Vaasan yliopisto, kevät 2014 / ORMS1010

Matemaattinen Analyysi

2 ~~1.~~ **harjoitus, viikko 11**

R1	ma	16–18	D115	(9.3.)
R2	ke	12–14	B209	(11.3.)

1. a) Minkä kokoinen on matriisi $\boldsymbol{M}$, jolle

$$\boldsymbol{M}\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}1\\7\end{pmatrix},\quad \boldsymbol{M}\begin{pmatrix}1\\-1\\0\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}\quad \text{ja}\quad \boldsymbol{M}\begin{pmatrix}0\\-1\\1\end{pmatrix}=\begin{pmatrix}-1\\3\end{pmatrix}.$$

b) Määritä kaikki matriisissa $\boldsymbol{M}$ olevat luvut.

a) Selvästi M on 2×3 matriisi!

b) $M=\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23}\end{pmatrix}$

$$M\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}1\\7\end{pmatrix} \Leftrightarrow \begin{cases} m_{11} \quad + m_{13} = 1 & (1) \\ m_{21} \quad + m_{23} = 7 & (4)\end{cases}$$

$$M\begin{pmatrix}1\\-1\\0\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix} \rightarrow \begin{cases} m_{11}-m_{12} = 2 & (2) \\ m_{21}-m_{22} = 2 & (5)\end{cases}$$

$$M\begin{pmatrix}0\\-1\\1\end{pmatrix}=\begin{pmatrix}-1\\3\end{pmatrix} \rightarrow \begin{cases} -m_{12}+m_{13} = -1 & (3) \\ -m_{22}+m_{23} = 3 & (6)\end{cases}$$

$$\begin{array}{c c} & \begin{array}{ccc} m_{11} & m_{12} & m_{13} \end{array} \\ \begin{array}{c}(1)\\(2)\\(3)\end{array} & \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 2 \\ 0 & -1 & 1 & -1 \end{array}\right) \cdot 1 \end{array} \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & -1 \end{array}\right) \cdot 1 \cdot (-1)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 2 & -2 \end{array}\right) \begin{array}{l} \rightarrow m_{11} = 2 \\ \rightarrow m_{12} = 0 \\ \rightarrow m_{13} = -1 \end{array}$$

$$\begin{array}{c c} & \begin{array}{ccc} m_{21} & m_{22} & m_{23} \end{array} \\ \begin{array}{c}(4)\\(5)\\(6)\end{array} & \left(\begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 1 & -1 & 0 & 2 \\ 0 & -1 & 1 & 3 \end{array}\right) \cdot 1 \end{array} \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 0 & -1 & -1 & -5 \\ 0 & -1 & 1 & 3 \end{array}\right) \cdot 1 \cdot (-1)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 2 & 8 \end{array}\right) \begin{array}{l} \rightarrow m_{21} = 3 \\ \rightarrow m_{22} = 1 \\ \rightarrow m_{23} = 4 \end{array}$$

$\therefore \quad M = \begin{pmatrix} 2 & 0 & -1 \\ 3 & 1 & 4 \end{pmatrix}$

Tarkistus

$M\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -1 \\ 3 & 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ ✓

$M\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -1 \\ 3 & 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ ✓

$M\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -1 \\ 3 & 1 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ ✓

2. Olkoon

$$f(x,y) = 5 + 2x - y + x^2 + xy.$$

Laske dw/dt, kun $w(t) = f(1+t, 2-3t)$.
(ohje: voit sijoittaa $f(x,y)$:n lausekkeeseen x:n paikalle $1+t$ ja y:n paikalle $2-3t$ ja sieventtä ja derivoida lausekkeen t:n suhteen. Toinen tapa on käyttää kaavaa

$$\frac{dw}{dt} = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt}$$

Tulosten tulisi olla samat.)

(1) $w(t) = f(1+t, 2-3t) = 5 + 2(1+t) - (2-3t) + (1+t)^2 + (1+t)(2-3t)$
$= 5 + 2 + 2t - 2 + 3t + 1 + 2t + t^2 + 2 - 3t + 2t - 3t^2$
$= 8 + 6t - 2t^2$

$\rightarrow \frac{dw}{dt} = 6 - 4t$

(2) $\frac{dw}{dt} = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt}$

$= (2 + 2x + y)\cdot 1 + (-1 + x)\cdot(-3)$
$= 5 - x + y$
$= 5 - (1+t) + (2-3t)$
$= 6 - 4t$

3. Laske matriisin $\boldsymbol{B}$ ominaisarvot ja ominaisvektorit

$$\boldsymbol{B} = \begin{pmatrix} 7 & 2 \\ 2 & 4 \end{pmatrix}$$

(Ohje: Tarkista saamasi arvot arvot λ_j, $\boldsymbol{v}_j$, $(j = 1,2)$ sijoittamalla ne ominaisarvoyhtälöön $\boldsymbol{B}\boldsymbol{v}_j = \lambda_j \boldsymbol{v}_j$.)

Karakteristinen yhtälö

$$\det(B - \lambda I) = 0$$

$$\Leftrightarrow \begin{vmatrix} 7-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

$$\Leftrightarrow (7-\lambda)(4-\lambda) - 2\cdot 2 = 0$$

$$\Leftrightarrow \lambda^2 - 11\lambda + 28 - 4 = 0$$

$$\Leftrightarrow \lambda^2 - 11\lambda + 24 = 0$$

$$\Leftrightarrow (\lambda - 3)(\lambda - 8) = 0$$

$$\lambda = \frac{11 \pm \sqrt{11^2 - 4\cdot 1\cdot 24}}{2\cdot 1} = \frac{11 \pm 5}{2}$$

$$\lambda_1 = 3 \text{ ja } \lambda_2 = 8$$

$\rightarrow$ Ominaisarvot $\lambda_1 = 3$ ja $\lambda_2 = 8$

Ominaisvektorit

$\boxed{\lambda_1 = 3}$

$$B\bar{x} = \lambda_1 \bar{x} \Leftrightarrow \begin{pmatrix} 7 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 3\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} 7x_1 + 2x_2 = 3x_1 \\ 2x_1 + 4x_2 = 3x_2 \end{cases} \Leftrightarrow \begin{cases} 4x_1 + 2x_2 = 0 \\ \cancel{2x_1 + x_2 = 0} \end{cases} \rightarrow x_2 = -2x_1$$

$$\bar{x}_1 = \begin{pmatrix} a \\ -2a \end{pmatrix}, \; a \neq 0$$

$\boxed{\lambda_2 = 8}$

$$B\bar{x} = \lambda_2 \bar{x} \Leftrightarrow \begin{pmatrix} 7 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 8\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} 7x_1 + 2x_2 = 8x_1 \\ 2x_1 + 4x_2 = 8x_2 \end{cases} \Leftrightarrow \begin{cases} -x_1 + 2x_2 = 0 \\ \cancel{2x_1 - 4x_2 = 0} \end{cases} \rightarrow x_1 = 2x_2$$

$$\bar{x}_2 = \begin{pmatrix} 2b \\ b \end{pmatrix}$$

$$V: \begin{cases} \lambda_1 = 3, & \bar{x}_1 = \begin{pmatrix} a \\ -2a \end{pmatrix}, \; a \neq 0 \\ \lambda_2 = 8, & \bar{x}_2 = \begin{pmatrix} 2b \\ b \end{pmatrix}, \; b \neq 0 \end{cases}$$

4. Laske matriisin $\boldsymbol{S}$ ominaisarvot ja ominaisvektorit, kun

$$\boldsymbol{S} = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

(Ohje: Tarkista saamasi arvot arvot λ_j, $\boldsymbol{v}_j$, $(j = 1,2)$ sijoittamalla ne ominaisarvoyhtälöön $\boldsymbol{S}\boldsymbol{v}_j = \lambda_j \boldsymbol{v}_j$.)

Ominaisarvot

$$\begin{vmatrix} 2-\lambda & 3 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & -1-\lambda \end{vmatrix} = 0$$

$$\Leftrightarrow \quad 0 - 0 + (-1-\lambda)\begin{vmatrix} 2-\lambda & 3 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$$\Leftrightarrow \quad -(\lambda+1)(\lambda-2)(\lambda-1) = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = 1, \quad \lambda_3 = 2$$

Ominaisvektorit

$\boxed{\lambda_1 = -1}$ $\quad S\bar{x} = -1 \cdot \bar{x} \Leftrightarrow \begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = -1 \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\Leftrightarrow \begin{cases} 2x_1 + 3x_2 = -x_1 \\ x_2 = -x_2 \\ x_1 - x_3 = -x_3 \end{cases} \Leftrightarrow \begin{cases} 3x_1 + 3x_2 = 0 \\ 2x_2 = 0 & \rightarrow x_2 = 0 \\ x_1 = 0 & \rightarrow x_1 = 0 \end{cases}$$

x_3 vapaa

$$\therefore \quad \bar{x}_I = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}, \quad a \neq 0$$

$\boxed{\lambda_2 = 1}$ $\quad S\bar{x} = 1 \cdot \bar{x} \Leftrightarrow \begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 1 \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\Leftrightarrow \begin{cases} 2x_1 + 3x_2 = x_1 \\ x_2 = x_2 \\ x_1 - x_3 = x_3 \end{cases} \Leftrightarrow \begin{cases} x_1 + 3x_2 = 0 & \rightarrow x_1 = -3x_2 \\ 0 = 0 \\ x_1 - 2x_3 = 0 & \rightarrow x_1 = 2x_3 \end{cases}$$

$$\therefore\ \bar{x}_{II} = \begin{pmatrix} 6b \\ -2b \\ 3b \end{pmatrix},\ b \neq 0$$

$\boxed{\lambda_3 = 2}$

$$S\bar{x} = 2 \cdot \bar{x} \Leftrightarrow \begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 2\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} 2x_1 + 3x_2 = 2x_1 \\ x_2 = 2x_2 \\ x_1 - x_3 = 2x_3 \end{cases} \Leftrightarrow \begin{cases} x_2 = 0 \\ -x_2 = 0 \\ x_1 - 3x_3 = 0 \end{cases} \rightarrow \begin{matrix} x_2 = 0 \\ x_1 = 3x_3 \end{matrix}$$

$$\therefore\ \bar{x}_{III} = \begin{pmatrix} 3c \\ 0 \\ c \end{pmatrix},\ c \neq 0$$

$$V: \begin{cases} \lambda_1 = -1, & \bar{x}_I = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix},\ a \neq 0 \\ \lambda_2 = 1, & \bar{x}_{II} = \begin{pmatrix} 6b \\ -2b \\ 3b \end{pmatrix},\ b \neq 0 \\ \lambda_3 = 2, & \bar{x}_{III} = \begin{pmatrix} 3c \\ 0 \\ c \end{pmatrix},\ c \neq 0 \end{cases}$$

5. Olkoon

$$\boldsymbol{Q} = \begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ 2 & 4 & 2 & 2 & 0 \\ 1 & 2 & 1 & 1 & -1 \end{pmatrix}.$$

Lausumme vektorin $\vec{w} = (1 \quad 3 \quad 0)^T$ usealla eri tavalla matriisin $\boldsymbol{Q}$ sarakkeiden lineaarikombinaationa.

$$\vec{w} = c_1\vec{q}_1 + c_2\vec{q}_2 + \ldots + c_5\vec{q}_5 \tag{1}$$

Selvitämme ensin mitä yhtälöitä kertoimien tulee toteuttaa:

$$\left(\begin{array}{ccccc|c} [1] & 2 & 0 & 2 & 1 & 1 \\ 2 & 4 & 2 & 2 & 0 & 3 \\ 1 & 2 & 1 & 1 & -1 & 0 \end{array}\right)\begin{array}{|c|c} \cdot 2 & 1 \\ \leftarrow_- & \\ & \leftarrow_- \end{array}$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 2 & 0 & 2 & 1 & 1 \\ 0 & 0 & [2] & -2 & -2 & 1 \\ 0 & 0 & 1 & -1 & -2 & -1 \end{array}\right)\begin{array}{|c|c} & \\ \cdot 1 & \cdot 0.5 \\ & \leftarrow_- \end{array}$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 2 & 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & -2 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1.5 \end{array}\right)$$

$$\sim \left(\begin{array}{ccccc|c} [1] & 2 & 0 & 2 & 1 & 1 \\ 0 & 0 & [2] & -2 & -2 & 1 \\ 0 & 0 & 0 & 0 & [1] & 1.5 \end{array}\right)$$

$$\sim \left\{\begin{array}{rcrcrcrcrcr} c_1 & + & 2c_2 & & & + & 2c_4 & + & c_5 & = & 1 \\ & & & & 2c_3 & - & 2c_4 & - & 2c_5 & = & 1 \\ & & & & & & & & c_5 & = & 1.5 \end{array}\right.$$

$$\sim \left\{\begin{array}{rcrcrcrcrcr} c_1 & & & + & c_5 & = & 1 & - & 2c_2 & - & 2c_4 \\ & & 2c_3 & - & 2c_5 & = & 1 & & & + & 2c_4 \\ & & & & c_5 & = & 1.5 & & & & \end{array}\right.$$

a) Etsi nyt lineaarikombinaation (1) kertoimille arvot siten, että ainakin yksi kerroin on iso ($c_k \geq 100$, jollekin k.)
b) Etsi tehtävän lineaarikombinaatiolle (1) pienet kertoimet siten, että ($|c_k| \leq 2$, kaikille k.)

$$\left\{\begin{array}{l} c_1 = -0{,}5 - 2c_2 - 2c_4 \\ 2c_3 = 4 + 2c_4 \\ c_5 = 1{,}5 \end{array}\right| :2$$

$$\left\{\begin{array}{l} c_1 = -0{,}5 - 2c_2 - 2c_4 \\ c_3 = 2 + c_4 \\ c_5 = 1{,}5 \end{array}\right.$$

a) valitaan $c_2 = 0$ ja $c_4 = 100$, jolloin

$$c_1 = -0{,}5 - 2\cdot 0 - 2\cdot 100 = -202{,}5$$
$$c_3 = 2 \quad 100 = 102$$
$$c_5 = 1{,}5$$

$$\therefore \bar{c} = \begin{pmatrix} -202{,}5 \\ 0 \\ 102 \\ 100 \\ 1{,}5 \end{pmatrix}$$

b) valitaan $c_2 = 0$ ja $c_4 = -1$, jolloin

$$c_1 = -0{,}5 - 2\cdot 0 - 2\cdot(-1) = 1{,}5$$
$$c_3 = 2 + (-1) = 1$$
$$c_5 = 1{,}5$$

$$\therefore \bar{c} = \begin{pmatrix} 1{,}5 \\ 0 \\ 1 \\ -1 \\ 1{,}5 \end{pmatrix}$$

6. a) Perustele tehtävän 5 lineaarikombinaatiota (1) koskeva havainto: ”Ainakin yksi lineaarikombinaation kertoimista on negatiivinen”.
b) Etsi nyt tehtävän 5 lineaarikombinaation (1) kertoimille arvot siten, että kertoimien summa on 0 ja kertoimien neliöiden summa on niin pieni kuin mahdollista.

$$\sum_{k=1}^{5} c_k = 0, \quad \text{ja} \quad \sum_{k=1}^{5} c_k^2 = \text{pieni}.$$

a) $\begin{cases} c_1 = -0{,}5 - 2c_2 - 2c_4 \\ c_3 = 2 + c_4 \\ c_5 = 1{,}5 \end{cases}$

Jos $c_2 \geq 0$ ja $c_4 \geq 0$, niin $c_1 = -0{,}5 - 2c_2 - 2c_4 < 0$

b) $c_1 + c_2 + c_3 + c_4 + c_5$

$$\begin{aligned} &= -0{,}5 - 2c_2 - 2c_4 && = c_1 \\ &\quad + c_2 && = c_2 \\ &\quad + 2 + c_4 && = c_3 \\ &\quad + c_4 && = c_4 \\ &\quad + 1{,}5 && = c_5 \\ &= 3 - c_2 \end{aligned}$$

$\therefore \; \Sigma c_k = 0 \longrightarrow c_2 = 3$

$\longrightarrow \begin{cases} c_1 = -0{,}5 - 2\cdot 3 - 2c_4 = -6{,}5 - 2c_4 \\ \quad -6{,}5 - 2c_4 \\ c_2 = 3 \\ c_3 = 2 + c_4 \\ c_4 = c_4 \\ c_5 = 1{,}5 \end{cases}$

Silloin

$$\begin{aligned} S = \Sigma c_k^2 &= c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 \\ &= (-6{,}5 - 2c_4)^2 + 3^2 + (2 + c_4)^2 + c_4^2 + 1{,}5^2 \\ &= 6{,}5^2 + 26c_4 + 4c_4^2 + 9 + 4 + 4c_4 + c_4^2 + c_4^2 + 1{,}5^2 \\ &= 57{,}5 + 30c_4 + 6c_4^2 \end{aligned}$$

$$\frac{dS}{dc_4} = 30 + 12c_4 = 0 \iff c_4 = \frac{-30}{12} = -2{,}5$$

Siis

$$\begin{cases} c_1 = -6{,}5 - 2\cdot(-2{,}5) = -1{,}5 \\ c_2 = \qquad\qquad = 3 \\ c_3 = 2 + (-2{,}5) \quad = -0{,}5 \\ c_4 = \quad -2{,}5 \quad = -2{,}5 \\ c_5 = \qquad\qquad = 1{,}5 \end{cases}$$

$$\sum c_k^2 = 1{,}5^2 + 3^2 + 0{,}5^2 + 2{,}5^2 + 1{,}5^2 = 20$$

Entä, jos vaatimus $\sum c_k = 0$ poistetaan?

$$S = \sum c_k^2 = (-0{,}5 - 2c_2 - 2c_4)^2 + c_2^2 + (2 + c_4)^2 + c_4^2 + 1{,}5^2$$

$$= 0{,}5^2 + 4c_2^2 + 4c_4^2 + 2c_2 + 2c_4 + 8c_2c_4 + c_2^2 + 4 + 4c_4 + c_4^2 + c_4^2 + 1{,}5^2$$

$$= 0{,}5^2 + 1{,}5^2 + 4 + 2c_2 + 2c_4 + 4c_4 + 4c_2^2 + 4c_4^2 + 8c_2c_4 + c_2^2 + c_4^2 + c_4^2$$

$$= 6{,}5 + 2c_2 + 6c_4 + 5c_2^2 + 8c_2c_4 + 6c_4^2$$

$$= 6{,}5 + (2\ 6)\begin{pmatrix} c_2 \\ c_4 \end{pmatrix} + (c_2\ c_4)\begin{pmatrix} 5 & 4 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} c_2 \\ c_4 \end{pmatrix}$$

$$\frac{\partial S}{\partial c_2} = 2 + 10c_2 + 8c_4 = 0$$

$$\frac{\partial S}{\partial c_4} = 6 + 8c_2 + 12c_4 = 0$$

$$\rightarrow \begin{cases} 10c_2 + 8c_4 = -2 \\ 8c_2 + 12c_4 = -6 \end{cases} \quad \begin{vmatrix} \cdot 3 \\ \cdot (-2) \end{vmatrix} \quad \begin{vmatrix} \cdot (-4) \\ \cdot 5 \end{vmatrix}$$

$$\begin{cases} 30c_2 + 24c_4 = -6 \\ -16c_2 - 24c_4 = 12 \end{cases}$$

$$14c_2 = 6 \rightarrow c_2 = \frac{6}{14}$$

$$\begin{cases} -40c_2 - 32c_4 = 8 \\ 40c_2 + 60c_4 = -30 \end{cases}$$

$$28c_4 = -22 \rightarrow c_4 = -\frac{22}{28} = -\frac{11}{14}$$

$$\therefore \begin{cases} c_1 = -0{,}5 - 2\cdot\frac{6}{14} - 2\cdot\left(\frac{-11}{14}\right) = \frac{3}{14} \approx 0{,}214 \\ c_2 = \quad = \frac{6}{14} \approx 0{,}429 \\ c_3 = 2 + \left(\frac{-11}{14}\right) = \frac{17}{14} \approx 1{,}214 \\ c_4 = \quad = -\frac{11}{14} \approx -0{,}786 \\ c_5 = \quad = \frac{21}{14} \approx 1{,}500 \end{cases}$$

$$\sum c_k^2 = \left(\frac{3}{14}\right)^2 + \left(\frac{6}{14}\right)^2 + \left(\frac{17}{14}\right)^2 + \left(\frac{-11}{14}\right)^2 + \left(\frac{21}{14}\right)^2 = \frac{896}{196} \approx 4{,}571$$

Tarkistus

$$\frac{3}{14}\begin{pmatrix}1\\2\\1\end{pmatrix} + \frac{6}{14}\begin{pmatrix}2\\4\\2\end{pmatrix} + \frac{17}{14}\begin{pmatrix}0\\2\\1\end{pmatrix} - \frac{11}{14}\begin{pmatrix}2\\2\\1\end{pmatrix} + \frac{21}{14}\begin{pmatrix}1\\0\\-1\end{pmatrix} = \begin{pmatrix}1\\3\\0\end{pmatrix}$$