Vaasan yliopisto, Avoin yliopisto kevät 2008

Talousmatematiikan perusteet, ORMS1030

11. harjoitus, (9.4.2008)

1. Ratkaise yhtälöryhmä

$$\begin{cases} x + y = 3 \\ 3x + 5y = 13 \end{cases}$$

$$\left(\begin{array}{cc|c} \textcircled{1} & 1 & 3 \\ 3 & 5 & 13 \end{array}\right) \begin{matrix} \\ -3 \end{matrix}$$

$$\sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 2 & 4 \end{array}\right) \rightarrow \begin{array}{ll} x + y = 3 & (1) \\ 2y = 4 & (2) \end{array}$$

$$(2) \rightarrow \underline{y = 2}$$

$$(1) \rightarrow x + 2 = 3$$

$$\Leftrightarrow \underline{x = 1}$$

Vastaus: $x = 1$, $y = 2$

2. Ratkaise yhtälöryhmä

$$\begin{cases} 2x + y + z = 3 \\ 4x + 5y - 3z = 3 \\ x + 2y - 2z = 2 \end{cases}$$

$$\left(\begin{array}{ccc|c} \textcircled{1} & 2 & -2 & 2 \\ 4 & 5 & -3 & 3 \\ 2 & 1 & 1 & 3 \end{array}\right) \begin{matrix} \\ -4 \\ -2 \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & 2 \\ 0 & \textcircled{-3} & 5 & -5 \\ 0 & -3 & 5 & -1 \end{array}\right) -1$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & 2 \\ 0 & -3 & 5 & -5 \\ 0 & 0 & 0 & 4 \end{array}\right) \checkmark \rightarrow \underline{R_j = \emptyset}$$

3. Ratkaise yhtälöryhmä

$$\begin{cases} 2x + y = 2 \\ 4x + 5y = 8 \\ x + 2y = 3 \end{cases}$$

$$\left(\begin{array}{cc|c} \textcircled{2} & 1 & 2 \\ 4 & 5 & 8 \\ 1 & 2 & 3 \end{array}\right) \begin{matrix} -2 \\ \downarrow \\ \end{matrix} \quad \begin{matrix} -\frac{1}{2} \\ \downarrow \\ \end{matrix} \sim \left(\begin{array}{cc|c} 2 & 1 & 2 \\ 0 & \textcircled{3} & 4 \\ 0 & 1{,}5 & 2 \end{array}\right) \begin{matrix} \\ \\ -\frac{1}{2} \\ \downarrow \end{matrix}$$

$$\sim \left(\begin{array}{cc|c} 2 & 1 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{array}\right) \text{ :/.} \quad \Rightarrow \quad \begin{cases} 2x + y = 2 & (1) \\ 3y = 4 & (2) \end{cases}$$

$$(2) \Rightarrow \quad \underline{y = 4/3}$$

$$(1) \Rightarrow \quad 2x + \frac{4}{3} = \frac{6}{3}$$

$$\Leftrightarrow \quad \underline{x = \frac{1}{3}}$$

Vastaus: $x = \frac{1}{3}$ ja $y = \frac{4}{3}$

4. Etsi ainakin yksi ratkaisu yhtälöryhmälle

$$\begin{cases} x + 2y + z = -1 \\ 3x + 5y - 2z = 4 \end{cases}$$

$$\left(\begin{array}{ccc|c} \textcircled{1} & 2 & 1 & -1 \\ 3 & 5 & -2 & 4 \end{array}\right) \begin{matrix} \\ \downarrow \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & -1 & -5 & 7 \end{array}\right)$$

$$\rightarrow \begin{cases} x + 2y + z = -1 \\ -y - 5z = 7 \end{cases} \Leftrightarrow \begin{cases} x + 2y = -1 - a & (1) \\ -y = 7 + 5a & (2) \\ z = a & (3) \end{cases}$$

$$(2) \Rightarrow \quad y = -7 - 5a$$

$$(1) \Rightarrow \quad x + 2(-7 - 5a) = -1 - a$$

$$\Leftrightarrow \quad x = 13 + 9a$$

V: $\begin{cases} x = 13 + 9a \\ y = -7 - 5a \\ z = a \end{cases}$

$a \in \mathbb{R}$

(4) Kanta ratkaisut

"x = 0"

$$\begin{cases} x = 0 \\ 2y + z = -1 \\ 5y - 2z = 4 \end{cases} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2/9 \\ -13/9 \end{pmatrix}$$

"y = 0"

$$\begin{cases} y = 0 \\ x + z = -1 \\ 3x - 2z = 4 \end{cases} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2/5 \\ 0 \\ -7/5 \end{pmatrix}$$

"z = 0"

$$\begin{cases} z = 0 \\ x + 2y = -1 \\ 3x + 5y = 4 \end{cases} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ -7 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ 2/9 \\ -13/9 \end{pmatrix} + \beta \begin{pmatrix} 2/5 \\ 0 \\ -7/5 \end{pmatrix} + (1 - \alpha - \beta) \begin{pmatrix} 13 \\ -7 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 13 \\ -7 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 0 - 13 \\ 2/9 + 7 \\ -13/9 \end{pmatrix} + \beta \begin{pmatrix} 2/5 - 13 \\ +7 \\ -7/5 \end{pmatrix}$$

$$= \begin{pmatrix} 13 \\ -7 \\ 0 \end{pmatrix} + \frac{\alpha}{9} \begin{pmatrix} -117 \\ 65 \\ -13 \end{pmatrix} + \frac{\beta}{5} \begin{pmatrix} -63 \\ 35 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} 13 \\ -7 \\ 0 \end{pmatrix} + \left(\frac{13\alpha}{9} + \frac{7\beta}{5} \right) \begin{pmatrix} -9 \\ 5 \\ -1 \end{pmatrix}$$

5. Ratkaise yhtälöryhmä

$$\begin{cases} x_1 + x_2 = 1 \\ x_1 + x_2 + x_3 = 2 \\ x_2 + x_3 + x_4 = 3 \\ x_3 + x_4 + x_5 = 4 \\ x_4 + 2x_5 = 5 \end{cases}$$

$$\left(\begin{array}{ccccc|c} \textcircled{1} & 1 & & & & 1 \\ 1 & 1 & 1 & & & 2 \\ & 1 & 1 & 1 & & 3 \\ & & 1 & 1 & 1 & 4 \\ & & & 1 & 2 & 5 \end{array}\right) \begin{matrix} \\ -1 \\ \\ \\ \\ \end{matrix} \sim \left(\begin{array}{ccccc|c} 1 & 1 & & & & 1 \\ 0 & 0 & \textcircled{1} & & & 1 \\ & 1 & 1 & 1 & & 3 \\ & & 1 & 1 & 1 & 4 \\ & & & 1 & 2 & 5 \end{array}\right) \begin{matrix} \\ \\ -1 \\ \\ \\ \end{matrix}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & & & & 1 \\ & 1 & 1 & 1 & & 3 \\ & & 1 & & & 1 \\ & & & \textcircled{1} & 1 & 3 \\ & & & 1 & 2 & 5 \end{array}\right) \begin{matrix} \\ \\ \\ \\ -1 \\ \end{matrix} \sim \left(\begin{array}{ccccc|c} 1 & 1 & & & & 1 \\ & 1 & 1 & 1 & & 3 \\ & & 1 & & & 1 \\ & & & 1 & 1 & 3 \\ & & & & 1 & 2 \end{array}\right)$$

$$\begin{aligned} x_1 + x_2 &= 1 && (1) \\ x_2 + x_3 + x_4 &= 3 && (2) \\ x_3 &= 1 && (3) \\ x_4 + x_5 &= 3 && (4) \\ x_5 &= 2 && (5) \end{aligned}$$

$(5) \rightarrow x_5 = 2$

$(3) \rightarrow x_3 = 1$

$(4) \rightarrow x_4 + 2 = 3 \rightarrow x_4 = 1$

$(2) \rightarrow x_2 + 1 + 1 = 3 \rightarrow x_2 = 1$

$(1) \rightarrow x_1 + 1 = 1 \rightarrow x_1 = 0$

$$V: \begin{cases} x_1 = 0 \\ x_2 = 1 \\ x_3 = 1 \\ x_4 = 1 \\ x_5 = 2 \end{cases}$$

Olkoon

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & -1 & 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 5 & 2 \\ -1 & -1 \\ 2 & 1 \end{pmatrix}$$

6. Laske, jos lauseke on järkevä. (Huom kaikki lausekkeet eivät nyt ole hyvin määriteltyjä!) a) $\mathbf{A}+\mathbf{B}$, b) $\mathbf{A}+\mathbf{C}$, c) $\mathbf{b}+\mathbf{C}$

$$A+B = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 0 \\ 2 & 6 & 0 \\ 1 & -1 & 1 \end{pmatrix}$$

$A+C$ ei ole olemassa

$3\times 3 \quad 3\times 2$

$b+C$ ei ole olemassa

?

7. Laske, jos lauseke on järkevä. (Huom kaikki lausekkeet eivät nyt ole hyvin määriteltyjä!) a) $\mathbf{AB}$, b) $\mathbf{BA}$, c) $\mathbf{AC}$, d) $\mathbf{CA}$, e) $\mathbf{C}^T\mathbf{A}$

$$AB = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -6 & -2 \\ 3 & 10 & 3 \\ 1 & -1 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -4 & 0 \\ 6 & 9 & -2 \\ -1 & -5 & 1 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ -1 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 5 & 0 \\ 2 & 1 \end{pmatrix}$$

CA ei ole olemassa

$3\times 2 \quad 3\times 3$

$$C^TA = \begin{pmatrix} 5 & -1 & 2 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -13 & 3 \\ 0 & -7 & 2 \end{pmatrix}$$