

4. Ratkaise

$$\begin{aligned} \min \quad & g(x, y) = x^2 + 2y + 3y^2 \\ \text{ehdolla} \quad & x + y = 2 \end{aligned}$$

Lagrangen funktio $L = x^2 + 2y + 3y^2 + \lambda(x + y - 2)$

$$\begin{cases} L_x = 0 \\ L_y = 0 \\ L_\lambda = 0 \end{cases} \Leftrightarrow \begin{cases} 2x + \lambda = 0 \\ 2 + 6y + \lambda = 0 \\ x + y - 2 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x + \lambda = 0 \\ 6y + \lambda = -2 \\ x + y = 2 \end{cases}$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} 2 & 0 & 1 & -2 \\ 0 & 6 & 1 & -2 \\ 1 & 1 & 0 & 2 \end{array} \right) \begin{array}{l} \cdot 2 \\ \cdot 3 \\ \cdot 2 \end{array} \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & 1 & -4 \\ 0 & 6 & 1 & -2 \end{array} \right) \begin{array}{l} \\ \cdot 3 \\ \cdot 2 \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & 1 & -4 \\ 0 & 0 & 4 & -14 \end{array} \right) \rightarrow \begin{cases} x + y = 2 & (1) \\ -2y + \lambda = -4 & (2) \\ 4\lambda = -14 & (3) \end{cases}$$

$$(3) \rightarrow \lambda = -3,5$$

$$(2) \rightarrow -2y + (-3,5) = -4 \rightarrow y = 0,25$$

$$(1) \rightarrow x + 0,25 = 2 \rightarrow x = 1,75$$

Vastaus:

$$\begin{cases} x^* = 1,75 \\ y^* = 0,25 \\ g(x^*, y^*) = 3,75 \end{cases}$$

$$g(1,75; 0,25) = 1,75^2 + 2 \cdot 0,25 + 3 \cdot 0,25^2 = 3,75$$

II tapa

$$\begin{aligned} y = 2 - x \rightarrow g(x, y) &= x^2 + 2(2 - x) + 3(2 - x)^2 \\ &= x^2 + 4 - 2x + 12 - 12x + 3x^2 \\ &= 4x^2 - 14x + 16 = \hat{g}(x) \end{aligned}$$

$$\begin{aligned} \hat{g}'(x) &= 8x - 14 \\ \hat{g}''(x) &= 8 > 0 \end{aligned}$$

minimikohda $8x^* - 14 = 0$

$$x^* = 1,75$$

$$y^* = 2 - x^* = 0,25$$