Vaasan kesäyliopisto, kesä 2011

Talousmatematiikan perusteet, ORMS1030

10. harjoitus, (ma 22.8.2011)

1. Erään tuotekorin osalta tiedetään vuosien 2000 ja 2010 hinnat ja ostojen määrät. Perusajankohta on nyt 2000 ja vertailuajankohta 2010

tuote	2000 p_0	q_0	a_0	2010 p_t	q_t	a_t
1	10,00	200		15,00	300	
2	2,00	500		8,00	100	
3	30,00	20		15,00	100	

a) Laske Laspeyres'in hinta- ja volyymi-indeksit.
b) Laske Paashenin hinta- ja volyymi-indeksit.

a) $$P^L = \frac{\sum p_t q_{t_0}}{\sum p_{t_0} q_{t_0}} \cdot 100 = \frac{15 \cdot 200 + 8 \cdot 500 + 15 \cdot 20}{10 \cdot 200 + 2 \cdot 500 + 30 \cdot 20} \cdot 100 = 203$$

$$Q^L = \frac{\sum q_t p_{t_0}}{\sum q_{t_0} p_{t_0}} \cdot 100 = \frac{300 \cdot 10 + 100 \cdot 2 + 100 \cdot 30}{200 \cdot 10 + 500 \cdot 2 + 20 \cdot 30} \cdot 100 = 172$$

b) $$P^P = \frac{\sum p_t q_t}{\sum p_{t_0} q_t} \cdot 100 = \frac{15 \cdot 300 + 8 \cdot 100 + 15 \cdot 100}{10 \cdot 300 + 2 \cdot 100 + 30 \cdot 100} \cdot 100 = 110$$

$$Q^P = \frac{\sum q_t \cdot p_t}{\sum q_{t_0} \cdot p_t} \cdot 100 = \frac{300 \cdot 15 + 100 \cdot 8 + 100 \cdot 15}{200 \cdot 15 + 500 \cdot 8 + 20 \cdot 15} \cdot 100 = 79$$

2. Laske integraalit

a) $$\int (2x^3 + 6x^2 - 4x + 3)\, dx$$

b) $$\int_1^4 10x\, dx$$

a) $$\int (2x^3 + 6x^2 - 4x + 3)\, dx$$
$$= \frac{1}{2}x^4 + 2x^3 - 2x^2 + 3x + C$$

b) $$\int_1^4 10x\, dx = \Big/_1^4\, 5x^2 = 5 \cdot 4^2 - 5 \cdot 1^2$$
$$= 5 \cdot 16 - 5 = 75$$

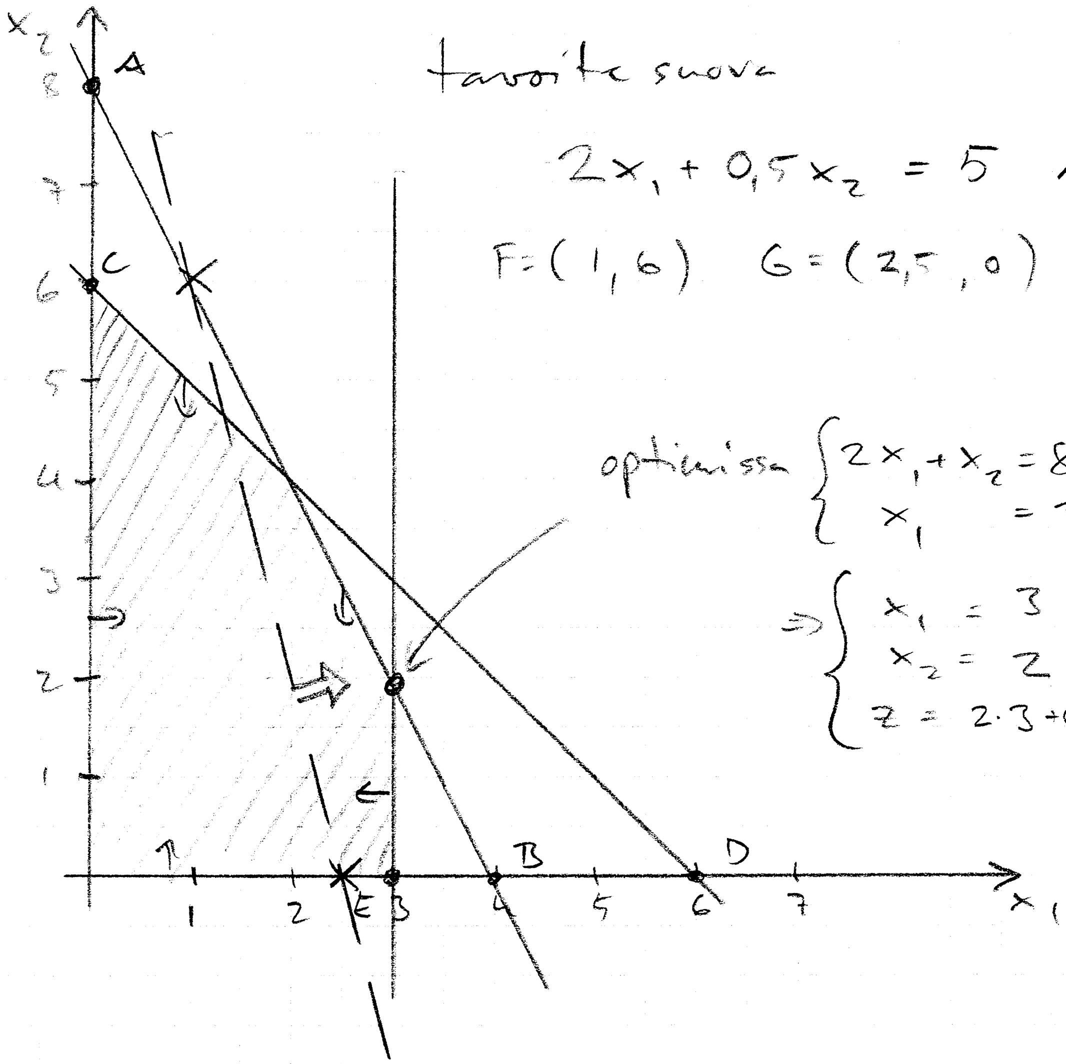
3. Ratkaise LP-malli

$$\begin{array}{lrcrcl} \max z = & 2x_1 & + & 0.5x_2 & & \\ \text{ehdoin} & 2x_1 & + & x_2 & \leq & 8 \\ & x_1 & + & x_2 & \leq & 6 \\ & x_1 & & & \leq & 3 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

1. rajoite $2x_1 + x_2 \leq 8$ $\downarrow$ $A = (0,8)$ $B = (4,0)$

2. rajoite $x_1 + x_2 \leq 6$ $\downarrow$ $C = (0,6)$ $D = (6,0)$

3. rajoite $x_1 \leq 3$ $\leftarrow$ $E = (3,0)$



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$2x_1 + 0{,}5x_2 = 5$ $\nearrow$

$F = (1,6)$ $G = (2{,}5, 0)$

optimissa $\begin{cases} 2x_1 + x_2 = 8 \\ x_1 = 3 \end{cases}$

$\Rightarrow \begin{cases} x_1 = 3 \\ x_2 = 2 \\ z = 2 \cdot 3 + 0{,}5 \cdot 2 = 7 \end{cases}$

Vastaus optimissa $x_1 = 3$, $x_2 = 2$ ja $z = 7$

3. Ratkaise yhtälöryhmä

$$\begin{cases} x - 2y + 2z = -3 \\ -x + y - z = 1 \\ 2x - 3y + z = -8 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & -3 \\ -1 & 1 & -1 & 1 \\ 2 & -3 & 1 & -8 \end{array}\right) \begin{array}{c} \cdot 1 \quad \cdot 2 \\ + \\ - \end{array} \rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 2 & -3 \\ 0 & -1 & 1 & -2 \\ 0 & 1 & -3 & -2 \end{array}\right) \begin{array}{c} \cdot 1 \\ + \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & -3 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & -2 & -4 \end{array}\right) \rightarrow \begin{cases} x - 2y + 2z = -3 & (1) \\ -y + z = -2 & (2) \\ -2z = -4 & (3) \end{cases}$$

$(3) \rightarrow z = 2$

$(2) \rightarrow -y + 2 = -2 \rightarrow y = 4$

$(1) \rightarrow x - 2\cdot 4 + 2\cdot 2 = -3$

$x = -3 + 8 - 4$

$x = 1$

V: $x = 1$, $y = 4$, $z = 2$

tarkistus

$$\begin{cases} 1 - 2\cdot 4 + 2\cdot 2 = -3 & ✓ \\ -1 + 4 - 2 = 1 & ✓ \\ 2\cdot 1 - 3\cdot 4 + 2 = -8 & ✓ \end{cases}$$

4. Laske det(**M**), $\mathbf{M}^T$ ja $\mathbf{M}^{-1}$ matriisille

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & 3 \\ 1 & 1 & -1 \\ 5 & 1 & 1 \end{pmatrix}.$$

a) $$\det(M) = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 1 & -1 \\ 5 & 1 & 1 \end{vmatrix} = +1 \cdot \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} - (-1) \begin{vmatrix} 1 & -1 \\ 5 & 1 \end{vmatrix} + 3 \cdot \begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix}$$
$$= (1\cdot 1 - (-1)\cdot 1) + (1\cdot 1 - (-1)\cdot 5) + 3(1\cdot 1 - 1\cdot 5)$$
$$= (1+1) + (1+5) + 3(1-5)$$
$$= 2 + 6 - 12$$
$$= \underline{\underline{-4}}$$

b) $$M^T = \begin{pmatrix} 1 & 1 & 5 \\ -1 & 1 & 1 \\ 3 & -1 & 1 \end{pmatrix}$$

c)

$$\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2 \qquad \begin{vmatrix} 1 & -1 \\ 5 & 1 \end{vmatrix} = 6 \qquad \begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = -4$$

$$\begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} = -4 \qquad \begin{vmatrix} 1 & 3 \\ 5 & 1 \end{vmatrix} = -14 \qquad \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

$$\begin{vmatrix} -1 & 3 \\ 1 & -1 \end{vmatrix} = -2 \qquad \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} = -4 \qquad \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

$$M^{-1} = \frac{1}{\det(M)} \begin{pmatrix} +|M_{11}| & -|M_{21}| & +|M_{31}| \\ -|M_{12}| & +|M_{22}| & -|M_{32}| \\ +|M_{13}| & -|M_{23}| & +|M_{33}| \end{pmatrix}$$

$$= \frac{1}{-4} \begin{pmatrix} +(2) & -(-4) & +(-2) \\ -(6) & +(-14) & -(-4) \\ +(-4) & -(2) & +(2) \end{pmatrix}$$

$$= \frac{1}{2} \cdot \begin{pmatrix} -1 & -2 & 1 \\ 3 & 7 & -2 \\ 2 & 1 & -1 \end{pmatrix}$$