Integral Transforms (2018)

Exercise 5/Week 47
NOTE: The exam is on Monday 26.11.2018, at 14:00-16:00 (lecture hall: F141).
Contents: The material in Lecture Notes, see also the corresponding material in the book of Kreyszig, and the weekly Exercises 1–5.
(Cf. Finnish Lecture Notes: Chapters 4, 6–8 (excluding Section 7.8)).

1. By using Fourier transforms derive for the differential equation

$$f' + af = g,$$

where g is a given fixed function and $a > 0$ is a constant, the following formula for its solution

$$f(x) = e^{-ax} \int_{-\infty}^{x} e^{at} g(t)\, dt.$$

Hint: Apply the Fourier transform calculated in Exercise 4.3.

2. Apply the result in the previous exercise to derive for the solution of the initial value problem

$$y' + ay = g; \qquad y(x_0) = y_0,$$

the following formula

$$y(x) = e^{-a(x-x_0)} \left(y_0 + \int_{x_0}^{x} e^{a(t-x_0)} g(t)\, dt \right).$$

Hint: The general solution for the corresponding nonhomogeneous equation has the form $Ce^{-ax} + f(x)$, where $f(x)$ is the solution given in the previous exercise and C is a constant determined with the aid of the given initial value condition.

3. Find a solution to the nonhomogeneous differential equation

$$f' + 2f = \cos t,$$

a) by using the formula derived in Exercise 5.1 above;
b) by Fourier transforming the equation and applying $\mathcal{F}$-transformation tables in Appendix A.

4. Using the $\mathcal{Z}$ transform solve the difference equation

$$y(n) + 2y(n-1) = \left(\frac{1}{3}\right)^n; \quad y(-1) = 0. \qquad (n \geq 0)$$

5. Using the $\mathcal{Z}$ transform solve the difference equation

$$y(n) + \frac{3}{5}y(n-1) - \frac{4}{25}y(n-2) = 3\delta(n-1); \quad y(-1) = y(-2) = 0. \qquad (n \geq 0)$$

Note: In Exercises 3 – 5 check correctness of the solution (including initial values) by substitution into the given equation.