4. Examples

Example 4.1 Implementation of the Example 3.1 in SAS. In SAS we can use the `Proc Model` procedure.

Simulate data from t-distribution with $\nu = 6$.
SAS:

```
data tdist;
do i = 1 to 500;
 y = tinv(ranuni(1258),6);
 z = 1;
 output;
end;
run;

proc model data = tdist;
 endogenous y;
 parms nu 5;
 eq.h1 = y**2 - nu/(nu-2);
 eq.h2 = y**4 - 3*nu**2/((nu-2)*(nu-4));
 fit h1 h2/gmm;
 instruments z / noint;
run;
```

Results:

The MODEL Procedure

Nonlinear GMM Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE
h1	0.5	499.5	5671.5	11.3544	3.3696
h2	0.5	499.5	8834480	17686.6	133.0

Nonlinear GMM Parameter Estimates

Parameter	Estimate	Approx Std Err	t Value	Approx Pr > |t|
nu	5.374285	0.3931	13.67	<.0001

Number of Observations		Statistics for System	
Used	500	Objective	0.004917
Missing	0	Objective*N	2.4584

Thus $\widehat{\nu} = 5.37$ with standard error 0.339. $J = 2.4584$ with 1 degree of freedom and p-value 0.117. Thus the data does not reject the moment conditions implied by the model.

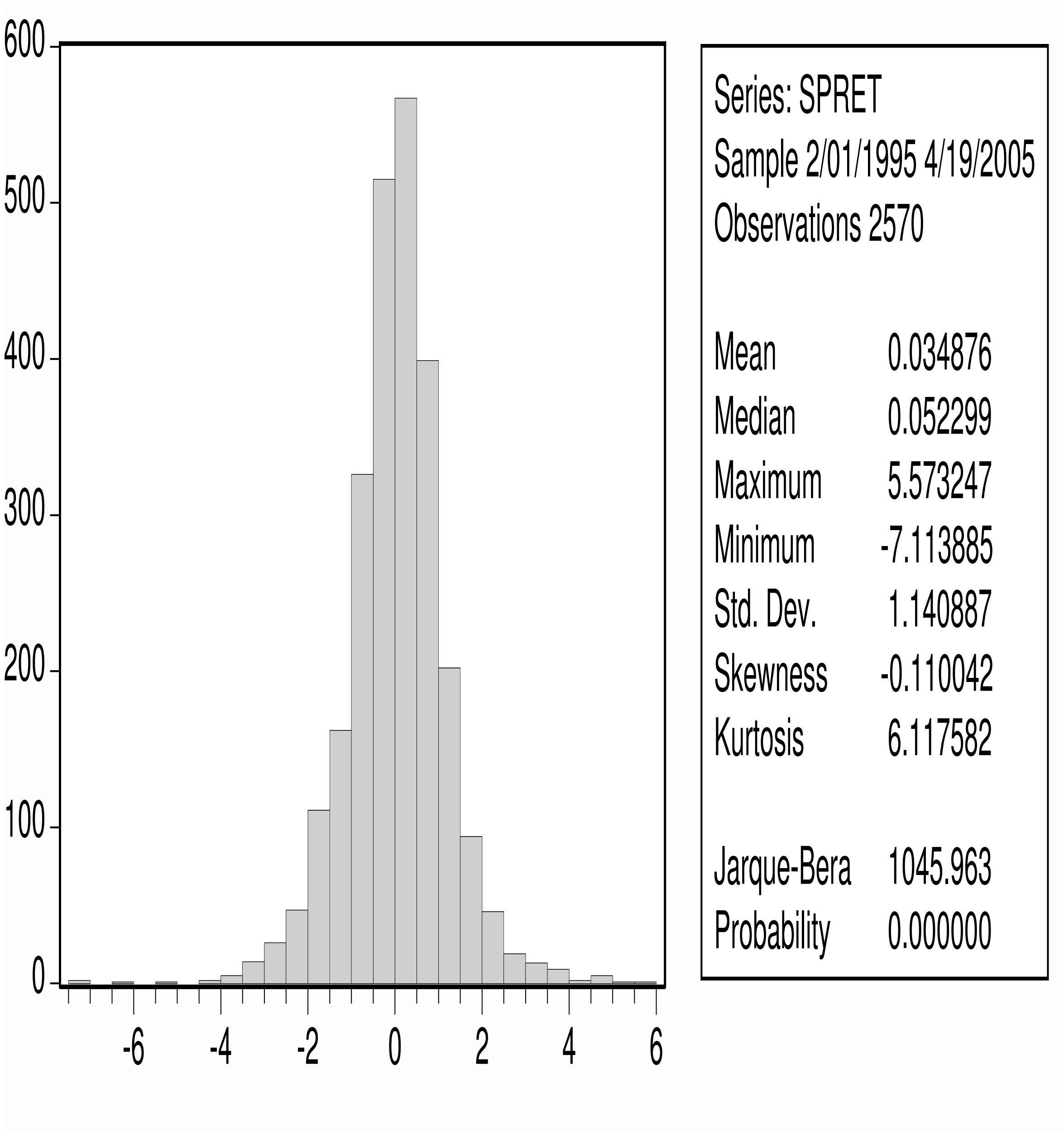
Example 4.2 Normality of SP500 (daily) returns, y_t. If normality holds, the moment conditions:

$$\begin{aligned} y_t - \mu &= 0 \\ (y_t - \mu)^2 - \sigma^2 &= 0 \\ (y_t - \mu)^3/\sigma^3 &= 0 \\ (y_t - \mu)^4/\sigma^4 - 3 &= 0 \end{aligned}$$

where $\mu = \mathrm{E}[y_t]$, $\sigma^2 = \mathrm{Var}[y_t]$. Data is obtained from finance.yahoo.com web site with sample period Jan 2, 1995 to May 19, 2005.
In EViews open Object → New object ... → System → System
and write commands (c(1) = mean, c(2) = standard deviation)

```
@inst c
param c(1) 0 c(2) 1.0
spret - c(1)
(spret - c(1))^2 - c(2)^2
((spret - c(1))/c(2))^3
((spret - c(1))/c(2))^4 - 3
```

Sample statistics



Select Estimate → GMM to get results

System: GMM_NORMALITY
Estimation Method: Generalized Method of Moments
Date: 04/20/05 Time: 02:51
Sample: 2/01/1995 4/19/2005
Included observations: 2570
Total system (balanced) observations 10280
Kernel: Bartlett, Bandwidth: Fixed (8), No prewhitening
Iterate coefficients after one-step weighting matrix
Convergence not achieved after: 1 weight matrix, 506 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.020709	0.020425	1.013894	0.3107
C(2)	1.286071	0.018708	68.74604	0.0000

Determinant residual covariance	74161.73
J-statistic	0.050103

Equation: SPRET - C(1)
Instruments: C
Observations: 2570

S.E. of regression	1.140975	Sum squared resid	3344.388
Durbin-Watson stat	2.022964		

Equation: (SPRET - C(1))^2 - C(2)^2
Instruments: C
Observations: 2570

S.E. of regression	2.964218	Sum squared resid	22563.95
Durbin-Watson stat	1.594675		

Equation: ((SPRET - C(1))/C(2))^3
Instruments: C
Observations: 2570

S.E. of regression	7.010289	Sum squared resid	126202.2
Durbin-Watson stat	2.244303		

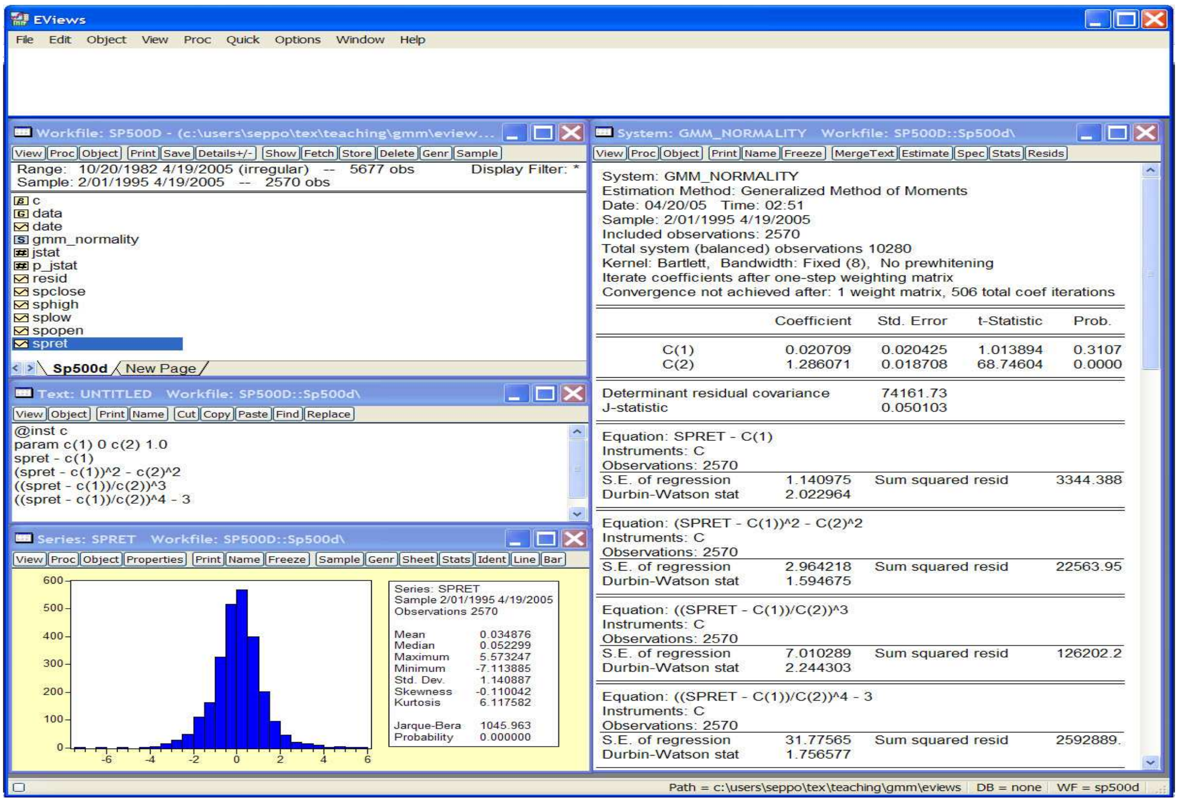
Equation: ((SPRET - C(1))/C(2))^4 - 3
Instruments: C
Observations: 2570

S.E. of regression	31.77565	Sum squared resid	2592889.
Durbin-Watson stat	1.756577		

Note that the J-statistic in EViews is not multiplied by number of observations. Thus

$$J = 2570 \times 0.050103 \approx 128.8,$$

which is highly statistically significant ($df = 4-2 = 2$), and thus rejects the normality hypothesis.



Let us next test whether a t-distribution with location parameter μ, scale parameter σ^2, and degrees of freedom parameter ν fits better. The density function is

$$f(y) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu\sigma^2}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{(y-\mu)^2}{\sigma^2(\nu-2)}\right)^{-\frac{1}{2}(\nu+1)}$$

with

$$\mathrm{E}[y] = \mu,$$

$$\mathrm{Var}[y] = \frac{\nu\sigma^2}{\nu-2},$$

and

$$\mathrm{E}\left[(y-\mu)^4\right] = \frac{3\nu^2\sigma^4}{(\nu-2)(\nu-4)}.$$

The implied moment conditions are

$$\begin{aligned} \mathrm{E}\left[y-\mu\right] &= 0 \\ \mathrm{E}\left[(y-\mu)^2-\sigma^2\tfrac{\nu}{\nu-2}\right] &= 0 \\ \mathrm{E}\left[(y-\mu)^3\right] &= 0 \\ \mathrm{E}\left[(y-\mu)^4-\tfrac{3\nu^2\sigma^4}{(\nu-2)(\nu-4)}\right] &= 0 \end{aligned}$$

EViews estimation produces with commands (c(1) = mean, c(2) = scale, c(3) = df)

```
@inst c
param c(1) 0 c(2) 1.0 c(3) 7.0
spret - c(1)
(spret - c(1))^2 - c(2)^2*c(3)/(c(3)-2)
(spret - c(1))^3
(spret - c(1))^4 - (3*c(2)^4)*(c(3)^2)/((c(3)-2)*(c(3)-4))
```

System: GMM_T
Estimation Method: Generalized Method of Moments
Date: 04/25/05 Time: 00:29
Sample: 2/01/1995 4/19/2005
Included observations: 2570
Total system (balanced) observations 10280
Kernel: Bartlett, Bandwidth: Fixed (8), No prewhitening
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 8 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.040571	0.019412	2.089975	0.0366
C(2)	0.933509	0.029089	32.09109	0.0000
C(3)	6.132127	0.436527	14.04754	0.0000
Determinant residual covariance		2346223.		
J-statistic		0.000233		

Equation: SPRET - C(1)
Instruments: C
Observations: 2570

S.E. of regression	1.140902	Sum squared resid	3343.955
Durbin-Watson stat	2.023225		

Equation: (SPRET - C(1))^2 - C(2)^2*C(3)/(C(3)-2)
Instruments: C
Observations: 2570

S.E. of regression	2.945789	Sum squared resid	22275.58
Durbin-Watson stat	1.616382		

Equation: (SPRET - C(1))^3
Instruments: C
Observations: 2570

S.E. of regression	14.94928	Sum squared resid	574122.4
Durbin-Watson stat	2.241108		

Equation: (SPRET - C(1))^4 - (3*C(2)^4)*(C(3)^2)/((C(3)-2)*(C(3)-4))
Instruments: C
Observations: 2570

S.E. of regression	87.50198	Sum squared resid	19654482
Durbin-Watson stat	1.761688		

The J-statistic is $J = T \times J_{\text{EViews}} = 2570 \times 0.000233 = 0.598$ with p-value 0.439 ($df = 1$), which indicates the return distribution seems to behave like a the t-distribution at least up to the first four moments with estimates $\widehat{\mu} = 0.041$, $\widehat{\sigma} = 0.934$, and $\widehat{\nu} = 6.132$.

Example 4.4 Estimation of Dynamic Rational Expectations Model.

Hansen and Singleton (1982). Generalized instrumental variables method of nonlinear rational expectations models. *Econometrica* 50, 1269–1286. Errata: *Econometrica* 52, 267–268.

Theoretical background, data and SAS-code for a similar problem can be found from.

`http://support.sas.com/rnd/app/examples/ets/harvey/index.htm`