Tampereen kesäyliopisto, syksy 2010

Talousmatematiikan perusteet, ORMS1030

7. harjoitus, (to 2.12.2010)

1. Ratkaise yhtälöryhmä

$$\begin{cases} x + 3y = 8 \\ 2x - 10y = 0 \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & 3 & 8 \\ 2 & -10 & 0 \end{array}\right) \begin{matrix} \cdot 2 \\ \leftarrow \end{matrix}$$

$$\left(\begin{array}{cc|c} 1 & 3 & 8 \\ 0 & -16 & -16 \end{array}\right) :(-16)$$

$$\left(\begin{array}{cc|c} 1 & 3 & 8 \\ 0 & 1 & 1 \end{array}\right) \rightarrow \begin{cases} x+3y=8 & (1) \\ y=1 & (2) \end{cases}$$

(2) $\rightarrow$ $y = 1$

(1) $\rightarrow$ $x + 3 \cdot 1 = 8 \rightarrow x = 5$

Vastaus: $x = 5$, $y = 1$

Tarkistus $\begin{cases} 5 + 3 \cdot 1 = 8 & \checkmark \\ 2 \cdot 5 - 10 \cdot 1 = 0 & \checkmark \end{cases}$

2. Ratkaise yhtälöryhmä

$$\begin{cases} 2y - z = 8 \\ x - 2y + 3z = -1 \\ 3x + y - z = 12 \end{cases}$$

$$\left(\begin{array}{ccc|c} \textcircled{1} & -2 & 3 & -1 \\ 0 & 2 & -1 & 8 \\ 3 & 1 & -1 & 12 \end{array}\right) \begin{matrix} \cdot 3 \\ \downarrow \\ \leftarrow - \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & \textcircled{2} & -1 & 8 \\ 0 & 7 & -10 & 15 \end{array}\right) \begin{matrix} \cdot \frac{7}{2} \\ \leftarrow - \end{matrix}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 2 & -1 & 8 \\ 0 & 0 & -13/2 & -13 \end{array}\right) \rightarrow \begin{cases} x - 2y + 3z = -1 & (1) \\ 2y - z = 8 & (2) \\ -6{,}5z = -13 & (3) \end{cases}$$

(3) → $z = -13/-6{,}5 = 2$

(2) → $2y - 2 = 8 \rightarrow y = 5$

(1) → $x - 2 \cdot 5 + 3 \cdot 2 = -1 \rightarrow x = 3$

Vastaus: $x = 3$, $y = 5$, $z = 2$

Tarkistus: $2 \cdot 5 - 2 = 8$ ✓

$3 - 2 \cdot 5 + 3 \cdot 2 = -1$ ✓

$3 \cdot 3 + 5 - 2 = 12$ ✓

3. Ratkaise yhtälöryhmä

$$\begin{cases} 2x - y = 4 \\ x - 2y = -1 \\ -x + y = -1 \end{cases}$$

$$\left(\begin{array}{cc|c} \textcircled{1} & -2 & -1 \\ 2 & -1 & 4 \\ -1 & 1 & -1 \end{array}\right) \begin{matrix} \cdot 2 \\ \leftarrow - \\ \\ \leftarrow + \end{matrix} \begin{matrix} \cdot 1 \\ \downarrow \end{matrix} \sim \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 3 & 6 \\ 0 & -1 & -2 \end{array}\right) \begin{matrix} \cdot \frac{1}{3} \\ \leftarrow + \end{matrix} \; :3$$

$$\sim \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right) \rightarrow \begin{cases} x - 2y = -1 & (1) \\ y = 2 & (2) \\ 0 = 0 \end{cases}$$

(2) → $y = 2$

(1) → $x - 2 \cdot 2 = -1 \rightarrow x = 3$

V: $\begin{cases} x = 3 \\ y = 2 \end{cases}$

4. Ratkaise yhtälöryhmä

$$\begin{cases} 2x - y = 2 \\ x - 2y = -1 \\ -x + y = -1 \end{cases}$$

$$\left(\begin{array}{cc|c} \textcircled{1} & -2 & -1 \\ 2 & -1 & 2 \\ -1 & 1 & -1 \end{array}\right) \begin{matrix} \cdot 2 \\ \cdot 1 \\ \end{matrix} \sim \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & \textcircled{3} & 4 \\ 0 & -1 & -2 \end{array}\right) \cdot \frac{1}{3}$$

$$\left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 3 & 4 \\ 0 & 0 & 2/3 \end{array}\right) \leftarrow \text{epätos.: } (0 = 2/3) \quad V$$

V: Ratkaisujoukko on tyhjä

5. Ratkaise yhtälöryhmä

$$\begin{cases} x + 3y + z = 8 \\ 2x + 2y - 2z = -4 \end{cases}$$

$$\left(\begin{array}{ccc|c} \textcircled{1} & 3 & 1 & 8 \\ 2 & 2 & -2 & -4 \end{array}\right) \cdot 2$$

$$\sim \left(\begin{array}{ccc|c} 1 & 3 & 1 & 8 \\ 0 & -4 & -4 & -20 \end{array}\right) :(-4) \sim \left(\begin{array}{ccc|c} 1 & 3 & 1 & 8 \\ 0 & 1 & 1 & 5 \end{array}\right)$$

$$\rightarrow \begin{cases} x + 3y + z = 8 \\ y + z = 5 \end{cases} \Leftrightarrow \begin{cases} x + 3y = 8 - z & (1) \\ y = 5 - z & (2) \end{cases}$$

(0) $z = a$

(2) $\rightarrow$ $y = 5 - a$

(1) $\rightarrow$ $x + 3(5-a) = 8 - a \rightarrow x = -7 + 2a$

V: $x = -7 + 2a$, $y = 5 - a$, $z = a$

Tark. $(-7 + 2a) + 3(5 - a) + a = 8$ ✓

$2(-7 + 2a) + 2(5 - a) - 2a = -4$ ✓

6. Olkoon

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 5 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}.$$

Laske a) $A + B$ b) AC c) $C^T A$

a) $A + B = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 2 \end{pmatrix}$

$2\times2 \qquad 2\times2 \qquad 2\times2$

b) $AC = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$

$2\times2 \qquad 2\times3 \qquad 2\times3$

c) $C^T A = \begin{pmatrix} 3 & 1 \\ 2 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ 3 & -2 \\ 2 & -1 \end{pmatrix}$

$3\times2 \qquad 2\times2 \qquad 3\times2$

7. Olkoon A, A ja A samat matriisit kuin edellisessä tehtävässä. Mitkä seuraavista lausekkeista on mahdollista laskea? Laske arvot, kun se on mahdollista.

a) $A + C$ b) $A^T + B$ c) CA d) $A^T C$

a) $A + C$ ei voi laskea (väärän kokoiset)

$2\times2 \quad 2\times3$

b) $A^T + B = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 2 \end{pmatrix}$

$2\times2 \quad 2\times2$

c) CA ei voi laskea (väärän kokoiset

$2\times3 \quad 2\times2$

d) $A^T C = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 7 & 3 & 2 \\ -3 & -2 & -1 \end{pmatrix}$

8. Määritä rivioperaatioiden avulla käänteismatriisit matriiseille

$$\boldsymbol{M} = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 4 & 3 \\ 2 & 5 & 1 \end{pmatrix}, \qquad \boldsymbol{Q} = \begin{pmatrix} 1 & 2 \\ 1 & 4 \end{pmatrix}$$

$$(M|I) \sim \left(\begin{array}{ccc|ccc} \textcircled{1} & 2 & -1 & 1 & 0 & 0 \\ 1 & 4 & 3 & 0 & 1 & 0 \\ 2 & 5 & 1 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \cdot(-1) \\ \\ \cdot(-2) \end{matrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & \textcircled{2} & 4 & -1 & 1 & 0 \\ 0 & 1 & 3 & -2 & 0 & 1 \end{array}\right) \begin{matrix} \\ \cdot\frac{1}{2} \\ \cdot(-1) \end{matrix} \cdot 2 \qquad \left(\begin{array}{ccc|ccc} 1 & 0 & -5 & 2 & -1 & 0 \\ 0 & 1 & 2 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \textcircled{1} & -\frac{3}{2} & -\frac{1}{2} & 1 \end{array}\right) \begin{matrix} \cdot 5 \\ \cdot(-2) \\ \\ \end{matrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5{,}5 & -3{,}5 & 5 \\ 0 & 1 & 0 & 2{,}5 & 1{,}5 & -2 \\ 0 & 0 & 1 & -1{,}5 & -0{,}5 & 1 \end{array}\right)$$

$$\therefore \quad M^{-1} = \begin{pmatrix} -5{,}5 & -3{,}5 & 5 \\ 2{,}5 & 1{,}5 & -2 \\ -1{,}5 & -0{,}5 & 1 \end{pmatrix}$$

$$(Q|I) \sim \left(\begin{array}{cc|cc} \textcircled{1} & 2 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right) \begin{matrix} \cdot(-1) \\ \\ \end{matrix} \sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & \textcircled{2} & -1 & 1 \end{array}\right) \begin{matrix} \cdot(-1) \\ \cdot\frac{1}{2} \end{matrix}$$

$$\sim \left(\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right)$$

$$\therefore \quad Q^{-1} = \begin{pmatrix} 2 & -1 \\ -0{,}5 & 0{,}5 \end{pmatrix}$$