

2. välikoe (ylimääräinen tentti) torstai 2.5.2011

Ratkaise 3 tehtävää. Kokeessa saa olla mukana laskin ja taulukkokirja.

1. Laske determinantit

$$a = \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix}, \quad b = \begin{vmatrix} 2 & -3 & 1 \\ 6 & 5 & -1 \\ 0 & 4 & 7 \end{vmatrix}, \quad c = \begin{vmatrix} 1 & -1 & 8 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix}$$

$$a = \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix} = 2 \cdot 5 - 3 \cdot 2 = 10 - 6 = \underline{\underline{4}}$$

$$\begin{aligned} b &= \begin{vmatrix} 2 & -3 & 1 \\ 6 & 5 & -1 \\ 0 & 4 & 7 \end{vmatrix} = +2 \cdot \begin{vmatrix} 5 & -1 \\ 4 & 7 \end{vmatrix} - (-3) \begin{vmatrix} 6 & -1 \\ 0 & 7 \end{vmatrix} + 1 \cdot \begin{vmatrix} 6 & 5 \\ 0 & 4 \end{vmatrix} \\ &= 2 \cdot (35 - (-4)) + 3 \cdot (42 - 0) + (24 - 0) \\ &= 78 + 126 + 24 \\ &= \underline{\underline{228}} \end{aligned}$$

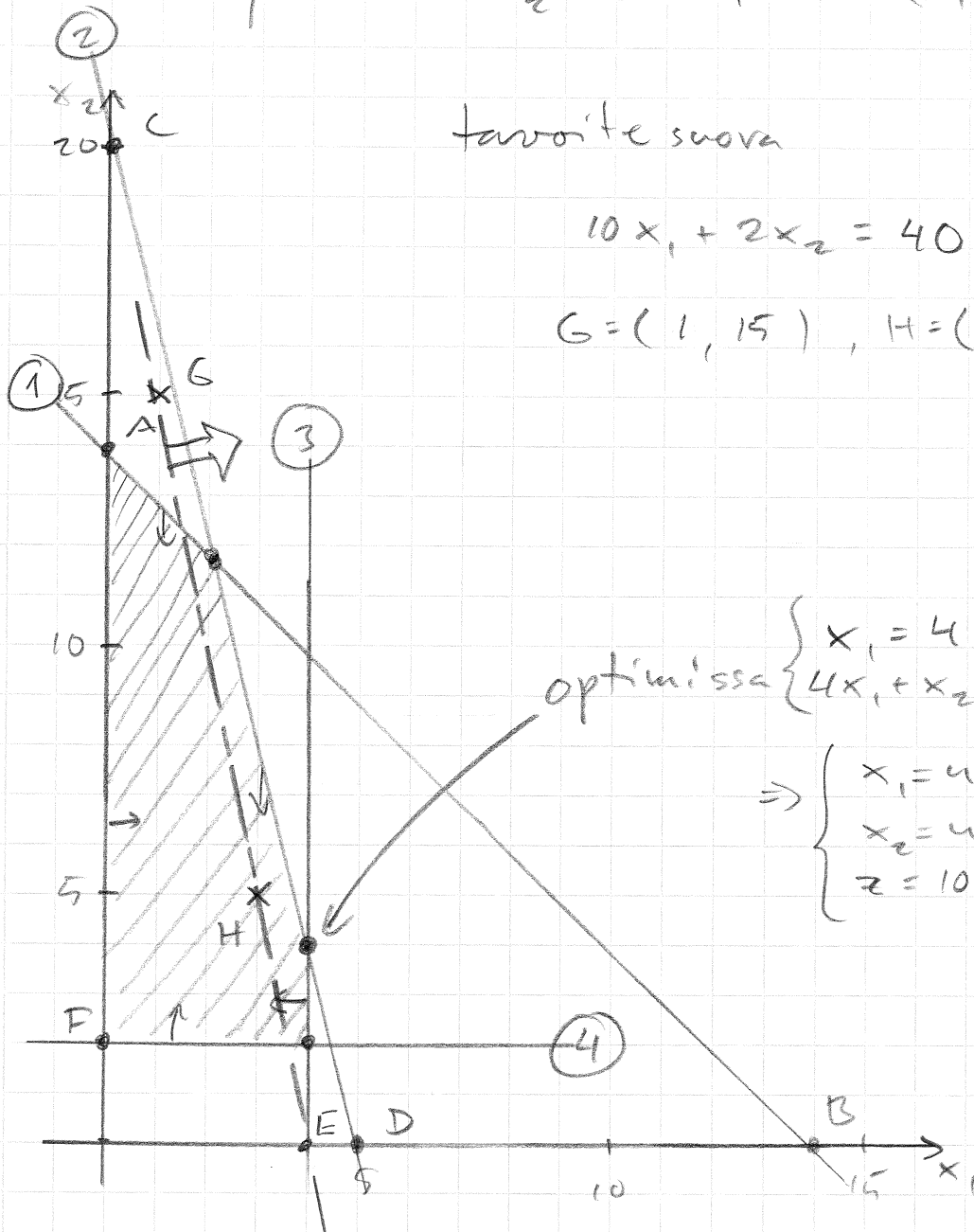
$$c = \begin{vmatrix} 1 & -1 & 8 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix} \xrightarrow{\cdot 2} \begin{vmatrix} 1 & -1 & 8 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 1 & 3 \end{vmatrix} \xrightarrow{\cdot \frac{1}{3}} \begin{vmatrix} 1 & -1 & 8 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{vmatrix} \xrightarrow{-}$$

$$= \begin{vmatrix} 1 & -1 & 8 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & \frac{4}{3} \end{vmatrix} = 1 \cdot 1 \cdot 3 \cdot \frac{4}{3} = \underline{\underline{4}}$$

2. Ratkaise LP-malli

$$\begin{aligned} \max z &= 10x_1 + 2x_2 \\ \text{ehdoin} \quad x_1 + x_2 &\leq 14 \\ 4x_1 + x_2 &\leq 20 \\ x_1 &\leq 4 \\ x_2 &\geq 2 \end{aligned}$$

1. rajoite $x_1 + x_2 \leq 14$ $\downarrow$ $A = (0, 14), B = (14, 0)$
 2. rajoite $4x_1 + x_2 \leq 20$ $\downarrow$ $C = (0, 20), D = (5, 0)$
 3. rajoite $x_1 \leq 4$ $\leftarrow$ $E = (4, 0)$
 4. rajoite $x_2 \geq 2$ $\uparrow$ $F = (0, 2)$



optimissa $\begin{cases} x_1 = 4 \\ 4x_1 + x_2 = 20 \end{cases}$

$$\Rightarrow \begin{cases} x_1 = 4 \\ x_2 = 4 \\ z = 10 \cdot 4 + 2 \cdot 4 = 48 \end{cases}$$

Vastaus: optimissa $\begin{cases} x_1 = 4 \\ x_2 = 4 \\ z = 48 \end{cases}$

3. Laske a) $\det(M)$, b) $A^T M$ ja c) M^{-1} , kun

$$M = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ 0 & 3 & -5 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 0 \\ -2 & 1 \\ 0 & 3 \end{pmatrix}.$$

$$\det(M) = \begin{vmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & -1 \\ 3 & -5 \end{vmatrix} - 0 + 0 \\ = 1 \cdot (-5 - (-3)) = \underline{\underline{-2}}$$

$$A^T M = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ 0 & 3 & -5 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & -3 & 5 \\ 0 & 10 & -16 \end{pmatrix}}}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & \textcircled{1} & -1 & 0 & 1 & 0 \\ 0 & 3 & -5 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \leftarrow^+ \\ \cdot 1 \cdot 3 \\ \leftarrow^- \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & \textcircled{-2} & 0 & -3 & 1 \end{array} \right) \begin{array}{l} \leftarrow^+ \\ \leftarrow^- \\ \cdot 1 \cdot \frac{1}{2} : (-2) \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 2,5 & -0,5 \\ 0 & 0 & 1 & 0 & 1,5 & -0,5 \end{array} \right)$$

$$\Rightarrow M^{-1} = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 2,5 & -0,5 \\ 0 & 1,5 & -0,5 \end{pmatrix}$$

4. Ratkaise yhtälöryhmä

$$\begin{cases} x - y + 3z = 10 \\ x + y + 4z = 20 \\ 2x + 2y - 2z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} \textcircled{1} & -1 & 3 & 10 \\ 1 & 1 & 4 & 20 \\ 2 & 2 & -2 & 0 \end{array} \right) \xrightarrow{\substack{\cdot 1 \quad \cdot 2 \\ \leftarrow \quad \leftarrow}} \left(\begin{array}{ccc|c} 1 & -1 & 3 & 10 \\ 0 & \textcircled{2} & 1 & 10 \\ 0 & 4 & -8 & -20 \end{array} \right) \xrightarrow{\substack{\cdot 2 \\ \leftarrow}}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 3 & 10 \\ 0 & 2 & 1 & 10 \\ 0 & 0 & -10 & -40 \end{array} \right) \rightarrow \begin{cases} x - y + 3z = 10 & (1) \\ 2y + z = 10 & (2) \\ -10z = -40 & (3) \end{cases}$$

$$(3) \rightarrow z = 4$$

$$(2) \rightarrow 2y + 4 = 10 \rightarrow y = 3$$

$$(1) \rightarrow x - 3 + 3 \cdot 4 = 10 \Rightarrow x = 1$$

$$\underline{\underline{\text{Vastaus} \begin{cases} x = 1 \\ y = 3 \\ z = 4 \end{cases}}}$$

$$\left(\begin{array}{l} \text{Tarkistus} \\ \begin{cases} 1 - 3 + 3 \cdot 4 = 10 & \checkmark \\ 1 + 3 + 4 \cdot 4 = 20 & \checkmark \\ 2 \cdot 1 + 2 \cdot 3 - 2 \cdot 4 = 0 & \checkmark \end{cases} \end{array} \right)$$