3.6 Residual analysis

Residual analysis provides the user various descriptive statistics and misspesification tests.

Johansen suggests using residuals from the unrestricted model to decide whether the model is acceptable or not.

The assumption is that the error terms are independent and

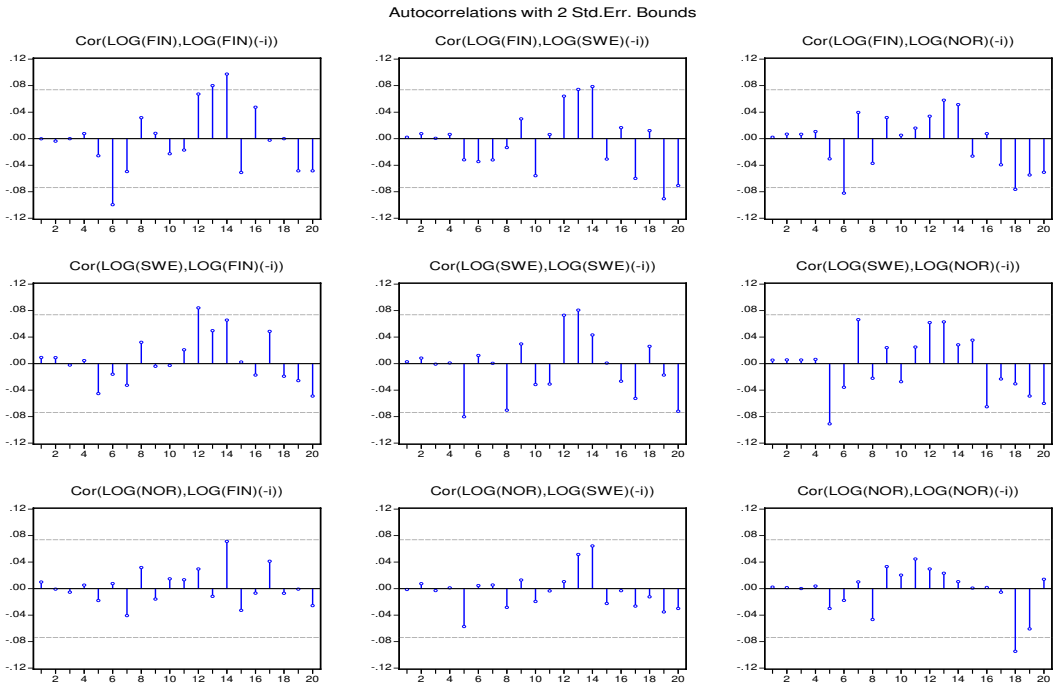
$$\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \Sigma). \tag{14}$$

This implies that one should check: the normality, autocorrelaion of the residuals, and heteroscedasticity.

Autocorrelation

Visual inspection of the correlograms gives quick overview whether there is left significant correlation in the residuals.

In the above example the VAR(5) residual autocorrelaitons are:



Thus, there is potentially some autocorrelation left into the residuals.

Using the portmanteau test (28) of Section 2.2 yields:

```
VAR Residual Portmanteau Tests for Autocorrelations
H0: no residual autocorrelations up to lag h
Date: 04/10/07   Time: 00:43
Sample: 8/27/1993 8/08/1996
Included observations: 737
```

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	0.188209	NA*	0.188465	NA*	NA*
2	0.472229	NA*	0.473258	NA*	NA*
3	0.565575	NA*	0.566985	NA*	NA*
4	0.682799	NA*	0.684849	NA*	NA*
5	9.412860	NA*	9.474541	NA*	NA*
6	22.38088	0.0077	22.54900	0.0073	9
7	34.21203	0.0119	34.49360	0.0109	18
8	46.69438	0.0107	47.11293	0.0096	27
9	49.30719	0.0688	49.75804	0.0633	36
10	55.81576	0.1295	56.35614	0.1194	45
11	62.33375	0.2040	62.97289	0.1886	54
12	71.83347	0.2085	72.62984	0.1904	63
13	83.91262	0.1592	84.92588	0.1415	72
14	94.74395	0.1410	95.96695	0.1226	81
15	99.84530	0.2242	101.1743	0.1977	90
16	107.9511	0.2530	109.4600	0.2219	99
17	117.1676	0.2572	118.8941	0.2228	108
18	131.9173	0.1636	134.0130	0.1345	117
19	142.8649	0.1446	145.2503	0.1156	126
20	151.8788	0.1521	154.5157	0.1200	135

```
*The test is valid only for lags larger than the VAR lag order.
df is degrees of freedom for (approximate) chi-square distribution
```

The Q-test indicate that there is obvious autocorrelation still in the residuals.

However, if we look at the residuals of the restricted ECM model, we have

VEC Residual Portmanteau Tests for Autocorrelations
H0: no residual autocorrelations up to lag h
Date: 04/10/07 Time: 00:51
Sample: 8/27/1993 8/08/1996
Included observations: 737

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	0.894271	NA*	0.895486	NA*	NA*
2	1.622145	NA*	1.625341	NA*	NA*
3	2.474462	NA*	2.481141	NA*	NA*
4	3.270679	NA*	3.281703	NA*	NA*
5	11.31190	0.2549	11.37785	0.2507	9
6	23.44896	0.1739	23.61453	0.1681	18
7	37.94858	0.0787	38.25319	0.0739	27
8	50.90791	0.0509	51.35473	0.0467	36
9	54.15326	0.1647	54.64021	0.1537	45
10	60.55402	0.2513	61.12901	0.2353	54
11	68.52547	0.2954	69.22124	0.2756	63
12	78.34483	0.2846	79.20312	0.2622	72
13	90.99733	0.2098	92.08281	0.1879	81
14	101.3846	0.1937	102.6712	0.1704	90
15	107.4499	0.2639	108.8626	0.2339	99
16	115.0620	0.3031	116.6436	0.2682	108
17	123.7651	0.3165	125.5522	0.2778	117
18	137.4486	0.2290	139.5782	0.1927	126
19	147.2028	0.2231	149.5905	0.1846	135
20	156.3008	0.2284	158.9423	0.1865	144

*The test is valid only for lags larger than the VAR lag order.
df is degrees of freedom for (approximate) chi-square distribution

The above tests suggest that there is in fact no material autocorrelation left behind.

This discrepancy may be due to some big jumps (kinds of outliers) in the return series that affect more in the unrestricted VAR.

Testing for Normality

The normality test relies usually on the skewness and kurtosis of the residuals.

In the previous example for the unrestricted VAR(5) test results are:

VAR Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
H0: residuals are multivariate normal
Sample: 8/27/1993 8/08/1996. Included observations: 737

Component	Skewness	Chi-sq	df	Prob.
1	-0.306533	11.54169	1	0.0007
2	-0.047667	0.279100	1	0.5973
3	0.117985	1.709895	1	0.1910
Joint		13.53068	3	0.0036

Component	Kurtosis	Chi-sq	df	Prob.
1	5.876937	254.1658	1	0.0000
2	5.182095	146.2189	1	0.0000
3	6.312256	336.9024	1	0.0000
Joint		737.2870	3	0.0000

Component	Jarque-Bera	df	Prob.
1	265.7074	2	0.0000
2	146.4980	2	0.0000
3	338.6123	2	0.0000
Joint	750.8177	6	0.0000

The error terms are obviously not normal.

The VECM estimation is to some extend robust for non-normality provided that the errors are symmetrically distributed.

Here even the symmetry is questionable.

Thus, caution must be exercised in interpreting the results.

Non-normality with fat tails is typical for return series.

Heteroscdasticity

In EViews there are available White heteroscedasticity test.

Choosing the White test with no cross terms yields

```
VAR Residual Heteroskedasticity Tests:
No Cross Terms (only levels and squares)
Sample: 8/27/1993 8/08/1996
Included observations: 737

   Joint test:
=========================
Chi-sq       df  Prob.
-------------------------
 369.1247   180  0.0000
=========================

   Individual components:
=============================================================
Dependent   R-squared    F(30,706)    Prob.   Chi-sq(30)  Prob.
-------------------------------------------------------------
res1*res1    0.106181    2.795633    0.0000  78.25531    0.0000
res2*res2    0.062923    1.580233    0.0261  46.37462    0.0286
res3*res3    0.090260    2.334867    0.0001  66.52170    0.0001
res2*res1    0.098325    2.566244    0.0000  72.46561    0.0000
res3*res1    0.069859    1.767484    0.0074  51.48593    0.0086
res3*res2    0.077629    1.980614    0.0015  57.21232    0.0020
=============================================================
```

Again the residual homoscedasticity is rejected.

A stylized fact is that return series have ARCH effect, which is causes here the heteroscedasticity.

However, there is some evidence that at least the cointegration rank tests are robust against moderate ARCH effects.