{aligned}
\begin{vmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 6 & -2 & -3 \end{vmatrix} &= +2 \cdot \begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix} - 5 \cdot \begin{vmatrix} 6 & 4 \\ 6 & -3 \end{vmatrix} + 7 \cdot \begin{vmatrix} 6 & 3 \\ 6 & -2 \end{vmatrix} \\
&= +2 \cdot (-9 - (-8)) - 5 \cdot (-18 - 24) + 7 \cdot (-12 - 18) \\
&= +2 \cdot (-1) - 5 \cdot (-42) + 7 \cdot (-30) \\
&= -2 + 210 - 210 \\
&= -2
\end{aligned}
$$

Käänteismatriisi:

$$
\begin{aligned}
& \left(\begin{array}{ccc|ccc} (2) & 5 & 7 & 1 & 0 & 0 \\ 6 & 3 & 4 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right) \begin{array}{|c|c} *(-3) & *(-3) \\ \leftarrow + & \\ & \leftarrow + \end{array} \\
\sim & \left(\begin{array}{ccc|ccc} 2 & 5 & 7 & 1 & 0 & 0 \\ 0 & -12 & -17 & -3 & 1 & 0 \\ 0 & -17 & -24 & -3 & 0 & 1 \end{array}\right) \begin{array}{|c} \\ \leftarrow + \\ *(-1) \end{array} \\
\sim & \left(\begin{array}{ccc|ccc} 2 & 5 & 7 & 1 & 0 & 0 \\ 0 & 5 & 7 & 0 & 1 & -1 \\ 0 & -17 & -24 & -3 & 0 & 1 \end{array}\right) \begin{array}{|c|c} & \leftarrow + \\ *3 & *(-1) \\ \leftarrow + & \end{array} \\
\sim & \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & -1 & 1 \\ 0 & 5 & 7 & 0 & 1 & -1 \\ 0 & -2 & -3 & -3 & 3 & -2 \end{array}\right) \begin{array}{|c} \\ \leftarrow + \\ *2 \end{array} \\
\sim & \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & -1 & 1 \\ 0 & (1) & 1 & -6 & 7 & -5 \\ 0 & -2 & -3 & -3 & 3 & -2 \end{array}\right) \begin{array}{c} \\ *2 \\ \leftarrow + \end{array} \\
\sim & \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -6 & 7 & -5 \\ 0 & 0 & (-1) & -15 & 17 & -12 \end{array}\right) \begin{array}{|c|c} & *0.5 \\ \leftarrow + & \\ *1 & *(-1) \end{array} \\
\sim & \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.5 & -0.5 & 0.5 \\ 0 & 1 & 0 & -21 & 24 & -17 \\ 0 & 0 & 1 & 15 & -17 & 12 \end{array}\right)
\end{aligned}
$$

Edellä rivioperaatioita tehtiin monin pienin askelin jotta tauluun saataisiin pieniä helppoja lukuja. seuraavassa sama matriisin kääntö on tehty suoraviivaisesti

$$
\left(\begin{array}{ccc|ccc} (2) & 5 & 7 & 1 & 0 & 0 \\ 6 & 3 & 4 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right)
\begin{array}{|c|c|c} *(-3) & *(-3) & *1/2 \\ \leftarrow + & & \\ & \leftarrow + & \end{array}
$$

$$
\sim \left(\begin{array}{ccc|ccc} 1 & 5/2 & 7/2 & 1/2 & 0 & 0 \\ 0 & (-12) & -17 & -3 & 1 & 0 \\ 0 & -17 & -24 & -3 & 0 & 1 \end{array}\right)
\begin{array}{|c|c|c} \leftarrow + & & \\ *(5/24) & *(-17/12) & *(-1/12) \\ & \leftarrow + & \end{array}
$$

$$
\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1/24 & -1/8 & 5/24 & 0 \\ 0 & 1 & 17/12 & 1/4 & -1/12 & 0 \\ 0 & 0 & (1/12) & 15/12 & -17/12 & 1 \end{array}\right)
\begin{array}{|c|c|c} \leftarrow + & & \\ & \leftarrow + & \\ *1/2 & *(-17) & *12 \end{array}
$$

$$
\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & -21 & 24 & -17 \\ 0 & 0 & 1 & 15 & -17 & 12 \end{array}\right)
$$

---

5. a) Määritä käänteismatriisi matriisille

$$
M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}
$$

b) Mitkä seuraavoista ominaisuuksista on matriisilla $\boldsymbol{M}$ tai sen käänteismatriisilla $\boldsymbol{M}^{-1}$. (Tarvittaessa Googlaa termien merkitys netistä.)

ominaisuus	english term	$\boldsymbol{M}$	$\boldsymbol{M}^{-1}$
alakolmio-muoto	lower triangular		
symmetrinen	symmetric		
persymmetrinen	persymmetric		
nauhamatriisi	band matrix		
Toeplitz	Toeplitz		
säännöllinen	of full Rank		

Ratkaisu: a)

$$
\left(\begin{array}{ccccc|ccccc}
(1) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1
\end{array}\right)\left|\begin{array}{l}
*(-1) \\ \leftarrow + \\ \\ \\ \\
\end{array}\right.
$$

$$
\sim \left(\begin{array}{ccccc|ccccc}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & (1) & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1
\end{array}\right)\left|\begin{array}{l}
\\ *(-1) \\ \leftarrow + \\ \\ \\
\end{array}\right.
$$

$$
\sim \left(\begin{array}{ccccc|ccccc}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & (1) & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1
\end{array}\right)\left|\begin{array}{l}
\\ \\ *(-1) \\ \leftarrow + \\ \\
\end{array}\right.
$$

$$
\sim \left(\begin{array}{ccccc|ccccc}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & (1) & 0 & -1 & 1 & -1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1
\end{array}\right)\left|\begin{array}{l}
\\ \\ \\ *(-1) \\ \leftarrow +
\end{array}\right.
$$

$$
\sim \left(\begin{array}{ccccc|ccccc}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 1 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 & 1
\end{array}\right)
$$

Siis

$$
\boldsymbol{M} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1
\end{pmatrix} \quad \longrightarrow \quad \boldsymbol{M}^{-1} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 \\
1 & -1 & 1 & 0 & 0 \\
-1 & 1 & -1 & 1 & 0 \\
1 & -1 & 1 & -1 & 1
\end{pmatrix}
$$

b)

Alakolmio-muoto (A lower triangular form) A square matrix is called lower triangular if all the entries above the main diagonal are zero.
Kaikki päädiagonalin yläpuolella olevat luvut ovat nollia.

Symmetrinen matriisi (A symmetric matrix) A symmetric matrix is a square matrix that is equal to its transpose.
$\boldsymbol{A} = \boldsymbol{A}^T$.

Persymmetrinen matriisi (A persymmetric matrix) Sanalle löytyy kaksi tulkintaa:
(1) A square matrix which is symmetric with respect to the northeast-to-southwest diagonal; or

(2) a square matrix such that the values on each line perpendicular to the main diagonal are the same for a given line (called also as a Hankel matrix).
(1) Neliömatriisi on persymmetrinen, jos se on symmetrinen oikealta ylhäältä vasemmalle alas kulkevan diagonaalin suhteen (esim $\boldsymbol{A}_1$), tai
(2) $n \times n$ neliömatriisi on persymmetrinen (hankel matriisi) jos

$$a_{i,j} = a_{i+p,j-p}, \forall i, j = 1, \ldots n, p \in \mathbb{Z}, \text{ joille}$$

i ja $i+p$ ovat kelvolliset rivi-indeksit ja j ja $j-p$ ovat kelvolliset sarakeindeksit. (esim $\boldsymbol{A}_2$)

$$\boldsymbol{A}_1 = \begin{pmatrix} 2 & -1 & 3 \\ 3 & 4 & -1 \\ -1 & 3 & 2 \end{pmatrix}, \qquad \boldsymbol{A}_2 = \begin{pmatrix} 2 & -1 & 4 \\ -1 & 4 & 2 \\ 4 & 2 & 5 \end{pmatrix},$$

Nauhamatriisi (A band matrix) A band matrix is a sparse matrix whose non-zero entries are confined to a diagonal band, comprising the main diagonal and zero or more diagonals on either side. (A matrix is called a band matrix or banded matrix if its bandwidth is reasonably small.)
$a_{ij} = 0$, jos $|i-j| > r$

$$\boldsymbol{A}_3 = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 4 & 2 & 0 & 0 \\ 0 & 2 & 5 & 7 & 0 \\ 0 & 0 & -4 & 1 & 2 \\ 0 & 0 & 0 & 2 & 3 \end{pmatrix}$$

Toeplitz matriisi (A Toeplitz matrix) a Toeplitz matrix or diagonal-constant matrix is a matrix in which each descending diagonal from left to right is constant.
Jokaisella diagonaalilla, ovat kaikki luvut samoja.

$$\boldsymbol{A}_3 = \begin{pmatrix} 2 & -1 & 0 & 7 & 0 \\ 3 & 2 & -1 & 0 & 7 \\ 4 & 3 & 2 & -1 & 0 \\ 9 & 4 & 3 & 2 & -1 \\ 2 & 9 & 4 & 3 & 2 \end{pmatrix}$$

Säännöllinen matriisi (A invertible matrix, A matrix of full rank) A matrix is full row rank when each of the rows of the matrix are linearly independent and full column rank when each of the columns of the matrix are linearly independent. For a square matrix these two concepts are equivalent and we say the matrix is full rank if all rows and columns are linearly independent. A square matrix is full rank if and only if its determinant is nonzero.
Säännöllinen matriisi on neliömatriisi, jolla on käänteismatriisi, säännöllisen matriisin sarakkeet muodostavat lineaarisesti riippumattoman systeemin, säännöllisen matriisin determinantti on nollasta eriävä.

ominaisuus	english term	$\boldsymbol{M}$	$\boldsymbol{M}^{-1}$
alakolmio-muoto	lower triangular	kyllä	kyllä
symmetrinen	symmetric	ei	ei
persymmetrinen (1)	persymmetric	kyllä	kyllä
Hankel (persymm.2)	Hankel	ei	ei
nauhamatriisi	band matrix	kyllä	ei
Toeplitz	Toeplitz	kyllä	kyllä
säännöllinen	of full Rank	kyllä	kyllä

---

6. Ratkaise $\boldsymbol{X}$ yhtälöstä $\boldsymbol{AXB}^T = \boldsymbol{C}$, missä

$$\boldsymbol{A} = \begin{pmatrix} -3 & 1 \\ 1 & 3 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 5 & 3 \\ 6 & 4 \end{pmatrix}, \quad \boldsymbol{C} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}.$$

Voit hyödyntää kaavaa:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Ratkaisu: Kaavan mukaan

$$\boldsymbol{A}^{-1} = \begin{pmatrix} -3 & 1 \\ 1 & 3 \end{pmatrix}^{-1} = \frac{1}{-3\cdot 3 - 1\cdot 1} \begin{pmatrix} 3 & -1 \\ -1 & -3 \end{pmatrix} = \begin{pmatrix} -0.3 & 0.1 \\ 0.1 & 0.3 \end{pmatrix}$$

$$\left(\boldsymbol{B}^T\right)^{-1} = \begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{1}{5\cdot 4 - 6\cdot 3} \begin{pmatrix} 4 & -6 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1.5 & 2.5 \end{pmatrix}$$

Nyt:

$$\begin{aligned}
&\boldsymbol{AXB}^T = \boldsymbol{C} \\
&\Leftrightarrow \boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{C}\left(\boldsymbol{B}^T\right)^{-1} \\
&\qquad = \begin{pmatrix} -0.3 & 0.1 \\ 0.1 & 0.3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ -1.5 & 2.5 \end{pmatrix} \\
&\qquad = \begin{pmatrix} -0.3 & 0.1 \\ 0.1 & 0.3 \end{pmatrix} \begin{pmatrix} 0.5 & -0.5 \\ 1.5 & -2.5 \end{pmatrix} \\
&\qquad = \begin{pmatrix} 0 & -0.1 \\ 0.5 & -0.8 \end{pmatrix}
\end{aligned}$$

Vastaus: $\boldsymbol{X} = \begin{pmatrix} 0 & -0.1 \\ 0.5 & -0.8 \end{pmatrix}$.

Tarkistus:

$$\begin{aligned}
\boldsymbol{AXB}^T &= \begin{pmatrix} -3 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 0 & -0.1 \\ 0.5 & -0.8 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix} \\
&= \begin{pmatrix} -3 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} -0.3 & -0.4 \\ 0.1 & -0.2 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \qquad \text{OK}
\end{aligned}$$

---

7. a) Ratkaise seuraava yhtälöryhmä rivioperaatioilla:

$$\begin{cases} \begin{array}{rcrcrcr} 2x & + & 2y & + & 4z & = & 14 \\ -2x & - & y & + & 3z & = & 6 \\ 6x & - & 2y & + & 4z & = & 2 \end{array} \end{cases}$$

b) Ratkaise a-kohdan yhtälöryhmä käyttäen hyväksi kerroinmatriisin LU-hajoitelmaa

$$\begin{pmatrix} 2 & 2 & 4 \\ -2 & -1 & 3 \\ 6 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -8 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 48 \end{pmatrix}$$

Ratkaisu:
a)

$$\left(\begin{array}{rrr|r} 2 & 2 & 4 & 14 \\ -2 & -1 & 3 & 6 \\ 6 & -2 & 4 & 2 \end{array}\right) \begin{array}{|r|r} *1 & *(-3) \\ \leftarrow + & \\ & \leftarrow + \end{array}$$

$$\sim \left(\begin{array}{rrr|r} 2 & 2 & 4 & 14 \\ 0 & 1 & 7 & 20 \\ 0 & -8 & -8 & -40 \end{array}\right) \begin{array}{|r} \\ *8 \\ \leftarrow + \end{array}$$

$$\sim \left(\begin{array}{rrr|r} 2 & 2 & 4 & 14 \\ 0 & 1 & 7 & 20 \\ 0 & 0 & 48 & 120 \end{array}\right) \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

$$\begin{aligned} (3) &\rightarrow z = 2.5 \\ (2) &\rightarrow y + 7 \cdot 2.5 = 20 \\ &\leftrightarrow y = 2.5 \\ (1) &\rightarrow 2x + 2 \cdot 2.5 + 4 \cdot 2.5 = 14 \\ &\leftrightarrow x = -0.5 \end{aligned}$$

Vastaus: $x = -1$, $y = 2.5$, $z = 2.5$
b)

$$
\begin{aligned}
&\begin{pmatrix} 2 & 2 & 4 \\ -2 & -1 & 3 \\ 6 & -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ 6 \\ 2 \end{pmatrix} \\
\Leftrightarrow &\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -8 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 2 & 2 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 48 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{=\begin{pmatrix} u & v & w \end{pmatrix}^T} = \begin{pmatrix} 14 \\ 6 \\ 2 \end{pmatrix} \\
\Leftrightarrow &\begin{cases} \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -8 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 14 \\ 6 \\ 2 \end{pmatrix} \\ \begin{pmatrix} 2 & 2 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 48 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \end{cases}
\end{aligned}
$$

Nyt

$$
\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -8 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 14 \\ 6 \\ 2 \end{pmatrix} \qquad \begin{matrix} (1) \\ (2) \\ (3) \end{matrix}
$$

$$
\begin{aligned}
(1) &\to u = 14 \\
(2) &\to -14 + v = 6 \\
&\leftrightarrow v = 20 \\
(3) &\to 3 \cdot 14 - 8 \cdot 20 + w = 2 \\
&\leftrightarrow w = 120
\end{aligned}
$$

$$
\begin{pmatrix} 2 & 2 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 48 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ 20 \\ 120 \end{pmatrix} \qquad \begin{matrix} (4) \\ (5) \\ (6) \end{matrix}
$$

$$
\begin{aligned}
(6) &\to z = 120/4802.5 \\
(5) &\to y + 7 \cdot 2.5 = 20 \\
&\leftrightarrow y = 2.5 \\
(4) &\to 2x + 2 \cdot 2.5 + 4 \cdot 2.5 = 14 \\
&\leftrightarrow x = -0.5
\end{aligned}
$$
