Integral Transforms (2018)

Exercise 2/Week 44

1. Calculate the inverse Laplace transform $\mathcal{L}^{-1}(\frac{1}{s^3-as^2};t)$ by applying

   a) partial fractions, b) the convolution theorem.

2. Using the Laplace transform solve the differential equation

$$y' + 4y = 6\cos t; \quad y(\pi) = 2.$$

3. Using the Laplace transform solve the differential equation

$$4y'' + 8y' + y = t; \quad y(0) = 0,\ y'(0) = 1.$$

4. Using the Laplace transform solve the integro-differential equation

$$y'(t) + 2t + \int_0^t y(u)\,du = 0; \quad y(0) = 1.$$

5. Using the Laplace transform solve the differential equation

$$y'' + 3y' + 2y = x(t); \quad y(0) = y'(0) = 0,$$

where

a) $x(t) = u(t-1) - u(t-2) = \begin{cases} 1, & 1 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$

b) $x(t) = \delta(t-1)$. (Here δ is the Dirac delta function, i.e., δ-distribution.)