A SIMPLE HEURISTIC FOR ASSIGNING DOORS TO TRAILERS IN CROSS-DOCKS

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ABSTRACT
Cross-docks are the facilities to which trailers arrive with goods that should be unloaded and then loaded (according to their destination) onto outbound trailers. Cross-docks eliminate inventory and reduce the total travel distance. Unlike general goods that could be routed to any demand destination, bundles of goods with specific destinations cannot be treated by the popular linear programming techniques, such as transshipment, transportation and assignment. Synchronous operation of a cross-dock means that all the trailers are available at the same time window called sort (usually 3 to 4 hours). Operating a cross-dock facility, requires assigning receiving doors and shipping doors to trailers. Due to the immense complexity of optimal formulations, dock-door assignment has been solved using heuristics.

This paper suggests a new heuristic approach for assigning dock doors to trailers. The heuristic has a complexity of O(n^2). This is a small fraction of the computations needed even for generating the cost matrix of a simple assignment formulation (when one side of the dock is arbitrarily fixed). Comparisons with solutions based on a popular (but much more complicated) heuristic, show that on the average the quality of the proposed solutions is at least as good as the other heuristic.

The paper also analyses existing formulations, and suggests a new formulation. In the process the reader gains insight into several important issues related to dock door assignment.

Keywords: Cross-docking

1. INTRODUCTION
Freight transportation using trailers is one of the major components of most supply chains. Cross-docks are transshipment facilities to which trailers arrive with freight that should be sorted and loaded directly onto outbound trailers (or staged in the load position while waiting for the outbound trailer). The freight to each destination is consolidated from several origins.
This consolidation of freight is called cross-docking. This operation is prevalent in large carriers of ground and overnight service.

Cross-docks are attractive for two main reasons [1]: (1) Cross-docking is a way to reduce inventory holding costs; and (2) For less-than-truckload (LTL) and small package carriers, cross-docking is a way to reduce transportation costs.

For ensuring efficient operation, all existing cross-docks are divided into load and unload positions called load/unload doors, or receiving/shipping doors. Such a door is the space required for the trailer when it parks against the dock for loading or unloading. These spaces are marked and designed in order to enable efficient planning and performing of the dock's operation. The classic and most popular configuration of a cross-dock facility is rectangular in shape with several tens of load/unload doors on each of the longer sides of the rectangle (the terms unload/load doors are interchangeable in this paper with receiving/shipping doors and inbound/outbound doors). For management purposes, it is customary to designate one side of the dock to inbound trailers and the other side to outbound trailers [2,3]. However, other studies present different or more general scheme of operation (e.g., [4]).

While some formulations (such as [5, 6, and 7]) consider many more factors like staffing, construction, and the transportation network, this paper is concerned only with the dock-doors' assignment and its effect on the workload at the cross-dock facility. This characterizes the cross-dock operational management decisions.

The rest of the paper proceeds as follows: Section 2 presents the classic single dock and its existing formulations and discusses their drawbacks. Section 3 presents existing heuristics, Section 4 presents the proposed heuristic, section 5 illustrates the proposed heuristic by using a small case study. Section 6 discusses the complexity of the proposed algorithm and section 7 concludes the paper.

2. EXISTING FORMULATIONS OF THE DOCK-DOOR ASSIGNMENT PROBLEM

The typical cross-dock operation is done on a dock facility with two-sided inbound/outbound configuration (as shown in figure 1). This configuration dedicates one side of the dock for inbound freight and the other side of the dock to outbound freight. While there are other cross-dock configurations, the two-sided inbound/outbound configuration is by far the most popular configuration, since its simplicity minimizes the chances of shipping items to a wrong destination.
In a typical cross-dock the inbound (i.e., the content of the incoming trailers) and the destination of every incoming package are known ([1]). The objective of the daily cross-dock’s operation plan is to assign the incoming trailers to the inbound door positions and the outgoing trailers to the outbound doors position, in order to minimize handling, i.e., the total distance traveled by the freight itself or by the forklifts. Note that this problem is separated from the problem of minimizing the travel distance of the trailers (which has also to determine the content of inbound trailers.

**Figure 1. The configuration of a single two-sided inbound/outbound cross-dock**

It is important to differentiate between cross-docks and sortation systems (or distribution centers). Any receiving and shipping system that uses a conveyor as a major freight moving tool ceases to be a cross-dock. The operation and the characteristics of the dock change when a conveyor is in place. For example, unloading is always done at the feeding end of the conveyor so that one receiving door serves a lot of arriving trailers. Thus, dock-door assignment problem is totally different in cross-docks then the door assignment in conveyor-based systems.
2.1 The Classic Formulation of the Door Assignment Problem

For the purpose of presenting the traditional formulations we offer the following notation scheme:

- \( I \) - Number of receiving doors (inbound unload doors)
- \( J \) - Number of shipping doors (outbound load doors)
- \( M \) - Number of origins \((I \geq M)\)
- \( N \) - Number of Destinations \((J \geq N)\)
- \( d_{ij} \) - The distance between receiving door \( i \), and destination door \( j \)
- \( x_{mi} \) - 0/1 indicator for assignment of an origin \( m \) to a receiving door \( i \)
- \( y_{nj} \) - 0/1 indicator for assignment of a destination \( n \) to a shipping door \( j \)
- \( w_{mn} \) - The number of forklift trips necessary for carrying the freight from origin \( m \) to destination \( n \). This number is a function of the freight volume sent from \( m \) to \( n \).

The model by Tsui and Chang, [2] is by far the most cited model for cross-docks' door assignment. This model suggests the following optimization formulation:

Minimize

\[
\sum_{j} \sum_{i} \sum_{n} \sum_{m} w_{mn} d_{ij} x_{mi} y_{nj}
\]

Subject to:

1. \( \sum_{m} x_{mi} = 1 \) for \( i = 1, 2, \ldots, I \) \((1)\)
2. \( \sum_{i} x_{mi} = 1 \) for \( m = 1, 2, \ldots, M \) \((2)\)
3. \( \sum_{n} y_{nj} = 1 \) for \( j = 1, 2, \ldots, J \) \((3)\)
4. \( \sum_{j} y_{nj} = 1 \) for \( n = 1, 2, \ldots, N \) \((4)\)

\( x_{mi} = 0 \) or \( 1 \) for all \( m, i \)
\( y_{nj} = 0 \) or \( 1 \) for all \( n, j \)

Constraint set (1) guarantees that each receiving door is assigned only one origin.
Constraint set (2) guarantees that each origin is assigned only one receiving-door.
Constraint set (3) guarantees that each shipping door is assigned only one destination.
Constraint set (4) guarantees that each destination is assigned only one shipping door.
The above formulation is known to be a bilinear program [8] with high complexity. This is a special (simple) version of the Quadratic Assignment Problem (QAP) which is known to be NP Complete [9, 10].

The above model totally disregards the weight and capacity issue and the fact that many trailers could come from a single origin and many can go to the same destination. These issues are treated in the next model (which deals directly with trailers and ignores the assignment of doors).

2.2 The Capacity Formulation for a Cross-dock

The weight and capacity are important factors and more than one trailer could be coming from the same origin and going to the same destination. One of the first works that included these considerations was done by [5]. Peck [5] proposes a “greedy” heuristics to improve productivity by assigning incoming and outgoing trailers to dock doors for minimizing the total distance material handlers travel during the transfer operations. A simulation of an LTL terminal showed 5% to 15% reduction in is transfer distance due to the heuristic.

Capacity considerations led also to the following model proposed by [11].

Notation scheme:
- \( I \) - Set of inbound (strip) trailers
- \( J \) - Set of outbound (destination) trailers
- \( x_{ij} \) - The weight (in pounds) of freight in inbound trailer \( i \) bound for destination trailer \( j \)
- \( d_{ij} \) - the distance between trailer \( i \) and trailer \( j \)
- \( b_j \) - The total weight of freight bound for destination trailer \( j \)
- \( c \) - The capacity (in pounds) of an inbound (strip) trailer
- \( \xi \) - The minimum number of trailers that must contain freight for any destination.

The formulation is:

\[
\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \quad \text{(5)}
\]

\[
\text{Subject to} \quad \sum_{i \in I} x_{ij} = b_j \quad \forall j \in J \quad \text{(6)}
\]

\[
\sum_{j \in J} x_{ij} = c \quad \forall i \in I \quad \text{(7)}
\]

\[
x_{ij} \leq \frac{b_j}{\xi} \quad \forall i \in I, \ j \in J \quad \text{(8)}
\]

\[
x_{ij} \geq 0 \quad \forall i \in I, \ j \in J \quad \text{(9)}
\]
However, a careful look at this model shows that there is no real door assignment at all: the four indexes \{m,i,j,n\} become two indexes \{i,j\}, and the trailer-to-door assignment is now assumed to be pre-assigned by the supervisors and the variables are the weights to be carried between input and output trailer pairs. Moreover, in the original problem each unit of freight has specific origin and destination. While in this transportation type model the freight is treated like general commodity (where the bj is analogous to demand and c to the supply). Thus, the door assignment was only treated via heuristic pair swap, and heuristic search techniques. For each door assignment, the optimal weights to be carried from each inbound to each outbound trailer was computed via the above fast LP.

2.3 Drawbacks of the Existing Formulations

Summarizing the main drawback of the existing formulations we observe that:

1. The underlying problem of the door assignment is the quadratic assignment problem (QAP) having non-polynomial complexity, so optimal formulations cannot be used as practical tools for large problems [10].

2. The door assignment formulations (e.g., [2]) ignore the trailer capacity and weights issues.

3. The weights formulations (e.g., [11]) are ignoring the door assignment issue and the fact that each good has fixed origin-destination which cannot be changed.

The first model [2] has some problematic assumptions. For example, they assume that one receiving door is assigned to each origin and one shipping door is assigned to each destination. However, this is unrealistic most real life cases: for example it often happens that the freight volume shipped to a destination exceeds from an origin or to all its destinations exceeds a full trailer load. The same is true for the volume that is received at a certain destination from all of its origins. Typically, constraints of the dock operation necessitate simultaneous unloading and loading of these trailers. So each origin and each destination may have multiple doors assigned to them.

Also, in some cases it may be desirable to allow inbound trips to collect freight from more than one origin on its way to the dock. In other cases, it may be desirable to allow shipping trips to more than one destination (i.e., loading freight of two or more destinations onto one trailer, in separate bulks).
The second model (by [11]), is not only leaving out the door assignment to trailers to be given by an outside source, it also prevents shipments from having specific destinations and treats them as a commodity that could be manipulated at will as long as the destinations get their demand (similar to the transshipment models).

3. EXISTING HEURISTICS OF THE DOCK-DOOR ASSIGNMENT PROBLEM

While the first formulation by [2] is very complex, if we arbitrarily fix all the $x_{mi}$ origins to unload-doors assignments, the problem becomes a standard assignment problem, which is easy to solve. The problem also becomes a standard assignment problem if all the $y_{nj}$ (destinations to load-doors assignments) are arbitrarily fixed [12].

Based on this insight, Tsui and Chang, [2] proposed a heuristic starting from an arbitrary door assignment for the origins, and solve a standard assignment problem for outbound doors to destinations. Then fix the destinations assignment, and solve the standard assignment problem for the origins. The algorithm continues to switch sides until convergence occurs. However, Tsui and Chang did not prove or show that the procedure actually converges. Even if it does converge, it is likely to find a local optimum. In 1992 Tsui and Chang [13] proposed a branch and bound approach for solving this problem having run time exponential growth as a function of problem size. However, their results were generally better than those of the heuristic in [2].

Both [2 and 13] totally disregard the weight and capacity issue and the fact that many trailers could come from a single origin and many can go to the same destination. There were other trials to solve the door dock assignment using genetic algorithms [14] tabu search [15], simulation[3], simulated annealing [16] and lagrangian relaxation [6]. Most heuristics require long run time for ensuring good solution. As could be seen below, the advantage of the proposed method is its simplicity and short run time (while not compromising on quality).

4. THE PROPOSED HEURISTIC SOLUTION

This section presents the proposed heuristic and illustrates it on a case study.

The following are the stages of the proposed algorithm:
4.1 Phase 1: Initialization

1. Cluster the origins and destinations of the largest 3 weight transfer (wmn) volumes in the "from-to" table.

2. Assign the trailer $m$ of the largest (wmn) from step 1 to the middle of the cross-dock on the inbound side, and the corresponding trailer(s) $n$ to the middle of the outbound side.

3. Arbitrarily assign the other two inbound trailers (chosen in step one) on each side of the middle trailer of step 2.

4. Compute (weight\*distance) of the alternatives of assigning the locations of the other two destination(s) (chosen in step one) on the sides of the outbound destination assigned in step 2. Choose the best alternative.

5. Define $I$ as iteration counter and assign $I=0$.

4.2 Phase 2: Iterations

1. $I:=I+1$
   If $I$ is even go to step 3. ($I=the$ number of iteration)
   Else: update the "from to" table to represent the current cluster of assigned inbound trailers using one row with the sum of the entries of the rows corresponding to these trailers

2. Choose the biggest entry (volume) from the assigned cluster's row while ignoring the cluster's column. Break ties arbitrarily. Add the corresponding destination to the assigned outbound cluster. Compare locating the destination to the right and to the left of the assigned outbound destinations cluster, and choose the assignment that minimizes the sum of (distance\*volume) between the trailer and the inbound side of the assigned cluster. If all $m$ trailers and $n$ destinations are assigned - then finish, Otherwise go to step 1 (of phase 2).

3. Update the "from to" table to represent the current cluster of assigned outbound trailers using one column with the sum of the entries of the columns corresponding to the outbound trailers.

4. Choose the biggest entry (volume) from the assigned cluster's column while ignoring the cluster's row. Break ties arbitrarily. Add the corresponding incoming trailer to the assigned inbound cluster. Compare locating the trailer to the right and to the left of the
assigned inbound cluster, and choose the assignment that minimizes the sum of (distance*volume) between the trailer and the outbound side of the assigned cluster. If all m trailers and n destinations are assigned - then finish, otherwise go to step 1 (of phase 2).

5. ILLUSTRATIVE CASE STUDY OF THE HEURISTIC

We chose an arbitrary illustrative small case of seven incoming trailers and a dock of 10 receiving doors and 10 shipping doors (overall 20 doors). The "from-to" matrix and the first cluster is depicted in table 1.

Table 1. The "from-to" matrix: the volume is given as percentage of trailer's capacity.

<table>
<thead>
<tr>
<th>Incoming Tracker</th>
<th>Destination</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0</td>
<td>5</td>
<td>70</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>5</td>
<td>0</td>
<td>15</td>
<td>55</td>
<td>5</td>
<td>20</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>80</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>15</td>
<td>0</td>
<td>35</td>
<td>10</td>
<td>25</td>
<td>10</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>75</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0</td>
<td>15</td>
<td>15</td>
<td>50</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>5</td>
<td>0</td>
<td>45</td>
<td>15</td>
<td>5</td>
<td>30</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>25</td>
<td>25</td>
<td>180</td>
<td>285</td>
<td>35</td>
<td>85</td>
<td>65</td>
<td>700</td>
</tr>
</tbody>
</table>

Assume that due to timing constraints; no more than two full trailers could be loaded (one after the other) on each load door. Thus, destination 4, that receives close to 3 full trailers, must have two separate load doors.

5.1 Phase 1: Initialization

1. The three largest entries in table 1 are highlighted: {1-3, 3-4, 5-4}
2. The largest entry is {3-4}, so trailer 3 is assigned to the middle door of the receiving (unload) dock and destination 4 is to the middle door of the shipping (load) dock.
3. Incoming trailers 5 and 1, are arbitrarily assigned to the doors on the sides of the incoming trailer 3 (assigned in step 2).
4. In this example the initial cluster has only two destinations: 3, 4 (the max is 3). Since destination 4 is already assigned, destination 3 could be assigned on each of its sides.
Obviously the freight travel distance is minimized by assigning destination directly against trailer 1. This is illustrated in Fig. 2.

![Figure 2. The assignment at the end of the initialization phase](image)

### 5.2 Phase 2: Iterations

The first iteration adds a destination to the assigned cluster. This destination is determined by the largest volume sent by the cluster to an unassigned destination.

**Iteration-1**

**Step 1:** \( I = 1 \); The updated volumes of the incoming cluster could be seen in the first row of table 2.

**Table 2. The first iteration assigns destination 7 to the destination cluster**

<table>
<thead>
<tr>
<th>Incoming Trailer #</th>
<th>Destination 1</th>
<th>Destination 2</th>
<th>Destination 3,4</th>
<th>Destination 5</th>
<th>Destination 6</th>
<th>Destination 7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,3,5}</td>
<td>0</td>
<td>10</td>
<td>225</td>
<td>0</td>
<td>25</td>
<td>40</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0</td>
<td>70</td>
<td>5</td>
<td>20</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>0</td>
<td>45</td>
<td>25</td>
<td>10</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>15</td>
<td>65</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>0</td>
<td>60</td>
<td>5</td>
<td>30</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>25</strong></td>
<td><strong>25</strong></td>
<td><strong>465</strong></td>
<td><strong>35</strong></td>
<td><strong>85</strong></td>
<td><strong>65</strong></td>
<td><strong>700</strong></td>
</tr>
</tbody>
</table>


Step 2:

In the case study, the incoming cluster \{1,3,5\} send the most (40% of a trailer) to destination 7. This could be seen in table 2. Thus destination 7 has to be assigned a load door. There are two configurations to choose from as seen in figure 3.

The comparison of the volumes time distance products of the two assignments with rectilinear simplification and length units equal to the door width:

**Assignment A:** \((20\times 2) + (10\times 3) + (10\times 4) = 40 + 30 + 40 = 110\)

**Assignment B:** \((20\times 3) + (10\times 2) + (10\times 1) = 60 + 20 + 10 = 90\)

Thus, assignment B is selected and the second iteration begins.

The second iteration adds an assignment of an incoming trailer to the assigned cluster. This inbound trailer is determined by the largest volume sent to the cluster by an unassigned inbound trailer.

**Iteration-2**

**Step 1:** \(I=2\); If \(I\) is even go to step 3.

**Step 3:** The updated volumes of the incoming cluster could be seen in table 3.

**Step 4:** The outbound cluster \{3,4,7\} receives the most (85% of a trailer) from trailer 6. This could be seen in table 3. Thus the inbound trailer 6 has to be assigned a load door. There are two configurations to choose from: either inbound door 3 or inbound door 7 (which is more favorable).
Table 3. The second iteration assigns trailer 6 to the unloading cluster

<table>
<thead>
<tr>
<th>Incoming Trailer #</th>
<th>Destination</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,3,5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>{1,3,5}</td>
<td>2 (3,4,7)</td>
<td>265</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>Total</td>
<td>25</td>
<td>530</td>
</tr>
</tbody>
</table>

Each iteration assigns either an inbound trailer or a destination. So after eight more iterations (5 unassigned destinations and 3 unassigned incoming trailers) breaking ties arbitrarily, we arrive at the final solution as shown in figure 4.

Figure 4. The final door assignment for the case study by the proposed heuristic solution.

6. DISCUSSION

The proposed heuristic algorithm assigns at least one door to a trailer per iteration. Thus, the iterations stop either all the doors or all trailers are assigned. Thus, defining \( x \) as the computed number of destination trailers \( (x \geq n) \) the maximal number of iterations is:

\[
\text{Min}\{m+x, i+j\}.
\]

Defining \( N = \text{Min}\{m+x, i+j\} \)

In each iteration the actions are:

1. Update of assigned shipped volume for a recently assigned trailer \((O(N))\)
2. Finding the minimal entry in a row or a column \((O(N))\)
3. Computing the total distance*volume for two alternative assignments \((2*O(N))\)
So having $N$ iterations, each having a complexity of $O(N)$, leads to the overall complexity of $O(N^2)$.

To compare the proposed heuristic to other possible solutions we chose 20 cases ("from-to" freight volume matrices) of 10 inbound by 10 outbound trailers and the same number of doors. For the comparison we ran the Tsui and Chang assignment heuristic [2] on each case and compared the results to the proposed solution. The results show average savings of about 5% (4.6%) in the total distance traveled.

7. CONCLUSION

This paper discusses the existing formulations of the single cross-dock door assignment problem, and suggests a new formulation. A heuristic algorithm is presented based on growing an initial cluster. The algorithm is illustrated via a case study and its complexity is $O(N^2)$. Comparison to other heuristic assignment shows average savings of about 5%. While previous formulations were explicitly bilinear, the current formulation shows that decisions on how to split a load among a group of same destination trailers - makes the problem even more complex, and it is not a bilinear problem anymore.

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