Vaasan yliopisto, kevät 2017

# **Talousmatematiikan perusteet**, ORMS1030

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**2. välikoe torstaina 6.4.2017** **A**

**Ratkaise 3 tehtävää.** Kokeessa saa olla mukana laskin ja taulukkokirja.

**A1.** Laske integraalit

$$\text{a)}\quad \int (8x^3 - 4x + 3)dx, \qquad \text{b)}\quad \int_1^3 (2 + 4x - 6x^2)dx$$

a) $\int (8x^3 - 4x + 3)dx = 2x^4 - 2x^2 + 3x + C$

b) $\int_1^3 (2 + 4x - 6x^2)dx = \Big/_1^3 (2x + 2x^2 - 2x^3)$

$= (2\cdot 3 + 2\cdot 3^2 - 2\cdot 3^3) - (2\cdot 1 + 2\cdot 1^2 - 2\cdot 1^3)$

$= (6 + 16 - 54) - (2 + 2 - 2)$

$= -32 - 2 = \underline{-34}$

**A2.** Ratkaise LP-malli

$$\begin{array}{lrcrcr} \max z = & 2x_1 & + & x_2 & & \\ \text{ehdoin} & 4x_1 & + & x_2 & \leq & 48 \\ & x_1 & + & x_2 & \leq & 15 \\ & x_1 & + & 2x_2 & \leq & 22 \\ & x_1 & & & \geq & 0 \\ & & & x_2 & \geq & 2 \end{array}$$

Anna vastauksena päätösmuuttujien arvot ja tavoitefunktion arvot optimissa.

1 raj. $4x_1 + x_2 \leq 48$ ↓ A: (7, 20), B: (12, 0)

2 raj $x_1 + x_2 \leq 15$ ↓ C: (0, 15), D: (15, 0)

3. raj $x_1 + 2x_2 \leq 22$ ↓ E: (0, 11), F: (16, 3)

4. raj $x_1 \geq 0$ → G: (0, 0)

5. raj $x_2 \geq 2$ ↑ H: (0, 2)

tavoitesuora $2x_1 + x_2 = 20$

I: (4, 12) J: (10, 0)

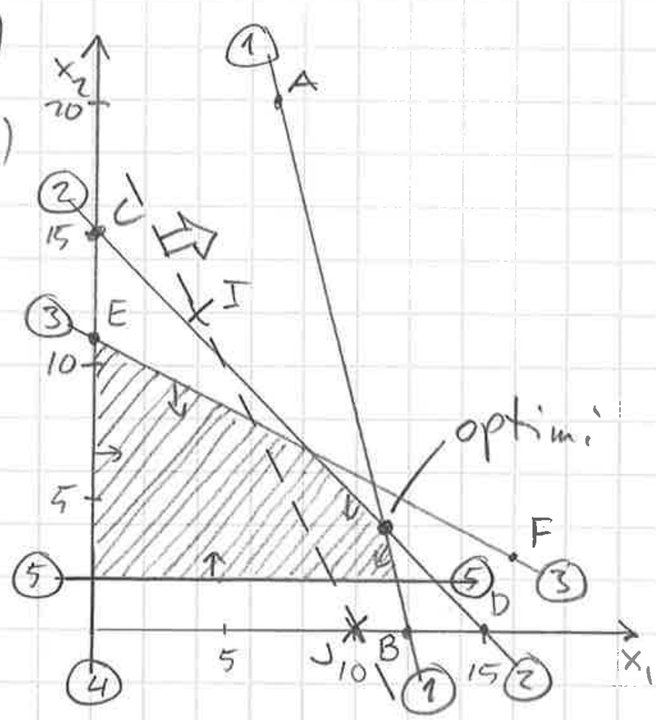
optimi $\begin{cases} ① \\ ② \end{cases} \Leftrightarrow \begin{cases} 4x_1 + x_2 = 48 \\ x_1 + x_2 = 15 \quad \cdot(-1) \end{cases}$

$\begin{cases} 3x_1 = 33 \\ x_1 + x_2 = 15 \end{cases} \Leftrightarrow \begin{cases} x_1 = 11 \\ x_2 = 4 \end{cases}$

$z = 26$

Vastaus: optimissa

$\underline{x_1 = 11,\ x_2 = 4 \text{ ja } z = 26}$



**A3.** Ratkaise yhtälöryhmä

$$\begin{cases} x + 2y + z = -5 \\ -2x - 3y - 2z = 8 \\ \quad - 2y + 2z = 8 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & -5 \\ -2 & -3 & -2 & 8 \\ 0 & -2 & 2 & 8 \end{array}\right) \begin{array}{l} \cdot 2 \\ + \\ \\ \end{array} \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & -5 \\ 0 & 1 & 0 & -2 \\ 0 & -2 & 2 & 8 \end{array}\right)$$

$$\Rightarrow \begin{cases} x + 2y + z = -5 & (1) \\ \quad y = -2 & (2) \\ \quad -2y + 2z = 8 & (3) \end{cases}$$

$(2) \to y = -2$

$(3) \to -2\cdot(-2) + 2z = 8 \to 2z = 4 \to z = 2$

$(1) \to x + 2\cdot(-2) + 2 = -5 \to x = -3$

Vastaus $x = -3,\ y = -2,\ z = 2$

tark $\begin{cases} -3 + 2\cdot(-2) + 2 = -5 & \text{ok} \\ -2\cdot(-3) - 3\cdot(-2) - 2\cdot 2 = 8 & \text{ok} \\ -2\cdot(-2) + 2\cdot 2 = 8 & \text{ok} \end{cases}$

Cramer:

$$D = \begin{vmatrix} 1 & 2 & 1 \\ -2 & -3 & -2 \\ 0 & -2 & 2 \end{vmatrix} = +1\cdot\begin{vmatrix} -3 & -2 \\ -2 & 2 \end{vmatrix} - 2\cdot\begin{vmatrix} -2 & -2 \\ 0 & 2 \end{vmatrix} + 1\cdot\begin{vmatrix} -2 & -3 \\ 0 & -2 \end{vmatrix} = (-6-4) - 2\cdot(-4-0) + (4-0) = 2.$$

$$D_1 = \begin{vmatrix} -5 & 2 & 1 \\ 8 & -3 & -2 \\ 8 & -2 & 2 \end{vmatrix} = +(-5)\begin{vmatrix} -3 & -2 \\ -2 & 2 \end{vmatrix} - 2\cdot\begin{vmatrix} 8 & -2 \\ 8 & 2 \end{vmatrix} + 1\cdot\begin{vmatrix} 8 & -3 \\ 8 & -2 \end{vmatrix} = -5\cdot(-6-4) - 2\cdot(16+16) + (-16+24) = -6$$

$$D_2 = \begin{vmatrix} 1 & -5 & 1 \\ -2 & 8 & -2 \\ 0 & 8 & 2 \end{vmatrix} = +1\cdot\begin{vmatrix} 8 & -2 \\ 8 & 2 \end{vmatrix} - (-5)\cdot\begin{vmatrix} -2 & -2 \\ 0 & 2 \end{vmatrix} + 1\cdot\begin{vmatrix} -2 & 8 \\ 0 & 8 \end{vmatrix} = (16+16) + 5\cdot(-4-0) + (-16-0) = -4$$

$$D_3 = \begin{vmatrix} 1 & 2 & -5 \\ -2 & -3 & 8 \\ 0 & -2 & 8 \end{vmatrix} = +1\cdot\begin{vmatrix} -3 & 8 \\ -2 & 8 \end{vmatrix} - 2\cdot\begin{vmatrix} -2 & 8 \\ 0 & 8 \end{vmatrix} + (-5)\cdot\begin{vmatrix} -2 & -3 \\ 0 & -2 \end{vmatrix} = (-24+16) - 2(-16-0) - 5\cdot(4-0) = 4$$

$$x = \frac{D_1}{D} = \frac{-6}{2} = -3, \quad y = \frac{D_2}{D} = \frac{-4}{2} = -2, \quad z = \frac{D_3}{D} = \frac{4}{2} = 2$$

**A4.** Olkoon

$$M = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 1 & -1 \\ -1 & 2 & 3 \end{pmatrix} \quad \text{ja} \quad Q = \begin{pmatrix} 0 & -1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

Laske $\det(M)$, $Q^T M Q$ ja $M^{-1}$.

a) $$\det(M) = \begin{vmatrix} 1 & 0 & 3 \\ -1 & 1 & -1 \\ -1 & 2 & 3 \end{vmatrix} = +1 \cdot \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} - 0 \cdot \begin{vmatrix} -1 & -1 \\ -1 & 3 \end{vmatrix} + 3 \cdot \begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix}$$
$$= (3-(-2)) - 0 + 3 \cdot (-2-(-1))$$
$$= 5 - 0 - 3 = \underline{\underline{2}}$$

b) $$Q^T M Q = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ -1 & 1 & -1 \\ -1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 0 & 2 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 5 \\ -3 & 3 \end{pmatrix}$$

c) $$\left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ -1 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \cdot 1 \; + \; + \rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 2 & 6 & 1 & 0 & 1 \end{array}\right) \cdot(-2) \; +$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 2 & -1 & -2 & 1 \end{array}\right) \; + \; + \; \cdot\left(-\tfrac{3}{2}\right) \; \cdot(-1) \; \cdot \tfrac{1}{2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/2 & 3 & -3/2 \\ 0 & 1 & 0 & 2 & 3 & -1 \\ 0 & 0 & 1 & -1/2 & -1 & 1/2 \end{array}\right) \rightarrow M^{-1} = \underline{\underline{\begin{pmatrix} 5/2 & 3 & -3/2 \\ 2 & 3 & -1 \\ -1/2 & 1 & 1/2 \end{pmatrix}}}$$

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Minorit (II tapa)

$$m_{11} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 5, \quad m_{12} = \begin{vmatrix} -1 & -1 \\ -1 & 3 \end{vmatrix} = -4, \quad m_{13} = \begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} = -1$$
$$m_{21} = \begin{vmatrix} 0 & 3 \\ 2 & 3 \end{vmatrix} = -6, \quad m_{22} = \begin{vmatrix} 1 & 3 \\ -1 & 3 \end{vmatrix} = 6, \quad m_{23} = \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} = 2$$
$$m_{31} = \begin{vmatrix} 0 & 3 \\ 1 & -1 \end{vmatrix} = -3, \quad m_{32} = \begin{vmatrix} 1 & 3 \\ -1 & -1 \end{vmatrix} = 2, \quad m_{33} = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1$$

$$M^{-1} = \frac{1}{2} \begin{pmatrix} (5) & -(-6) & (-3) \\ -(-4) & (6) & -(2) \\ (-1) & -(2) & (1) \end{pmatrix} = \begin{pmatrix} 2{,}5 & 3 & -1{,}5 \\ 2 & 3 & -1 \\ -0{,}5 & -1 & 0{,}5 \end{pmatrix}$$

**A5.** Oheisessa taulukossa on erään tuotteen hintaindeksejä. Laske hinnan keskimääräinen kasvuvauhti viiden vuoden jaksolle 2000-2005

| vuosi | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
|---|---|---|---|---|---|---|---|---|---|
| indeksi | 100 | 108 | 110 | 115 | 115 | 118 | 122 | 126 | 135 |

kasvutekijöiden keskiarvo on

$$\left(\frac{x_{2005}}{x_{2000}}\right)^{\frac{1}{5}} = \left(\frac{126}{110}\right)^{\frac{1}{5}} = 1{,}027532511$$

$$\hat{=}\ 1{,}0275$$

keskimääräinen kasvuvauhti on

2,75 % per vuosi