A MODEL FOR PLANNING OF PRODUCTION, INVENTORY AND COMBINED ROUTING OF DELIVERY AND PICK-UP

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This work studies the effectiveness of a new modeling approach that integrates decisions concerning production-inventory and transportation of both raw materials and finished products. We refer to this problem as the Production, Inventory, Distribution and Pick-up Routing Problem (PIDPRP). By permitting transportation of both finished products and raw materials in the same vehicle tour, we propose an alternative to the phenomena of empty truck movements. The problem is formulated as a mixed integer program with tours enumeration. A commercial solver is used to validate and solve a variety of small instances. The impact of some parameters of the problem on the solution time and on the properties of the solutions is analyzed. The integrated model is compared with three alternative decomposition approaches.

1. Introduction and literature review

The continuously growing competition among companies has led researchers and practitioners to seek improved operations of supply chains. While in the past, policies of different functions of the supply chain were managed separately, more recent studies examined the advantages of taking various decisions jointly. The majority of these works describe a limited number of joint decisions, for example integrating production and distribution or transportation and inventory.

Only few works studied joint decisions of production, inventory, distribution (supply quantity to retail points) and routing in a dynamic setting. Fumero and Vercellis (1999) considered a centralized system with a single production plant, multiple products, and multiple retail points, where inventory can be held either at the plant or at the retail location. They solved it using Lagrange relaxation and noted the advantage of the integrated solution over its decoupled counterpart. Tang et al. (2006), considered a problem of production-distribution network system with multiple suppliers and multiple destinations for single and multiple products. Their model allows direct shipment only, so there is no issue of routing. The production, inventory and assignment problem is solved by a two layers decomposition method that combines heuristics using Lagrange relaxation. Lei et al. (2006), studied a production, inventory and distribution routing problem with multiple production plants and multiple demand centers including possible inventory holding at any node. An initial feasible solution is obtained by restricting the tours to direct shipment only and solving a transportation problem. Next, optimal tours for each period are constructed. Boudia et al. (2007), considered a problem of a single plant, a single product, multiple customers and inventories at the plant and at the customers. An algorithm based on the GRASP (Greedy Randomized Adaptive Search Procedure) metaheuristic is developed and compared to a decoupled approach. For a detailed literature review the reader is referred to Tang et al. (2006).

This study can be seen as an extension of Fumero and Vercellis (1999) in the sense that the transportation of raw material is integrated in the model. A nice feature of the model is the opportunity to save deadheading by combining outgoing within ingoing transportation in the same tour. We believe that such a strategy is beneficial both from economic and environmental points of view.

To the best of our knowledge, this alternative was not considered previously. This setting is applicable to industries where the same decision maker is responsible for both inbound and outbound logistic and where retail points (or customers) and suppliers or intermodal terminals (such as sea ports) are located in the same geographical area.

Due to the complexity of the Production, Inventory, Distribution and Pick-up Routing Problem (PIDPRP), our approach for obtaining a solution is to build a mathematical model based on tour choices rather than arc choices as in the classical model. In Section 2, we present a linear integer mathematical model for the PIDPRP. In section 3, three alternative decomposition approaches to the PIDPRP are presented. In section 4, the results of our experiments with the PIDPRP model and the decomposed models are presented, compared and discussed. In section 5, we outline some directions for future research.

2. Mathematical model for the PIDPRP

Consider a centralized supply chain with several suppliers, a single manufacturing plant and a set of retail points. Several products are produced in the plant using several types of raw materials. A fleet managed by the firm is based in the plant and hence each tour of the vehicle must start and end there. Each vehicle is capable of transporting both raw materials and finished goods. A third party carrier that charges fee based on the destination and the volume of the shipments can also be used for transportation. A deterministic dynamic demand at each of the retail points for some finite planning horizon is given. The demand needs to be satisfied while minimizing the total production, inventory holding and transportation costs. In the notation below, all parameters and variables related to quantities are in *transportation units (TU)*.

Sets:

Production

P set of type of products, Z set of type of raw materials

T set of periods, R set of resources

Network

N_0 the depot, N_1 set of retail points, N_2 set of suppliers

N set of nodes (depot, suppliers, and retail points), $N = N_0 \cup N_1 \cup N_2$

Routing

K set of trucks

G set of possible tours (after column preprocessing)

N_g set of nodes that belong to route g

Parameters:

Production

d_{pnt} demand of retail point n for product p at period t

γ_{zp} quantity of material z required for the production of one unit of product p

c^F fixed cost of production at any period

s_r availability of resources of type r per period

u_{pr} resource r utilization by one unit of product p

M big number

Inventories

h_p^P the cost of one unit of product p in the plant inventory at any period

h_z^Z the cost of one unit of raw material z in the plant inventory at any period

$h_{pn}^{N_1}$ the cost of inventory of one unit of product *p* at inventory of retail point *n*, $n \in N_1$, at any period

Routing

v_k volume of truck *k*

l_g the length of tour $g \in G$

c_k^K the cost of truck *k* if it is used (ex: drivers) at any period

c_k^{km} the cost of truck *k* per kilometer (variable cost of transportation)

c_{zn}^Z the cost of one unit of raw material *z* purchased from supplier *n* (not including transportation), $n \in N_2$. May be infinite if z is not available from *n*, $n \in N_2$

c_n^0 transportation cost per unit, charged by the third party carrier between N_0 and *n*, $n \in N$

<u>Variables:</u>

Production

x_{pt}^P quantity of product *p* produced at period *t*

$$q_t = \begin{cases} 1 & \text{if there is production at period } t \\ 0 & \text{if not} \end{cases}$$

Inventories

x_{pt}^{HP} inventory level of product *p* in the plant at the end of period *t*

x_{zt}^{HZ} inventory level of raw material *z* in the plant at the end of period *t*

$x_{pnt}^{HN_1}$ inventory level of product *p* at $n \in N_1$ at the end of period *t*

Routing

y_{pnkt}^P quantity of product *p* shipped by truck *k* to $n \in N_1$ at period *t*

y_{znkt}^Z quantity of raw material *z* picked from $n \in N_2$ by truck *k* at period *t*

w_{pnt}^P quantity of product *p* shipped to $n \in N_1$ at period *t*, using the third party carrier

w_{znt}^Z quantity of raw material *z* shipped from $n \in N_2$ at period *t*, using the third party carrier

$$\delta_{gkt} = \begin{cases} 1 & \text{if the producer takes tour } g \text{ using truck } k \text{ at period } t \\ 0 & \text{if not} \end{cases}$$

Objective function:

$$\text{Min}\left\{\begin{array}{c}\sum_{z\in Z}\sum_{n\in N_2}\sum_{t\in T} c_{zn}^{Z}\left(\sum_{k\in K} y_{znkt}^{Z} + w_{znt}^{z}\right) + \sum_{t\in T} c^{F}\, q_t + \sum_{p\in P}\sum_{n\in N_1}\sum_{t\in T} x_{pnt}^{HN_1}\, h_{pn}^{N_1} \\ \text{(cost of raw material } \mathbf{(1)} + \text{ fixed cost of production } \mathbf{(2)} + \text{ inventory cost at retail points } \mathbf{(3)}\text{)} \\ +\sum_{p\in P}\sum_{t\in T} x_{pt}^{HP} h_p^{P} + \sum_{z\in Z}\sum_{t\in T} x_{zt}^{HZ} h_z^{Z} + \sum_{g\in G}\sum_{k\in K}\sum_{t\in T}\delta_{gkt}\left(l_g c_k^{km} + c_k^{K}\right) \\ \text{(inventory cost of products at plant } \mathbf{(4)} + \text{ inventory cost of raw material at plant } \mathbf{(5)} \\ + \text{ transportation cost for routing and trucks used } \mathbf{(6)}\text{)} \\ + \sum_{p\in P}\sum_{n\in N_1}\sum_{t\in T} c_n^{0}\, w_{pnt}^{P} + \sum_{z\in Z}\sum_{n\in N_2}\sum_{t\in T} c_n^{0}\, w_{znt}^{Z} \\ \text{(cost of transportation of product by a third party carrier } \mathbf{(7)} \\ + \text{ cost of transportation of raw material by a third party carrier } \mathbf{(8)}\text{)}\end{array}\right\}$$

Subject to:

Production

1) Resources

$$\sum_{p\in P} u_{pr}\, x_{pt}^{P} \le s_r \qquad ,\forall r\in R, \forall t\in T$$

2) Product and production

$$\sum_{p\in P} x_{pt}^{P} \le M q_t \qquad ,\forall t\in T$$

Balance inventories & product & distribution

3) Inventory balance and demand

$$x_{pt}^{P} = x_{pt}^{HP} + \sum_{n\in N_1}\sum_{k\in K} y_{pnkt}^{P} + \sum_{n\in N_1} w_{pnt}^{P} - x_{p(t-1)}^{HP} \qquad ,\forall p\in P, \forall t\in T$$

4) Inventory balance and raw material

$$\sum_{n\in N_2}\left(\sum_{k\in K} y_{znk(t-1)}^{Z} + w_{zn(t-1)}^{Z}\right) = \sum_{p\in P}\gamma_{zp}\, x_{pt}^{P} + x_{zt}^{HZ} - x_{z(t-1)}^{HZ} \;,\forall z\in Z, \forall t\in T$$

5) Respect of the shipped quantity of products

$$\sum_{k\in K} y_{pnkt}^{P} + w_{pnt}^{P} + x_{pn(t-1)}^{HN_1} = d_{pnt} + x_{pnt}^{HN_1} \;,\forall p\in P, \forall n\in N_1, \forall t\in T$$

Routing

6) Link between shipment to any retail point and the corresponded tour

$$\sum_{p\in P} y_{pnkt}^{P} \le v_k \sum_{\substack{g\in G\\ n\in N_g}} \delta_{gkt} \qquad ,\forall n\in N_1, \forall k\in K, \forall t\in T$$

7) Link between pick-up from any supplier and the corresponded tour

$$\sum_{z\in Z} y^{Z}_{znkt} \le v_k \sum_{\substack{g\in G\\ n\in N_g}} \delta_{gkt} \qquad ,\forall n\in N_2, \forall k\in K, \forall t\in T$$

8) Volume capacity to retail points

$$\sum_{p\in P}\sum_{n\in N_1} y^{P}_{pnkt} \le v_k \sum_{g\in G} \delta_{gkt} \qquad ,\forall k\in K, \forall t\in T$$

9) Volume capacity from suppliers

$$\sum_{z\in Z}\sum_{n\in N_2} y^{Z}_{znkt} \le v_k \sum_{g\in G} \delta_{gkt} \qquad ,\forall k\in K, \forall t\in T$$

10) Truck and period (the same truck is used once or is not used at any period)

$$\sum_{g\in G} \delta_{gkt} \le 1 \qquad ,\forall k\in K, \forall t\in T$$

11) Any node is visited by at most one truck at any period

$$\sum_{\substack{g\in G\\ n\in N_g}} \sum_{k\in K} \delta_{gkt} \le 1 \qquad ,\forall n\in (N_1\cup N_2), \forall t\in T$$

Binary and non-negativity constraints

12) $\delta_{gkt} \in (0,1)$ $,\forall g\in G, \forall k\in K, \forall t\in T$ 13) $q_t \in (0,1)$ $,\forall t\in T$

14) $x^{P}_{pt} \ge 0$ $,\forall p\in P, \forall t\in T$ 15) $x^{HP}_{pt} \ge 0$ $,\forall p\in P, \forall t\in T$

16) $x^{HZ}_{zt} \ge 0$ $,\forall z\in Z, \forall t\in T$ 17) $x^{HN_1}_{pnt} \ge 0$ $,\forall p\in P, \forall n\in N_1,\ \forall t\in T$

18) $y^{P}_{pnkt} \ge 0$ $,\forall p\in P, \forall n\in N_1, \forall k\in K, \forall t\in T$

19) $y^{Z}_{znkt} \ge 0$ $,\forall z\in Z,, \forall n\in N_2, \forall k\in K, \forall t\in T$

20) $w^{P}_{pnt} \ge 0$ $,\forall p\in P,, \forall n\in N_1,\ \forall t\in T$

21) $w^{Z}_{znt} \ge 0$ $,\forall z\in Z, \forall n\in N_2,\ \forall t\in T$

3. Three decomposed models

The PIDPRP is highly intractable and thus in practice the production, inventory and transportation decisions are frequently taken in a decomposed manner; this decomposition can be done in various ways. In this section we present three alternative decomposition approaches: PIP (Production Inventory Problem) & DPRP (Distribution Pick-up Routing Problem), PIP & DRP (Distribution Routing Problem) & PRP (Pick-up Routing Problem), PIDRP (Production Inventory Distribution Routing Problem) & PRP.

3.a PIP & DPRP:

In this model, production decisions are made first. Then distributions to retail points and pick-up from suppliers are determined together, while the results of the production model are taken as given. In the PIP, we consider a production inventory problem. The demand is determined by the total demand of all the retail locations and the (typically lower) holding cost of the plant is used. In the DPRP, routing decisions are made, so as to minimize the total transportation cost and the extra *echelon inventory holding cost* incurred by transferring products to retailer points before the demand is realized and by transferring raw material before they are needed for production.

PIP:

Objective function: we keep expressions (2), and (4).

Constraints: constraints (1), (2), (13), (14), and (15) of the PIDPRP and the following inventory balance constraint that replaces constraint (3).

$$x_{pt}^{HP} = x_{pt}^{P} - \sum_{n \in N_1} d_{pnt} + x_{p(t-1)}^{HP} \quad, \forall p \in P, \forall t \in T \quad \textbf{(3)}$$

DPRP:

Additional parameters: the values of x_{pt}^{P}, x_{pt}^{HP} from the optimal solution of the PIP are taken as parameters.

Objective function: we keep expressions (1), (5), (6), (7), (8) and we add $x_{pnt}^{HN_1}\left(h_{pn}^{N_1} - h_{p}^{P}\right)$ to account the extra holding cost of the lower echelon. Note that h_{p}^{P} was already incurred in PIP.

Constraints: we keep the following constraints: (4) – (11), (12), (16) - (21).We add a new constraint that keeps the total inventory level at the retail location not greater than the total upper echelon inventory upon which we decided in the solution of PIP.

$$x_{pt}^{HP} \geq \sum_{n \in N_1} x_{pnt}^{HN_1} \quad, \forall p \in P, \forall t \in T \quad \textbf{(22)}$$

3.b PIP & DRP & PRP:

Here we solve the PIP and decouple the DPRP into two distinct routing problems, one for the finished products and one for the raw materials. Consequently the opportunity to save deadhead is lost.

DRP:

Additional parameters: the values of the variables x_{pt}^{HP} from the optimal solution of PIP are taken as parameters.

Objective function: we keep expressions (6) and (7). As in DPRP, expression (3) is replaced by $x_{pnt}^{HN_1}\left(h_{pn}^{N_1} - h_p^P\right)$.

Constraints: we keep constraints: (5), (6), (8), (10), (11), (12), (17), (18), (20), and (22).

PRP:

Sets: here the node set is $N = N_0 \cup N_2$, and G includes only tours over this set.

Additional parameters: the values of the variables x_{pt}^{P} from the optimal solution of PIP are taken as parameters.

Objective function: we keep expressions (1), (5), (6), (8).

Constraints: we keep the following constraints: (4), (7), (9), (10), (11), (12), (16), (19) and (21).

3.c PIDRP & PRP:

Decisions on production, inventory and routing of products are made first. Then the routing of the raw material is determined. In PIDRP, we consider a production inventory distribution and routing problem similar to the one studied by Fumero and Vercellis (1999).

PIDRP:

Sets: here the node set is $N = N_0 \cup N_1$, and G includes only tours over this set.

Objective function: we keep expressions (2), (3), (4), (6), (7).

Constraints: we keep the following constraints: (1), (2), (3), (5), (6), (8), (10), (11), (12), (13), (14), (15), (17), (18), and (20).

4. Results

In this section we present results of the PIDPRP and compare them to the three decomposed models presented in the previous section. We used a 7 nodes network with a plant, four retail points and two suppliers. The plant produces two types of products using three types of raw materials. The fleet includes two types of trucks. We suppose there is an infinite quantity of trucks (6 trucks in our test problems). All possible 63 tours, which satisfy the problem's constraints, are enumerated. In the preprocessing phase we solve the TSP problem for each of these tours. The planning horizon consists of six periods. The mathematical model was programmed in OPL and solved using ILOG CPLEX 10.2 on Intel Pentium 4, 3.6 Ghz with 2Mb of Ram. Table number 3 shows the number of variables (continuous and binary), constraints and non-zero coefficients for each of the models.

	PIDPRP	DRP	PIP	PRP	DPRP	PIDRP
Continuous variables	742	397	28	321	714	425
Binary variables	2274	540	6	108	2268	546
Constraints	2794	315	43	225	500	344
Non zero coefficients	25563	5088	106	1692	25134	5487

Table 3: number of variables, constraints and non zero coefficient for any model

Tables 4-9 bellow show the results of six experiments. The results of each experiment are within 0.01% of optimality. Table 4 represents the results of a basic experiment, and each of the subsequent tables represents a certain change in the model parameters as specified at any table reference.

	PIDPRP		PIP & DPRP		PIP & DRP & PRP		PIDRP & PRP	
COSTS	value	%	value	%	value	%	value	%
Purchase of raw materials	83.22	8.22	89.34	8.33	88.34	8.2	88.34	8.2
Production	440	43.5	440	41.05	440	40.85	440	40.85
Inv. of finished products	207.84	20.55	144.6	13.48	144.6	13.42	144.6	13.42
Inv. of raw materials	78.64	7.77	167	15.58	167	15.5	167	15.5
Transportation by fleet	171.56	16.96	170.04	15.86	179.88	16.7	179.88	16.7
Transportation by carrier	30.1	2.97	60.86	5.68	57.36	5.32	57.36	5.32
Total cost	1011.36	100	1071.84	100	1077.19	100	1077.19	100
Comparison (%)			5.98		6.51		6.51	
Solver time (seconds)	2557.19		2.77		0.03		9.59	

Table 4: experiment 1 – basic experiment

	PIDPRP		PIP & DPRP		PIP & DRP & PRP		PIDRP & PRP	
COSTS	value	%	value	%	value	%	value	%
Purchase of raw materials	82.4	9.53	81.1	9.29	80.1	9.04	80.1	9.04
Production	450	51.78	360	41.25	360	40.62	360	40.62
Inv. of finished products	61.6	7.09	133.6	15.31	133.6	15.07	133.6	15.07
Inv. of raw materials	77.89	8.96	97.87	11.21	86.97	9.81	86.97	9.81
Transportation by fleet	196.83	22.64	200.04	22.92	213.07	24.04	213.07	24.04
Transportation by carrier	30.1	3.46	0	0	12.6	1.42	12.6	1.42
Total cost	869.16	100	872.61	100	886.34	100	886.34	100
Comparison (%)			0.39		1.97		1.97	
Solver time (seconds)	6720.82		104.18		1.53		4.51	

Table 5: experiment 2 – inventory costs of raw materials are lower (1/6, 1/4, 1/2), inventory costs at retail points are higher (x2, x1.5, constant), production cost is lower (5/6), fixed costs of trucks are higher (x1.1, constant)

	PIDPRP		PIP & DPRP		PIP & DRP & PRP		PIDRP & PRP	
COSTS	value	%	value	%	value	%	value	%
Purchase of raw materials	89.72	9.00	93.3	9.19	98.34	9.63	98.34	9.63
Production	440	44.16	440	43.33	440	43.09	440	43.1
Inv. of finished products	189.4	19.01	144.6	14.23	144.6	14.16	144.6	14.16
Inv. of raw materials	58.7	5.89	87	8.57	87	8.52	87	8.52
Transportation by fleet	184.11	18.48	186.11	18.33	186.84	18.29	186.84	18.29
Transportation by carrier	34.47	3.46	64.36	6.34	64.36	6.30	64.36	6.3
Total cost	996.39	100	1015.37	100	1021.14	100	1021.14	100
Comparison (%)			1.90		2.48		2.48	
Solver time (seconds)	7862		3.52		3.57		14.5	

Table 6: experiment 3 - quantity of raw materials available in inventory at period zero is lower (4/5), the volume of one type of trucks is lower (5/6)

	PIDPRP		PIP & DPRP		PIP & DRP & PRP		PIDRP & PRP	
COSTS	value	%	value	%	value	%	value	%
Purchase of raw materials	83.22	8.46	89.34	8.68	88.34	8.53	88.34	8.53
Production	400	40.69	400	38.86	400	38.64	400	38.64
Inv. of finished products	212.84	21.65	144.6	14.05	144.6	13.96	144.6	13.96
Inv. of raw materials	75.74	7.705	150.2	14.59	150.2	14.51	150.2	14.51
Transportation by fleet	181.06	18.42	184.13	17.89	194.55	18.79	194.55	18.79
Transportation by carrier	30.1	3.06	60.86	5.913	57.36	5.54	57.36	5.54
Total cost	982.96	100	1029.13	100	1035.05	100	1035.05	100
Comparison (%)			4.69		5.29		5.29	
Solver time (seconds)	4700		3.27		0.53		8.005	

Table 7: experiment 4 – resources of production are lower (2/3), costs of trucks per unit of distance are higher (x1.1), costs of inventory of raw materials are lower (1/2, constants), fixed cost of production is lower (9/10)

	PIDPRP		PIP & DPRP		PIP & DRP & PRP		PIDRP & PRP	
COSTS	value	%	value	%	value	%	value	%
Purchase of raw materials	72.84	10.22	72.94	10.23	72.84	9.99	76.18	10.36
Production	350	49.1	350	49.1	350	47.999	280	38.08
Inv. of finished products	61.6	8.64	61.6	8.64	61.6	8.449	139.6	18.98
Inv. of raw materials	32.98	4.62	32.98	4.62	32.98	4.529	39.12	5.32
Transportation by fleet	195.43	27.41	195.43	27.41	211.84	29.05	200.09	27.21
Transportation by carrier	0	0	0	0	0	0	0.21	0.028
Total cost	712.85	100	712.85	100	729.26	100	735.20	100
Comparison (%)			0		2.30		3.13	
Solver time (seconds)	12849		87.53		1.765		9.79	

Table 8: experiments 5 - quantities of raw materials available in inventory at period zero are higher (6/5), resources of production are lower (2/3), costs of inventory of raw material are lower (1/10), inventory costs at retail points are higher (x2, x1.5, constant), fixed cost of production is lower (3/5)

	PIDPRP		PIP & DPRP		PIP & DRP & PRP		PIDRP & PRP	
COSTS	value	%	value	%	value	%	value	%
Purchase of raw materials	70.1	9.69	70.1	9.37	74.66	9.71	74.66	9.71
Production	300	41.47	300	40.1	300	39.03	300	39.03
Inv. of finished products	35.25	4.87	0	0	0	0	0	0
Inv. of raw materials	133.35	18.43	187.4	25.05	187.4	24.38	187.4	24.38
Transportation by fleet	184.36	25.48	190.11	25.41	204.63	26.62	204.63	26.72
Transportation by carrier	0.42	0.058	0.42	0.056	1.89	0.25	1.89	0.25
Total cost	723.47	100	748.03	100	768.58	100	768.58	100
Comparison (%)			3.39		6.23		6.23	
Solver time (seconds)	82048		52.12		3.85		5.07	

Table 9: experiments 6 - quantities of raw materials available in inventory at period zero are lower (constants, 4/5), proportions of raw materials to produce items are lower (3/5, constants), resources of production are lower (2/3), fixed cost of production is lower (1/2), cost of transportation by a carrier is lower (1/2, constants)

The first main observation is that, at any experiment the total cost of the PIP & DPRP is the closest to the total cost of the PIDPRP. In the PIP & DPRP, production decisions are separated from routing decisions, however decisions regarding pick-up and delivery are made simultaneously, thus addressing the deadheading issue. Thus it appears that PIP & DPRP could be generally an efficient alternative to PIDPRP.

The second main observation is that for all the experiments, except experiment 5, the total costs of PIP & DRP & PRP and PIDRP & PRP are exactly the same. The explanation is that the production planning of PIP and PIDRP are the same for all these experiments as we observed in the results and in these specific instances the production planning has the main influence on the other decisions. In experiment 5, the total costs PIDRP & PRP are higher than the total costs for PIP & DRP & PRP. The explanation is that in experiment 5, the production planning is different for PIP and PIDRP. Then we obtained a higher cost for PRP with the input of the production plan obtained by the PIDRP than the cost of PRP with the input of the production plan obtained by the PIP. Although in experiment 5, the total costs of PIP & DRP were lower than the total cost of the PIDRP, the respective cost of PRPs made the difference. We learn from this result that a more integrated model does not necessarily lead to a better solution.

We observe that the relative differences between the optimal solution of PIDPRP and the three decomposed models are on average 2.725% for PIP & DPRP, 4.13% for PIP & DRP & PRP and 4.268% for PIDRP & PRP. These differences are not negligible in industries with

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small profit margins. It shows the effectiveness of integrating the pick-up of raw materials to the others decisions.

Solution times of PIDPRP are very high compared to those of the decomposed models. From table 3, we observe that, although the non-zero coefficients of PIDPRP (25563) and PIP & DPRP (25240) are similar, their solver times are a lot different.

Of course, all the previous observations are based on results that are applicable for the small problems we tested, with the specific values that were used. More experiments are needed to check whether similar conclusions hold in other cases.

5. Conclusion

In this paper, we presented an original modeling approach that takes joint decisions of production, inventory, distribution, pick-up and routing (PIDPRP). We have shown the effectiveness of PIDPRP in comparison to three decomposed models. However, more extensive numerical tests are still needed to establish this observation.

Further research is needed to devise faster algorithms that can tackle larger instances of this problem. One promising direction with this regard is to reduce the number of enumerated tours by removing dominated and non promising ones. In addition it may possible to decompose the problem in a different way, in order to reduce the solution time while keeping more of the qualities of the integrated approach.

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