Multivariate Volatility Models, U of Tampere 2003

Exercises, Wednesday, February 19, 2003

1. Let S_t, $t = 1, \ldots, T$ denote daily stock prices of a given share. Daily (log) stock returns (in percents) are defined as

$$r_t = 100 \times (\log(S_t + D_t) - \log(S_t)),$$

where D_t denotes cash dividends paid on day t.

Assuming that the returns are not serially correlated. That is $\text{corr}[r_t, r_s] = 0$ if $s \neq t$.

Assume further that the variance of the daily returns is a constant $\sigma^2 = \text{var}[r_t]$ for all t.

Show that the annual variance can be simply obtained from the daily variance as $\sigma_A^2 = n\sigma^2$, where n is the number of days in a year (assume for simplicity that all days are trading days).

2. Using the law of iterated expectations[1] show that in the case of the simple ARCH(1) with $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$, the unconditional variance is

$$\text{var}[u] = \sigma^2 = \frac{\alpha_0}{1 - \alpha_1}.$$

Assuming further that $u_t | \mathcal{F}_{t-1} \sim N(0, \sigma_t^2)$, find the third and fourth moments of u_t. How do they compare to the case of a normal distribution.

3. Get from the web site, http://finance.yahoo.com > (International) Indices, weekly S&P 500 (USA) S&P TSX composite (Canada) indices form January 1990 up to present.

[1] Let $\mathcal{F}_t$ denote the information available at time t such that $F_t \subset F_{t+1}$ (mathematically $\mathcal{F}_t$ constitutes an increasing set of it sigma algebras), and let $E_t[S_{t+1}] = E[S_{t+1}|\mathcal{F}_t]$ denote the conditional expectation of S_{t+1} given information $\mathcal{F}_t$. The law of iterated expectations says

$$E_t E_{t+1}[S_{t+h}] = E_t[S_{t+h}].$$

Note that it is alway assumed that S_t is observed at time point t, and hence is considered to belong to the information set $\mathcal{F}_t$. Consequently $E_t[S_t] = S_t$. Note also that the prediction $E_t[S_{t+h}]$ is done at time point t, and is thus of course known once it is done. Thus it also belongs to the information set $\mathcal{F}_t$, and hence also in $\mathcal{F}_{t+j}$ for all $j > 0$. These imply (as a heuristic proof) the above law of iterated expectations.

a) Define the return series.

b) Calculate simple sample statistics.

c) Compute rolling (annual) volatilities (standard deviations) based on 52 weeks for both series, and plot the time series. Interpret the results.

d) Make a similar graph for the contemporaneous correlation of the index returns, and interpret the results in relation to the changes on the stock markets.

4. Estimate univariate ARCH, GARCH and EGARCH models for the index returns of the previous exercise. Do also appropriate diagnostic checks of the models. Which one is best fitting. Specify also the mean process of the returns.