Short term volatility changes and active portfolio management

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Abstract

In this paper the more traditional active portfolio management discussed in Treynor and Black (1973), Rosenberg (1979), Rudd and Clasing (1982), and Pynnönen (1995) is extended to utilize the changing volatility in addition to the predicted short term residual returns. It is shown that taking this new aspect into account enables investors to perform a more detailed analysis of the sources of required return that should be gained by active portfolio management. Furthermore, the approach enables investors to predict future end period volatility in the investment horizon on the basis of current information. In this paper the necessary formulas are derived for practical portfolio analysis. In addition, in contrast to the above papers, the possible change in the market position is taken explicitly into account, and its impact on active management is included in the analysis. This situation becomes relevant when short selling is not allowed.

Keywords: Finance; Active portfolio management; Optimization; ARCH; Volatility

1. Introduction

Modern portfolio theory (MPT) serves as the scientific approach to investment. The basic theory is established by the pioneering work of Harry Markowitz (1959). Latest developments in this area can be found from Rudd and Clasing (1982), Elton and Gruber (1995), Grinold and Kahn (1995) and Pynnönen (1995). The key idea of the theory is an attempt to model the relation between risk and reward. With the model investors can quantitatively maximize the expected reward consistent with their willingness to bear risk. To maintain optimality in the short run this requires frequent rebalancing of the portfolio in accordance to the market information. This process is usually referred as *active portfolio management*.

The main ingredients in adjusting the portfolio in response to the new information are the return and risk (volatility or standard deviation) estimates coupled with the investor's risk aversion. Although it has been long known to the investment community that the return series in speculative markets tend to have short term volatility clusterings in the pass of time [Mandelbrot (1963), Fama (1965)], the risk component has traditionally been considered time invariant [Rosenberg (1979), Pynnönen (1995), Treynor and Black (1973), Rudd and Clasing (1982), Grinold and Kahn (1995)].

Probably one of the main reasons for this has been that time-varying volatility was not successful modeled until Engle (1982). He found an elegant and operationally simple way to model volatility clusterings via Autoregressive Conditional Heteroscedasticity (ARCH) [for the latest development in this area see, Bollerslev *et al.* (1992)]. The ARCH-models are successfully applied in modeling short term volatility changes by Engle (1982), Akgiray (1989), Randolph (1991), Schlag (1991), Booth *et al.* (1992), Engle and Susmel (1993), and Glosten *et al.* (1993), to mention a few.

Because active portfolio management is short term policy, it is natural to expect that the short term shocks that affect the return, and hence the short term portfolio revision, are also related to the short term volatility. This suggests that the short term shocks should be taken into account in the active revision policy not only on the return side but also on the volatility side. The main benefit is that the investor can predict the future level of volatility on the basis of the currently available information, and device out the required return that should be gained to compensate the current level of riskiness (volatility) if the portfolio is revised.

In this paper we shall extend the more traditional active portfolio management discussed in Treynor and Black (1973), Rosenberg (1979), Rudd and Clasing (1982), and Pynnönen (1995) to exploit the changing volatility and the short term residual returns anticipated by the investor. Taking these new aspects into account enables investors to perform a more detailed analysis of the sources of required return that should be gained by active portfolio management. In this paper we shall derive the required formulas for portfolio analysis. Furthermore, contrary to the above referred papers, in this paper we shall also allow a possible change in the market position to be taken into account, and its impact on active management is included in the formulas. This point becomes relevant when short selling is not allowed.

2. Active and Normal Portfolio

We shall assume that the *excess return* series (excess over/under the risk free return),

r_{it}, of the ith share can be modeled by the *market model*

$$r_{it} = \alpha_{it} + \beta_i r_{mt} + \epsilon_{it}, \tag{2.1}$$

where r_{mt} is the market excess return, ϵ_{it} is the residual return with zero mean and conditional (with respect to past information) variance, ω_{it}^2, following a generalized ARCH-process, GARCH. The parameter α_{it} measures a temporal mispricing or expected residual return of the security, and β_i, the Beta of the stock, indicates the sensitivity of the security with respect to changes of the market portfolio.

It may be noted that adoption of a GARCH-type model for residuals implies that the ordinary least squares (OLS) is no more efficient in estimating Betas. The method of maximum likelihood (ML) must be used instead. Furthermore, one could also model time varying Beta in the GARCH framework, but here we shall, however, treat Beta as a constant during short term portfolio revision periods. For estimating varying Beta in the GARCH framework, the interested reader is referred to Bollerslev *et al.* (1988).

In addition to the constant Beta, we assume that the residual returns are uncorrelated with the market portfolio and with other securities. This implies that, if (2.1) is a portfolio consisting of q shares, and Q denotes the total number of shares on the markets, then the residual variance can be written as

$$\omega_{pt}^2 = \sum_{h=1}^{Q} (h_i^a)^2 \omega_{it}^2, \tag{2.2}$$

where $h_i^a = h_i - \beta_p w_i$ is the *active holding* with market weight w_i and h_i the actual proportion invested on the ith share. β_p denotes the portfolio Beta. Note that $h_i^a = -\beta_p w_i$, if share i is not in the portfolio. For further details see Pynnönen (1995).

The *active portfolio* which results from an active strategy, discussed in the next section, is indicated by the difference between the investor's current and *normal* portfolio. Here we assume that the normal portfolio is the investor's initial situation where he/she has allocated a fraction, β_n, of his/her funds to the risky market portfolio and the rest into the riskless alternative.

We assume that a quadratic utility function is a sufficient approximation for the investor's preferences, such that the indifference curves can be presented as

$$C(\mu, \sigma) = \mu - \lambda\sigma^2, \tag{2.3}$$

where C denotes the indifference curve, and λ denotes the investor's risk aversion parameter. Adopting this preference structure implies that long run optimality requires that the investor allocates

$$\beta_n = \frac{\mu_m}{2\lambda\sigma_m^2} \tag{2.4}$$

of his/her funds to the risky market and the rest, $1 - \beta_n$, to the riskless alternative [see e.g. Rosenberg (1979) and Pynnönen (1995) for further details]. Here μ_m is the market's expected excess return and σ_m^2 is the variance of the market return. Hence, knowing the long term policy one knows instantaneously also the investor's risk aversion parameter, λ. It is worth noting that the expected long term return, μ_n, for the risky normal portfolio is then $\mu_n = \beta_n\mu_m$ with variance $\sigma_n^2 = \beta_n^2\sigma_m^2$. This gives another important relation that in optimality it must hold that

$$2\beta_n\lambda = \frac{\mu_m}{\sigma_m^2}, \tag{2.5}$$

i.e., in optimality the mean/variance ratio must equal the (adjusted) risk aversion parameter, $2\beta_n\lambda$. Of course this is just restatement of relation (2.4), but the interpretation is important, when one revises the portfolio: Relation (2.5) must hold in every optimality situation!

It is natural to think that the risk aversion parameter is fairly constant for each individual over time, and hence so is β_n. Otherwise it would be impossible to make any rational plans for the future. Adopting this assumption, relations (2.4) and (2.5) provide the basic relations on which one can build the optimal active strategy, i.e., how the portfolio should be revised in response to the information affecting share prices.

Decomposition of the return in the manner of (2.1) implies that the expected return, μ_{pt}, and variance, σ_{pt}^2, of the investor's current risky portfolio on which he/she has allocated a fraction β_n of his/her money, become $\mu_{pt} = \beta_p\mu_m + \alpha_{pt}$, and $\sigma_{pt}^2 = \beta_p^2\sigma_m^2 + \omega_{pt}^2$. Here we denote the market parameters as time invariant, since changes in them are a matter of market timing, considered later in this paper.

Relation (2.5) tells us that in the optimum the current portfolio should be such that $2\beta_n\lambda = \mu_{pt}/\sigma_{pt}^2$. But then, because $\mu_{pt} = \beta_p\mu_m + \alpha_{pt}$, and $\sigma_{pt}^2 = \beta_p^2\sigma_m^2 + \omega_{pt}^2$, the residual return α_{pt} should be such that

$$2\beta_n\lambda = \frac{\beta_p\mu_m + \alpha_{pt}}{\beta_p^2\sigma_m^2 + \omega_{pt}^2}. \tag{2.6}$$

The special residual return, α_{pt}, that makes (2.6) true, is called the *required Alpha*, and we shall denote it as α_{pt}^R. Solving for it from (2.6) and using (2.5) gives,

$$\alpha_{pt}^R = \frac{\omega_{pt}^2}{\sigma_m^2}\mu_m + (\beta_p - 1)\beta_p\mu_m. \tag{2.7}$$

The last term on the right hand side of (2.7) represents the additional required return due to the changed exposure to the market risk of the revised portfolio. If the market risk does not change, i.e., $\beta_p = 1$, then the term disappears. Moreover, it is important to note that everything else on the right hand side of (2.7) is time invariant except the residual variance. Thus, using modern techniques for estimating the conditional risk measure, ω^2_{pt}, (2.7) provides a simple tool for an investor to see how much gain should be available in order to respond actively to a new piece of information.

Hence, allowing the residual risk dynamics enables the investor to exploit the most up to date information of the risk prospects prevailing in the market. This has obvious advantages over the earlier practice with constant (static) residual risk. Using the GARCH-machinery it is possible to estimate the impact of the new piece of information on the riskiness of a security. The latest evidence suggest that an asymmetric ARCH-model emphasizing negative shocks would be more appropriate than the symmetric one [see e.g. Ding *et al.* (1993)]. We shall, however, demonstrate our procedure with a low order symmetric GARCH-model, which has often proved to be a reasonable description of the risk dynamics.

Adopting this practice, suppose that the residual variances of the individual shares follow GARCH(1,1) processes, such that for the ith share we have the conditional variance

$$\omega^2_{it} = \theta_{i0} + \theta_{i1}\epsilon^2_{i,t-1} + \phi_i\omega^2_{i,t-1} \tag{2.8}$$

with standard stationarity assumptions. Then fairly simple algebra shows that the unconditional (long term) variance of the residuals is

$$\sigma^2_{\epsilon_i} = \mathrm{var}(\epsilon_{it}) = \frac{\theta_{i0}}{1-\theta_{i1}-\phi_i}. \tag{2.9}$$

Solving for θ_{i0}, we can write (2.8) as

$$\omega^2_{it} = (1-\theta_{i1}-\phi_i)\sigma^2_{\epsilon_i} + \theta_{i1}\epsilon^2_{i,t-1} + \phi\omega^2_{i,t-1}. \tag{2.10}$$

Hence, the short term volatility can be interpreted as a weighted sum of the long term volatility and current shocks. Consequently using only $\sigma^2_{\epsilon_i}$ as the end period volatility gives a very limited picture of the reality.

Although ARCH-processes aggregate over time they do not aggregate over shares. In spite of this, we can break down the conditional variance of the portfolio into a form emphasizing the long and short term variance components, as follows

$$\omega^2_{pt} = \mathrm{var}(\epsilon_{pt}|\Psi_{t-1}) = \sum_{i=1}^{Q}(h^a_i)^2\omega^2_{it} = \sigma^2_p(\theta,\phi) + \epsilon^2_{p,t-1}(\theta) + \omega^2_{p,t-1}(\phi), \tag{2.11}$$

where Ψ_t denotes the information set available at time point t, $\sigma^2_p(\theta,\phi) = \sum_{i=1}^{Q}(h^a_i)^2(1-\theta_{i1}-\phi_i)\sigma^2_{\epsilon_i}$, $\epsilon^2_{p,t-1}(\theta) = \sum_{i=1}^{Q}(h^a_i)^2\theta_{i1}\epsilon^2_{i,t-1}$, and $\omega^2_{p,t-1}(\phi) = \sum_{i=1}^{Q}(h^a_i)^2\phi_i\omega^2_{i,t-1}$.

Replacing ω^2_{pt} in (2.7) by the right hand side of (2.11) suggests even further decomposition of the required excess return such that

$$\begin{aligned} \alpha^R_{pt} &= \frac{\sigma^2_p(\theta,\phi)}{\sigma^2_m}\mu_m + \frac{\epsilon^2_{p,t-1}(\theta) + \omega^2_{p,t-1}(\phi)}{\sigma^2_m}\mu_m + (\beta_p - 1)\beta_p\mu_m \\ &= \alpha^{RL}_p + \alpha^{RS}_{pt} + \alpha^{RM}_p, \end{aligned} \tag{2.12}$$

where

$$\alpha^{RL}_p = \frac{\sigma^2_p(\theta,\phi)}{\sigma^2_m}\mu_m \tag{2.13}$$

denotes the stake of required alpha due to long term residual risk,

$$\alpha^{RS}_{pt} = \frac{\epsilon^2_{p,t-1}(\theta) + \omega^2_{p,t-1}(\phi)}{\sigma^2_m}\mu_m \tag{2.14}$$

denotes the stake of required alpha due to current risk prospects, and

$$\alpha^{RM}_p = (\beta_p - 1)\beta_p\mu_m \tag{2.15}$$

denotes the stake of required return due to change in market risk as a consequence of the revision policy.

Finally, suppose that the investor wants to predict volatility s periods ahead. Then using the iterated law of conditional expectations, we can write

$$\text{var}(\epsilon_{t+s}|\Psi_t) = \text{E}_t(\epsilon_{t+s}^2) = \text{E}_t\text{E}_{t+1}\cdots\text{E}_{t+s-1}(\epsilon_{t+s}^2), \tag{2.16}$$

where E_t denotes the conditional expectation given the information set Ψ_t. Now $\text{E}_{t+s-1}(\epsilon_{t+s}^2) = \theta_0 + \theta_1\epsilon_{t+s-1}^2 + \phi\omega_{t+s-1}^2$. Substituting this repeatedly gives finally

$$\begin{aligned} \text{var}(\epsilon_{t+s}|\Psi_t) &= \theta_0\sum_{i=0}^{s-2}(\theta_1+\phi)^i + (\theta_1+\phi)^{s-1}\omega_{t+1}^2 \\ &= \theta_0\frac{1-(\theta_1+\phi)^{s-1}}{1-(\theta_1+\phi)} + (\theta_1+\phi)^{s-1}\omega_{t+1}^2. \end{aligned} \tag{2.17}$$

3. Active Strategy

3.1 Market Timing

Market timing consists of swapping investor's money between the market portfolio and the risk free alternative. The swapping occurs in response to the general estimated direction of the market as a whole.

Relation (2.4) serves as a guide in responding to the new global market information. Traditionally it is thought that new information affects only the return side of the market portfolio. Nevertheless, as was discussed above, it is evident that the short term volatility also changes over short time periods. This short term volatility change brings a new aspect to market timing.

On the return side the response to the market forecast is optimally carried out by changing the market proportion in the portfolio in accordance with

$$\beta_T = \beta_n (1 + \frac{e}{\mu_m}) \tag{3.1}$$

where e is the expected change of the market in the near future. β_T is called the *target* Beta of market timing [see Pynnönen (1995) for further details].

The new risk position of the policy is

$$\sigma_T^2 = \beta_n^2 \sigma_m^2 + \mathrm{var}(\Delta\beta)(\sigma_m^2 + \mu_m^2), \tag{3.2}$$

where $\Delta\beta = \beta_T - \beta_n$ [Pynnönen (1995)].

Again, by straightforward algebra, in the optimality, the new strategy should result to a portfolio yielding

$$\alpha_T^R = \frac{\beta_n}{\mu_m} \mathrm{var}(e). \tag{3.3}$$

Here we have assumed that the market volatility is the long term volatility. Assume next that the uncertainty in the markets as a whole changes, but the investor does not have scenarios of the market direction different from the consensus. Then denoting the short term market volatility as σ_{mt}^2 ($= \mathrm{var}(r_{mt}|\Psi_{t-1})$), we get from the basic relation (2.4) the targeted market proportion as

$$(3.4) \qquad \beta_{vt} = \frac{\mu_m}{2\lambda\sigma_{mt}^2} = \beta_n \frac{\sigma_m^2}{\sigma_{mt}^2}.$$

Finally, combining the changing short term volatility with the investors forecasts about the future market expectations, the above two strategies, (3.1) and (3.4), can be combined to yield the market proportion for this kind of optimal market timing as

$$(3.5) \qquad \beta_{Tvt} = \frac{\mu_m + e}{2\lambda\sigma_{mt}^2} = \frac{\beta_n\sigma_m^2}{\sigma_{mt}^2}(1 + \frac{e}{\mu_m}) = \beta_{vt}(1 + \frac{e}{\mu_m}).$$

From a different perspective, using this in place of β_n in (3.3) the required alpha due to market timing with time varying market risk expectations becomes

$$(3.6) \qquad \alpha_{Tt}^R = \frac{\beta_{Tvt}}{\mu_m}\mathrm{var}(e).$$

This is again a useful tool to monitor whether the portfolio manager is optimally responding to the estimates of future prospects.

3.2 Stock selection

In stock selection the interest is in the future prospects of individual stocks. The main difference in this strategy compared to the previous one is diversification. That is, one can appraise at the same time several stocks and balance the portfolio with them in a given time point and across time, whereas in market timing the diversification occurs only between different time points. Another main difference is that the specific risk, (ω^2), must also be considered. Actually this, coupled with the alpha, are the key components in the strategy. Broadly speaking the objective in stock selection is to take a short position in stocks considered over valued and a long position in stocks considered undervalued. Nevertheless, short selling restrictions may prevent undertaking this action.

The most crucial objective in active management is to maintain asset holding at a level where the increase in risk can be exactly justified by an increase in return. Here we shall consider methods to achieve this goal. However, for the sake of simplicity we shall not take into account taxation and brokers' commissions, although especially in Finland they are remarkable and bigger than in countries where equity markets are well developed.

For a systematic and logically defendable active strategy the start off point is analysts' forecasts for current valuation and risk of each stock under consideration. In the framework of the market model, (2.1), this means estimation of stock betas, alphas (α_{it}), and residual variances (ω_{it}^2) for each stock, i.

Treating the expected returns, variances and covariances of the returns of the shares as fixed, the problem reduces to finding holdings on each share such that the highest indifference curve is reached. Without loss of generality we can assume for the sake of simplicity that the normal Beta, β_n, equals unity. More precisely, rewriting (2.3) as

$$C_t(h_1, \ldots, h_q) = \mu_{pt} - \lambda\sigma_{pt}^2 \tag{3.7}$$

where $\mu_{pt} = \sum_{i=1}^q h_i\mu_{it}$, and $\sigma_{pt}^2 = \beta_p^2\sigma_m^2 + \sum_{i=1}^q h_i^2\omega_{it}^2$. The maximum of (3.7) is found by setting the partial derivatives equal to zero, i.e.

$$\frac{\partial C}{\partial h_i} = \mu_i - 2\lambda(\beta_i\beta_p\sigma_m^2 + h_i\omega_{it}^2) = 0. \tag{3.8}$$

We can use (3.8) for two purposes. First if we suppose that the portfolio is given, then remembering that $\mu_{it} = \alpha_{it} + \beta_i\mu_m$, we can solve for α_{it}, resulting in what is called the *required alpha* for the ith share. Denoting these solutions as α_{it}^R, we get from (3.8)

$$\alpha_{it}^R = \frac{h_i\omega_{it}^2}{\sigma_m^2}\mu_m + \beta_i(\beta_p - 1)\mu_m, \tag{3.9}$$

where we have used the relation $2\lambda = \mu_m/\beta_n\sigma_m^2$ with $\beta_n = 1$. The required alpha gives the extra return that should be gained to merit the adopted position in the share. It should be noted that using (2.11), the required alpha could be broken down further into the long term, short term, and market required returns in the manner of (2.12).

Looking at the problem from the other end, suppose that we are equipped with an α estimate for each share. Then we should rebalance the portfolio such that the extra returns would be optimally exploited with respect to our willingness to bear risk. This is achieved by equating the required returns with the (risk adjusted) alpha estimates and solving for the holdings. Doing this yields

$$h_{it} = \frac{\sigma_m^2}{\omega_{it}^2\mu_m}(\alpha_i - \beta_i(\beta_p - 1)\mu_m). \tag{3.10}$$

Hence, summarizing, the above theory provides the investor, on the one hand, a powerful tool to evaluate the optimality of the current portfolio, and on the other hand, a device for optimal weighting when estimates of the share return and risk components are available.

4. Conclusions

In this paper we have considered how changing short term volatility can be taken into account in short term portfolio revision. Use of modern ARCH-technology allows the investor to evaluate the short term environment prevailing in the market.

Using the ARCH-model of volatility, we have shown on the portfolio level that breaking the volatility into long and short term components, one gets a device that enables the investor to analyze what amount of return should be gained from each of the risk sources (long term, short term and changed exposition to the market risk) to merit the adopted portfolio policy. This division can also be extended to share wise monitoring of a given portfolio.

Conversely given estimates of the extra returns of the shares and residual variances the optimal holdings with respect to the investor's preferences can also be solved such that the resulting portfolio corresponds to the required returns.

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