8. SEM Applications

8.1 MIMIC models

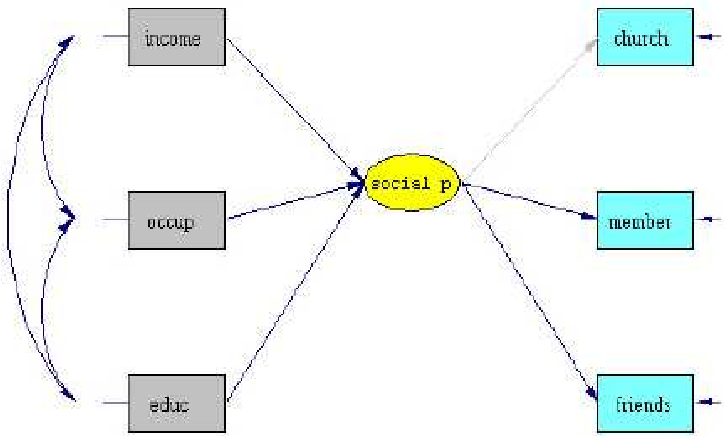
Multiple Indicators Multiple Causes (MIMIC) model involves using latent variables that are predicted by observed variables.

Example 8.1: Social status and social participation (Hodge and Traiman 1968, *American Sociological Review* 33. 723–740).

In the cited relationship between social status and social participation in a sample of 530 women was studies.

It was hypothesized that `income`, `occupation`, and `eduction` explain social participation. Social participation (`spart`) was measured by `church attendance` (`cattn`), `memberships` (`memb`), and `friends seen` (`friends`).

The follwowing model was specified:



Mathematical representation:

(1) $$\mathbf{y} = \boldsymbol{\lambda}\eta + \epsilon$$

(2) $$\eta = \gamma'\mathbf{x} + \zeta,$$

where $\mathbf{y} = (y_1, y_2, \ldots, y_p)'$ are indicators of the latent variable η and $\mathbf{x} = (x_1, x_2, \ldots, x_q)'$ are causes of η.

In particular equation (1) says that y's are *congeneric* measures of η.

SIMPLIS code:

```
! Example 8.1: Social Status and Social Participation
!              (Hodge and Treiman 1968)
Observed Variables:
income occup educ church member friends
Correlation Matrix:
1.000
 .304  1.000
 .305   .344  1.000
 .100   .156   .158  1.000
 .284   .192   .324   .360  1.000
 .176   .136   .226   .210   .265  1.000
Sample Size = 530
Latent Variables  'social p'
Relationships
church = 1.00*'social p'   ! Note. 1.00 implies a fixed coef
member = 'social p'
friends = 'social p'
'social p' = income occup educ
Path Diagram
End of Problem
```

Estimation results:

```
                   Goodness of Fit Statistics

                     Degrees of Freedom = 6
      Minimum Fit Function Chi-Square = 12.50 (P = 0.052)
Normal Theory Weighted Least Squares Chi-Square = 12.02 (P = 0.061)
        Estimated Non-centrality Parameter (NCP) = 6.02
     90 Percent Confidence Interval for NCP = (0.0 ; 20.00)
```

The goodness of fit is just borderline acceptable in terms of the χ^2-statistic.

Equations:

```
         Measurement Equations


church = 1.00*social p, Errorvar.= 0.78   , R = 0.22
                                  (0.058)
                                   13.61

member = 1.58*social p, Errorvar.= 0.46   , R = 0.54
        (0.24)                     (0.075)
         6.71                       6.10

friends = 0.86*social p, Errorvar.= 0.84   , R = 0.16
         (0.14)                     (0.058)
          6.03                       14.51

         Structural Equations

social p = 0.11*income + 0.045*occup + 0.16*educ,
          (0.028)       (0.026)       (0.031)
           3.82          1.73          4.93

            Errorvar.= 0.16   , R = 0.26
                      (0.037)
                       4.35
```

Overall, it seem that this model is not very successful because most of the relationships are rather poorly determined as revealed by the low R-squares.

Furthermore the low t-value of `occup` reveals that occupation may not be a significant determinant of social participation although income and education are.

8.2 Multi Group Analysis

Example 8.2: Nine psychological variables: Comparison of Measurement Models between Boys and Girls.

`vicperc`: Visual perception scores
`cubes`: Test of spatial visualization
`lozenges`: Test of spatial orientation

`paragraph`: Paragraph comprehension score
`sentence`: Sentence completion score
`wordmean`: Word meaning score

`addition`: Add test score
`countdot`: Counting groups of dots score
`sccaps`: Straight and curved capitals test score

Comparing Factor Models Between Groups

Sometimes it may be of interest to test whether the factor models are similar between different groups.

For example are the indicators measuring same underlying factors in different groups.

The comparison can be done on different levels.

Let

(3) $$\mathbf{x}^{(g)} = \Lambda^{(g)}\boldsymbol{\xi}^{(g)} + \boldsymbol{\delta}^{(g)}$$

denote the factor model for group g, $g = 1, 2$ (we assume for the sake of simplicity that we have only two groups).

The covariance matrices factor accordingly

(4) $$\Sigma^{(g)} = \Lambda^{(g)}\Phi^{(g)}\Lambda^{(g)\prime} + \Theta_{\delta}^{(g)}.$$

If there are no constraints across groups, each group can be analyzed separately.

If there are constraints across groups, the data must be analyzed simultaneously.

The analysis can be proceeded in a hierarchical manner:

Test first where the covariance matrices are equal:

$$H_{\Sigma} : \Sigma^{(1)} = \Sigma^{(2)}. \tag{5}$$

If H_{Σ} is accepted, then there is no need to proceed, because it implies that the factor model are identical in the groups.

If H_{Σ} is rejected, we can proceed in a hierarchical manner.

Assuming that the factor analysis model of the form (4) applies, test whether the factor patterns are of the same form across groups (i.e., equally many factors and the diagram is of the same form, i.e., $\Lambda^{(g)}$ have zeros (or fixed elements) in the same positions.

If this hypothesis is accepted we can impose various kinds of restrictions on individual parameters or parameter matrices across groups.

For example, that all factor structure coefficients are the same, i.e.,

$$H_{\Lambda} : \Lambda^{(1)} = \Lambda^{(2)}. \tag{6}$$

Example 8.2: Consider the Holzinger data. There are 72 boys and 73 girls in the sample.

Firs we test (5)

$$H_\Sigma : \Sigma^{(1)} = \Sigma^{(2)}$$

Testing for equality of the covariance matrices can be easiest done for example by SAS DISCRIM procedure (we'll return to this later).

The test is a chi-square test and the test statistic.

For the data the program calculates $\chi^2 = 58.2$ where $df = 45$. The p-value is 0.0897, which indicates that there is no strong empirical evidence that the covariance matrices would be different.

Thus, we can presume that there should not appear big differences in the factor model.

Next we proceed to test that model of three common factors ($q = 3$) applies both groups and that the factor patterns are of the form

i.e.,

$$\Lambda^{(g)} = \begin{pmatrix} 1 & 0 & 0 \\ * & 0 & 0 \\ * & 0 & 0 \\ 0 & 1 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \\ 0 & 0 & 1 \\ 0 & 0 & * \\ * & 0 & * \end{pmatrix} \tag{7}$$

where $*$ indicates a freely estimated parameter.

The factor covariances as well as the error variances are freely estimated.

That is, $\Phi^{(g)}$ and $\Theta^{(g)}$ are freely estimated. In each group there are 7 factor pattern parameters, 6 factor covariance parameters, and 9 error variance parameters, thus altogether $2 \times (7+6+9) = 44$ parameters to be estimated.

We have $2 \times 45 = 90$ covariances, implying that we will have $90 - 44 = 46$ degrees of freedom.

Thus, essentially our hypothesis is that there are three common factors and we can specify the null hypothesis as

$$H_q : q = 3 \tag{8}$$

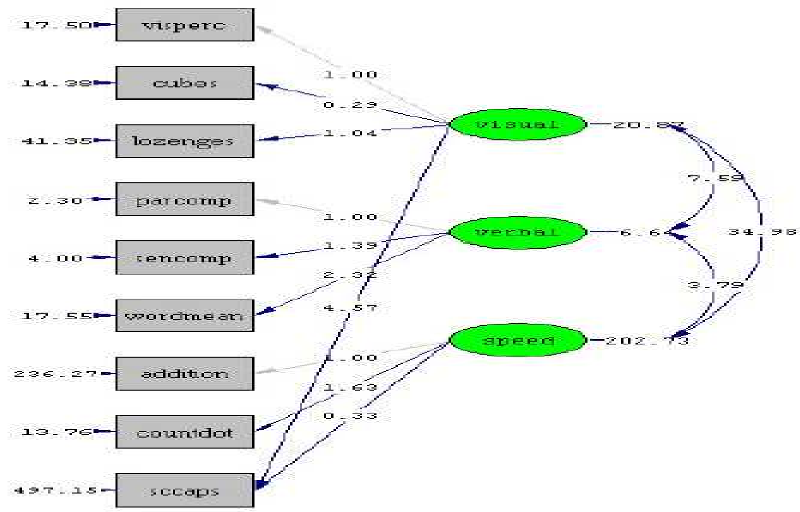
SIMPLIS commands:

```
Group boys
SYSTEM FILE from file holzinger_boys.dsf
Sample Size = 72
Latent Variables  visual verbal speed
Relationships
visperc = 1.00*visual
cubes = visual
lozenges = visual
parcomp = 1.00*verbal
sencomp = verbal
wordmean = verbal
addition = 1.00*speed
countdot = speed
sccaps = visual speed
Path Diagram
Method of Estimation: Generalized Least Squares
```

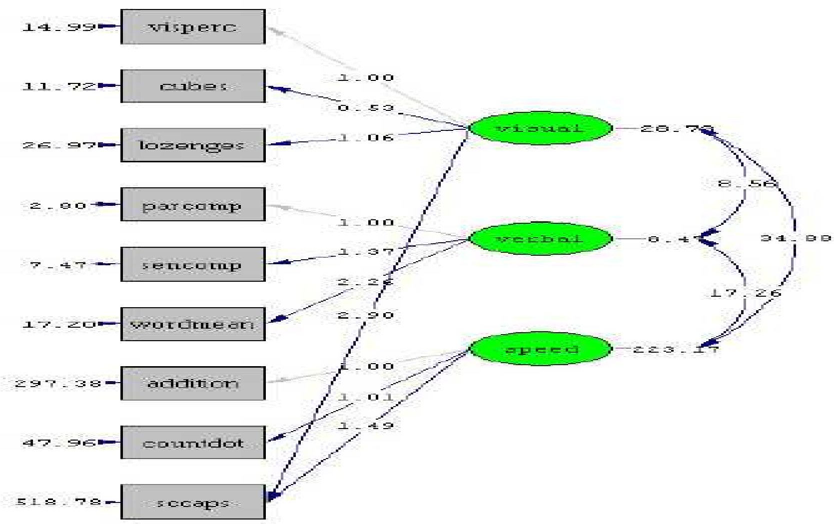
```
Group girls
SYSTEM FILE from file holzinger_girls.dsf
Sample Size = 73
Latent Variables  visual verbal speed
Relationships
cubes = visual
lozenges = visual
sencomp = verbal
wordmean = verbal
countdot = speed
sccaps = visual speed
Set the Variance of visual Free
Set the Covariances of verbal and visual Free
Set the Variance of verbal Free
Set the Covariances of speed and visual Free
Set the Covariances of speed and verbal Free
Set the Variance of speed Free
Set the Error Variance of visperc Free
Set the Error Variance of cubes Free
Set the Error Variance of lozenges Free
Set the Error Variance of parcomp Free
Set the Error Variance of sencomp Free
Set the Error Variance of wordmean Free
Set the Error Variance of addition Free
Set the Error Variance of countdot Free
Set the Error Variance of sccaps Free
End of Problem
```

Using LISREL (Generalized Least Squares), the estimated models for boys and girls are:

Boys:



Girls:




```
Chi-square = 42.94
Degrees of freedom = 46
Probability level = .60
```

The chi-square and associated p-value of 0.60 indicates that the factor model fits into the data, and we accept the null hypothesis (8).

Remark 8.1: Although we may reported the standardized estimates, it is important that in a multi-sample analysis with constraints over groups, one *must not* standardize the variables within groups (neither the observed nor the latent variables).

Next we test whether the factor patterns are the same. The null hypothesis is

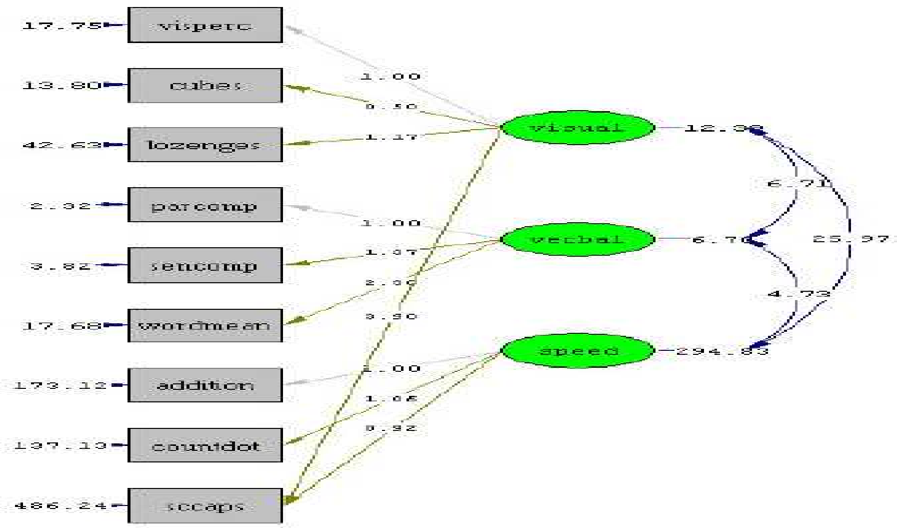
$$H_\Lambda : \Lambda^{(1)} = \Lambda^{(2)}. \tag{9}$$

```
Group boys
SYSTEM FILE from file holzinger_boys.dsf
Sample Size = 72
Latent Variables  visual verbal speed
Relationships
visperc = 1.00*visual
cubes = visual
lozenges = visual
parcomp = 1.00*verbal
sencomp = verbal
wordmean = verbal
addition = 1.00*speed
countdot = speed
sccaps = visual speed
Path Diagram
Method of Estimation: Generalized Least Squares
```

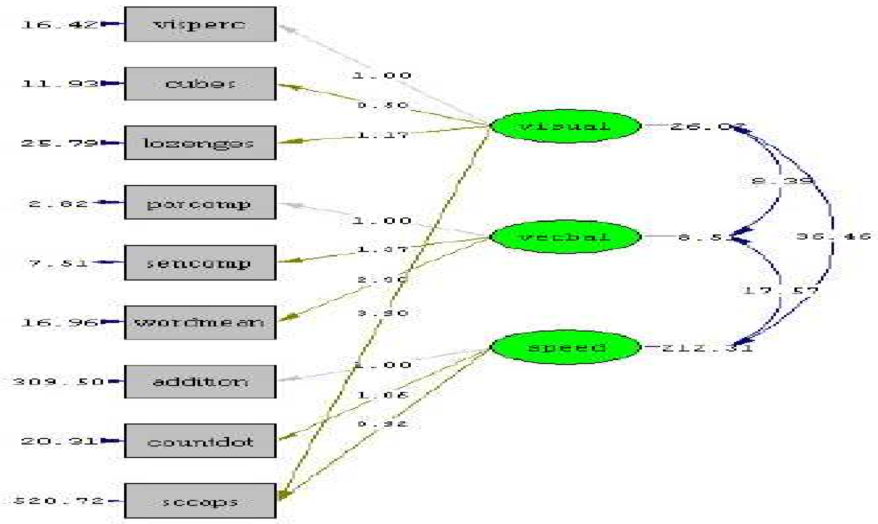
```
Group girls
SYSTEM FILE from file holzinger_girls.dsf
Sample Size = 73
Latent Variables  visual verbal speed
Relationships
! cubes = visual
! lozenges = visual
! sencomp = verbal
! wordmean = verbal
! countdot = speed
! sccaps = visual speed
Set the Variance of visual Free
Set the Covariances of verbal and visual Free
Set the Variance of verbal Free
Set the Covariances of speed and visual Free
Set the Covariances of speed and verbal Free
Set the Variance of speed Free
Set the Error Variance of visperc Free
Set the Error Variance of cubes Free
Set the Error Variance of lozenges Free
Set the Error Variance of parcomp Free
Set the Error Variance of sencomp Free
Set the Error Variance of wordmean Free
Set the Error Variance of addition Free
Set the Error Variance of countdot Free
Set the Error Variance of sccaps Free
End of Problem
```

Estimates (non-standardized) are as follows

Boys:



Girls:




```
Chi-square = 52.08
Degrees of freedom = 53
Probability level = .51
```

The chi-square difference is $52.08 - 42.94 = 9.14$ and the degrees of freedom difference is $53 - 46 = 7$ implying p-value 0.243 and we can accept the additional restrictions.

Given equality of the factor structures, we can proceed to test whether the error variances are the same

(10) Given $\Lambda^{(1)} = \Lambda^{(2)}$, $H_{\Lambda\Theta} : \Theta^{(1)} = \Theta^{(2)}$

The results are:

```
Chi-square = 62.67
Degrees of freedom = 62
Probability level  = .45
```

The chi-square difference is 10.59 and df difference 9, which correspond a p-value 0.30. The hypothesis is accepted.

Finally, $H_{\Lambda\Theta}$, we test whether the factor covariances Φ are the same in both groups, that is the null hypothesis

$$\text{(11)} \qquad \text{Given } H_{\Lambda\Theta},\ H_{\Lambda\Theta\Phi} : \Phi^{(1)} = \Phi^{(2)}$$

SIMPLIS commands:

```
Group boys
SYSTEM FILE from file holzinger_boys.dsf
Sample Size = 72
Latent Variables  visual verbal speed
Relationships
visperc = 1.00*visual
cubes = visual
lozenges = visual
parcomp = 1.00*verbal
sencomp = verbal
wordmean = verbal
addition = 1.00*speed
countdot = speed
sccaps = visual speed
Path Diagram
```

```
Group girls
SYSTEM FILE from file holzinger_girls.dsf
Sample Size = 73
Latent Variables  visual verbal speed
Relationships
! *** Commenting all the below lines implies that all the structures
! *** are invariant between the groups
! cubes = visual
! lozenges = visual
! sencomp = verbal
! wordmean = verbal
! countdot = speed
! sccaps = visual speed
! Set the Variance of visual Free
! Set the Covariances of verbal and visual Free
! Set the Variance of verbal Free
! Set the Covariances of speed and visual Free
! Set the Covariances of speed and verbal Free
! Set the Variance of speed Free
! Set the Error Variance of visperc Free
! Set the Error Variance of cubes Free
! Set the Error Variance of lozenges Free
! Set the Error Variance of parcomp Free
! Set the Error Variance of sencomp Free
! Set the Error Variance of wordmean Free
! Set the Error Variance of addition Free
! Set the Error Variance of countdot Free
! Set the Error Variance of sccaps Free
End of Problem
```

Results:

```
Chi-Square: 87.29
Degrees of freedom: 68
Probability level: 0.058
```

The difference of Chi-square is 24.82 with 6 degrees of freedom and p-value 0.000. Thus we reject these restrictions.

Summary of the results are in the table below:

Hypothesis	χ^2	*df*	$\Delta\chi^2$	Δ*df*	p-val	Decision
$H_q : q = 3$	42.94	46	na	na	0.601	Accepted
H_Λ	52.08	53	9.14	7	0.243	Accepted
$H_{\Lambda\Theta}$	62.67	62	10.59	9	0.305	Accepted
$H_{\Lambda\Theta\Phi}$	87.49	68	24.82	6	0.000	Rejected

Thus far the conclusion is that otherwise the structures are invariant (the same for boys and girls) but at least some of the latent variables have different correlations and/or variances.

Testing for these leads to nonlinear relationships such that in all cases the relationships $|\phi_{ij}| \leq \sqrt{\phi_{ii}\phi_{jj}}$ must be satisfied. Here $\phi_{ij} = \mathbb{C}\mathrm{ov}[\xi_i, \xi_j]$ are the factor covariances and $\phi_{ii} = \mathbb{V}\mathrm{ar}[\xi_i]$ and $\phi_{jj} = \mathbb{V}\mathrm{ar}[\xi_j]$ are the factor variances.

These kinds of restrictions can be imposed in LISREL

Testing for the equality of the factor covariances across groups, we first fix the covariances equal between the groups (without explicitly demanding the above restriction, thus hoping that the restrictions are satisfied as such).

We change the SIMPLIS commands for Girls (Boys do not need to be changed) by deleting the commenting marks (!) from appropriate lines:

```
Group girls
SYSTEM FILE from file holzinger_girls.dsf
Sample Size = 73
Latent Variables  visual verbal speed
Relationships
! cubes = visual
! lozenges = visual
! sencomp = verbal
! wordmean = verbal
! countdot = speed
! sccaps = visual speed
! Set the Variance of visual Free
 Set the Covariances of verbal and visual Free
! Set the Variance of verbal Free
 Set the Covariances of speed and visual Free
 Set the Covariances of speed and verbal Free
! Set the Variance of speed Free
! Set the Error Variance of visperc Free
! Set the Error Variance of cubes Free
! Set the Error Variance of lozenges Free
! Set the Error Variance of parcomp Free
! Set the Error Variance of sencomp Free
! Set the Error Variance of wordmean Free
! Set the Error Variance of addition Free
! Set the Error Variance of countdot Free
! Set the Error Variance of sccaps Free
End of Problem
```

Estimation produces:

```
Degrees of Freedom = 65
Chi-Square = 79.36
p-val = 0.11
```

This model is a restriction of the model under $H_{\ominus}$, thus the χ^2 difference from it is $79.36 - 87.49 = 16.69$ with 3 degrees of freedom and p-value 0.001, which implies rejection of the hypothesis.

Thus, we stop here and conclude that the model is otherwise invariant except the factor corvariances and variances.

Analysis of Latent Mean Structures

So far it has been assumed that the random variables (an in particular the latent variables) have zero means.

In single sample the latent variable means are unidentified (thus set to zero for convenience).

In multi-samples the individual means of the latent variables are again unidentified, but the group differences can be estimated (provided that the factors have the same scales across groups).

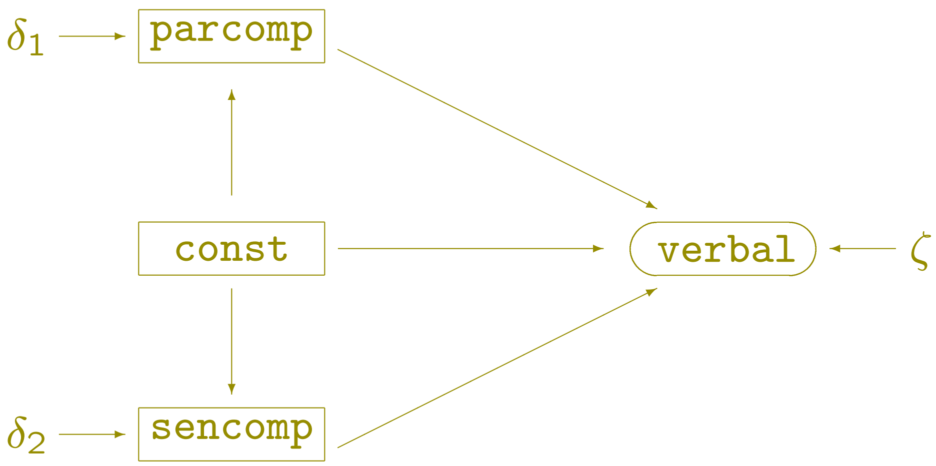
The idea is that the group differences in observed variables can be explain by the differences in the latent factor means.

This implies that one must assume that the factor structures (the factor regression coefficients) are invariant across groups.

The differences in the means can be measured by fixing the mean in one group equal to zero.

Example 8.3: Consider the Holzinger data in Example 8.2.

For example the mean for the latent variable `verbal` in path diagram can be indicated as follows.

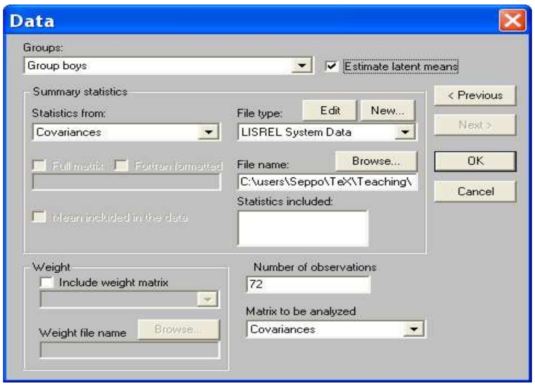


The `const` variable in LISREL is a dummy variable equal to 1. The path coefficients from the `const` are the means.

The factor pattern matrices Λ were found to be same between the groups.

With these preliminaries and setting the factor means for boys equal to zero (reference), the factor means can be estimated with LISREL:

Select from the **Setup** $\rightarrow$ **Data...**



check **Estimate latent means**.

Change the previous SIMPLIS commands as follows:

```
Group boys
SYSTEM FILE from file holzinger_boys.dsf'
Sample Size = 72
Latent Variables  visual verbal speed
Relationships
visperc = CONST 1.00*visual
cubes = CONST visual
lozenges = CONST visual
parcomp = CONST 1.00*verbal
sencomp = CONST verbal
wordmean = CONST verbal
addition = CONST 1.00*speed
countdot = CONST speed
sccaps = CONST visual speed
Path Diagram

Group girls
SYSTEM FILE from file holzinger_girls.dsf'
Sample Size = 73
Latent Variables  visual verbal speed
Relationships
visual = CONST
verbal = CONST
speed = CONST
Set the Variance of visual Free
Set the Covariances of verbal and visual Free
Set the Variance of verbal Free
Set the Covariances of speed and visual Free
Set the Covariances of speed and verbal Free
Set the Variance of speed Free
Set the Error Variance of countdot Free
End of Problem
```

The results are for Boys:

```
Group boys:
        Measurement Equations

 visperc = 29.93 + 1.00*visual, Errorvar.= 24.40, R = 0.45
          (0.72)                          (4.21)
           41.37                           5.80

   cubes = 24.96 + 0.45*visual, Errorvar.= 15.08, R = 0.21
          (0.42)  (0.093)                 (1.98)
           59.24   4.84                    7.61

lozenges = 16.36 + 1.13*visual, Errorvar.= 39.17, R = 0.40
          (0.85)  (0.19)                  (6.18)
           19.19   6.10                    6.34

 parcomp = 9.47  + 1.00*verbal, Errorvar.= 2.62 , R = 0.73
          (0.36)                          (0.57)
           26.67                           4.57

 sencomp = 18.23 + 1.29*verbal, Errorvar.= 7.14 , R = 0.62
          (0.48)  (0.11)                  (1.18)
           38.12   11.45                   6.06

wordmean = 16.22 + 2.20*verbal, Errorvar.= 20.72, R = 0.62
          (0.82)  (0.19)                  (3.43)
           19.84   11.47                   6.03

addition = 90.89 + 1.00*speed, Errorvar.= 316.34, R = 0.48
          (2.61)                         (50.48)
           34.80                          6.27

countdot = 109.49 + 1.09*speed, Errorvar.= 219.05, R = 0.61
          (2.66)   (0.18)                 (72.55)
           41.21    6.08                   3.02

sccaps = 193.75 + 3.49*visual + 1.08*speed, Errorvar.= 585.12, R = 0.55
        (3.93)   (0.74)        (0.22)                 (94.89)
         49.30    4.73          4.97                   6.17
```

For girls (same as boys, except errer variance for `countdot`):

```
 Group girls
         Measurement Equations

  visperc = 29.93 + 1.00*visual, Errorvar.= 24.40, R = 0.53
           (0.72)                          (4.21)
            41.37                           5.80

    cubes = 24.96 + 0.45*visual, Errorvar.= 15.08, R = 0.27
           (0.42)  (0.093)                 (1.98)
            59.24   4.84                    7.61

 lozenges = 16.36 + 1.13*visual, Errorvar.= 39.17, R = 0.47
           (0.85)  (0.19)                  (6.18)
            19.19   6.10                    6.34

  parcomp = 9.47  + 1.00*verbal, Errorvar.= 2.62 , R = 0.80
           (0.36)                          (0.57)
            26.67                           4.57

  sencomp = 18.23 + 1.29*verbal, Errorvar.= 7.14 , R = 0.70
           (0.48)  (0.11)                  (1.18)
            38.12   11.45                   6.06

 wordmean = 16.22 + 2.20*verbal, Errorvar.= 20.72, R = 0.70
           (0.82)  (0.19)                  (3.43)
            19.84   11.47                   6.03

 addition = 90.89 + 1.00*speed, Errorvar.= 316.34, R = 0.40
           (2.61)                         (50.48)
            34.80                          6.27

 countdot = 109.49 + 1.09*speed, Errorvar.= 36.28 , R = 0.88
           (2.66)    (0.18)                (37.38)
            41.21     6.08                  0.97

sccaps = 193.75 + 3.49*visual + 1.08*speed, Errorvar.= 585.12, R = 0.60
        (3.93)    (0.74)        (0.22)                (94.89)
         49.30     4.73          4.97                  6.17
```

Latent variable mean differences between boys and girls and goodness of fit statistics:

```
         Mean Vector of Independent Variables

              visual      verbal       speed
            --------    --------    --------
               -0.69        0.96       -1.40
              (0.96)      (0.52)      (3.02)
               -0.72        1.82       -0.47


       Global Goodness of Fit Statistics
             Degrees of Freedom = 67
             Chi-Square = 76.54 (P = 0.20)
```

The mean of the latent variable `verbal` is 0.96 units above the boys' mean with t-value 1.82 and p-value 0.069, which is significant at the 10% level.

With respect to the other latent variables there is no empirical evidence for mean differences.

8.3 Longitudinal Analysis

In a longitudinal analysis the same measurements are obtained form the same people (objects) at two or more occasions.

The adjacent observations generate often autocorrelation into the error terms.

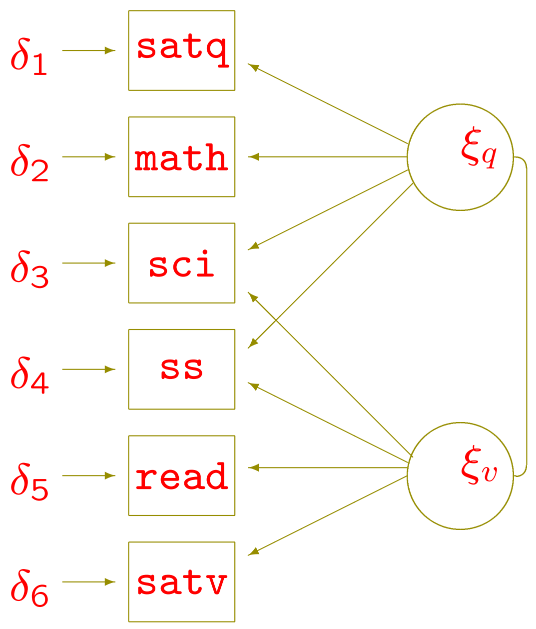
Example 8.4: Change in verbal and quantitative ability between grades 7 and 9.

Sample $n = 383$ (girls) measured in grades 7 and 9.

Variables:

`math`: achievement test in mathematics
`sci`: achievement test in science
`ss`: achievement test in social studies
`rea`: achievement test in reading
`satv`: scholastic aptitude test, verbal part
`satq`: scholastic aptitude test, quantitative pat

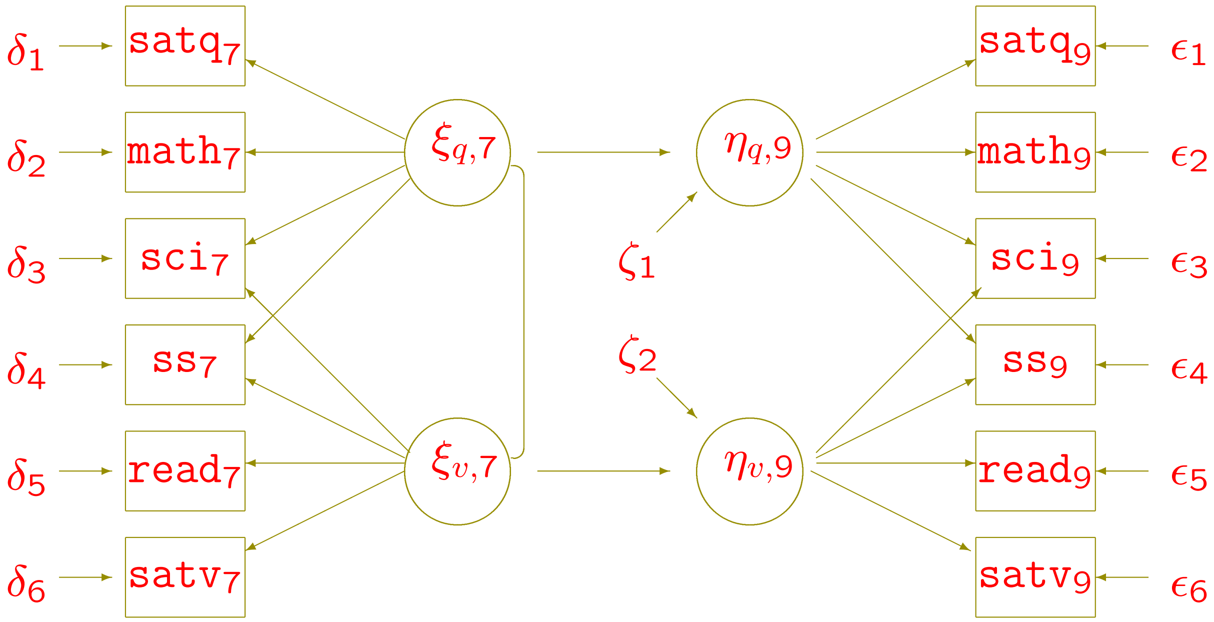
Earlier studies have shown that following measurement model for the ξ_q (quantitative) and ξ_v (verbal) abilities applies



It is postulated that 7th grade quantitative ability affects only the quantitative ability at the 9th grade. Same with the verbal abilities.

Thus, it is assumed that there are no cross-effects.

The model to be estimated is

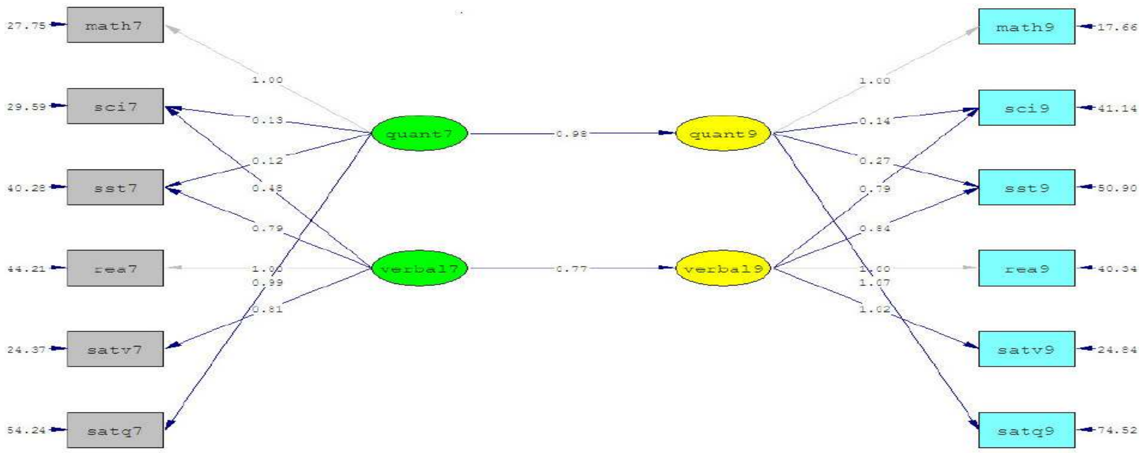


The correlation matrix

	math7	sci7	sst7	rea7	satv7	satq7	math9	sci9	sst9	rea9	satv9	satq9
math7	1											
sci7	0.6498	1										
sst7	0.6809	0.7362	1									
rea7	0.6959	0.7025	0.757	1								
satv7	0.6868	0.712	0.7844	0.8287	1							
satq7	0.7053	0.5971	0.6433	0.6088	0.6096	1						
math9	0.7364	0.6085	0.6528	0.6574	0.6533	0.6953	1					
sci9	0.6419	0.7339	0.7434	0.6794	0.6995	0.5606	0.6686	1				
sst9	0.6719	0.6905	0.7462	0.7239	0.7227	0.5994	0.7055	0.7391	1			
rea9	0.6400	0.6346	0.7178	0.7724	0.7512	0.5693	0.6644	0.6934	0.6927	1		
satv9	0.6837	0.7155	0.7557	0.7909	0.8830	0.5998	0.6911	0.7355	0.7400	0.7879	1	
satq9	0.6511	0.5164	0.5445	0.5445	0.5621	0.7145	0.7521	0.5774	0.6065	0.5903	0.5918	1
std	11.4008	9.2213	13.0459	14.6407	11.6230	12.4204	11.6608	11.4428	13.3248	12.1527	11.7089	14.5348

Fitting the model yields goodness-of-fit, Chi-square = 217.7 Degrees of freedom = 47 Probability level = 0.000, showing that the model does not fit.

Here we need to relax the error covariances as follows

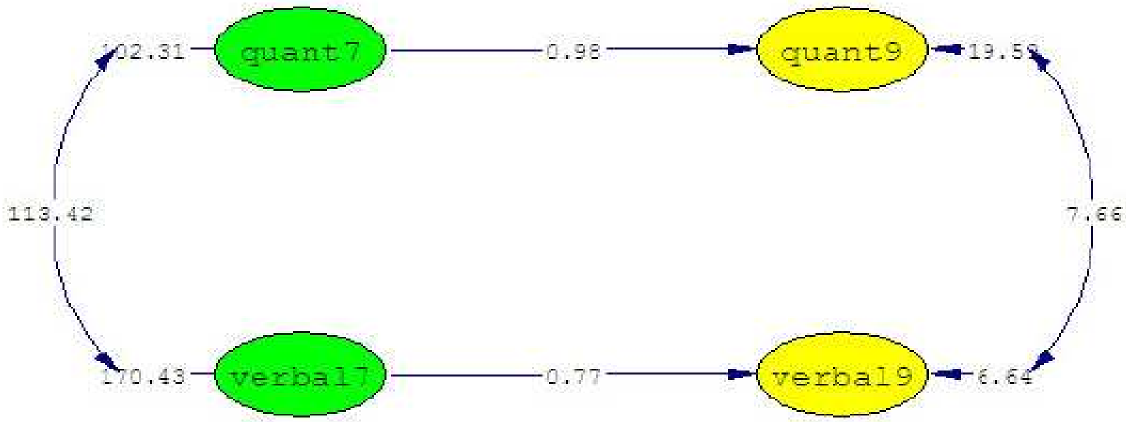


The goodness-of-fit improves to Chi-square = 65.6 Degrees of freedom = 40 Probability level = 0.007. However, the ratio $\chi^2/df = 65.6/40 = 1.64 < 2$ indicates a reasonable fit.

SIMPLIS commands:

```
Example 8.4: Change in Verbal and Quatitative Abilities
SYSTEM FILE from file ex84.DSF'
Sample Size = 383
Latent Variables  quant7 verbal7 quant9 verbal9
Relationships
 math7 = 1*quant7
 satq7 = quant7
 sci7 = quant7 verbal7
 sst7 = quant7 verbal7
 rea7 = 1*verbal7
 satv7 = verbal7
 math9 = 1*quant9
 satq9 = quant9
 sci9 = quant9 verbal9
 sst9 = quant9 verbal9
 rea9 = 1*verbal9
 satv9 = verbal9
quant9 = quant7
verbal9 = verbal7
Set the Error Variance of verbal9 free
Set the Error Covariance of quant9 and verbal9 Free
Set the Error Variance of quant9 free
Set the Error Variance of quant7 free
Set the Error Covariance of verbal7 and quant7 Free
Set the Error Variance of verbal7 free
Set the Error Covariance of math7 and math9 Free
Set the Error Covariance of sci7 and sci9 Free
Set the Error Covariance of sst7 and sst9 Free
Set the Error Covariance of rea7 and rea9 Free
Set the Error Covariance of satv7 and satv9 Free
Set the Error Covariance of satq7 and satq9 Free
Path Diagram
Admissibility Check = Off
End of Problem
```

Looking mode closely the structural model,




```
        Structural Equations

   quant9 = 0.98*quant7, Errorvar.= 19.59, R = 0.83
           (0.047)                 (3.85)
            20.93                   5.08

  verbal9 = 0.77*verbal7, Errorvar.= 6.64 , R = 0.94
           (0.032)                  (1.54)
            24.28                    4.32
```

we see that both of the coefficients are highly statistically significant.

Furthermode, the $R^2 = 0.94$ of the verbal equation suggests that verbal ability at the 7th grade almost perfectly predicts the ability at the 9th grade.

Example 8.5: Panel Model for Political Efficacy:

Political efficacy (see Example 1.1) investigated in two time points.

Is there political stability?

Other interesting questions, which we do not investigate here, are:

Has the political efficacy changed over time? Has the variance of efficacy increased over time?

Variables:

`nosay`: People like me have no say in what government does
`voting`: Voting is the only way that people like me can have any say about how the government runs things
complex: Sometimes politics and government seem so complicated that a person like me cannot really understand what is really going on
`nocare`: I don't think that public officials care much about what people like me think
`touch`: Generally speaking, those we elect to Congress in Washington lose touch with the people pretty quickly
`interest`: Parties are only interested in people's vote but not in their opinions.

The variables are measured in the ordinal scale and are supposed to reflect the underlying continuous variables that are assumed to be normally distributed.

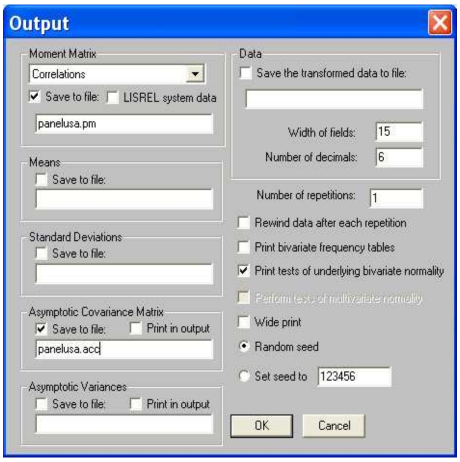
Data:

panelusa.psf

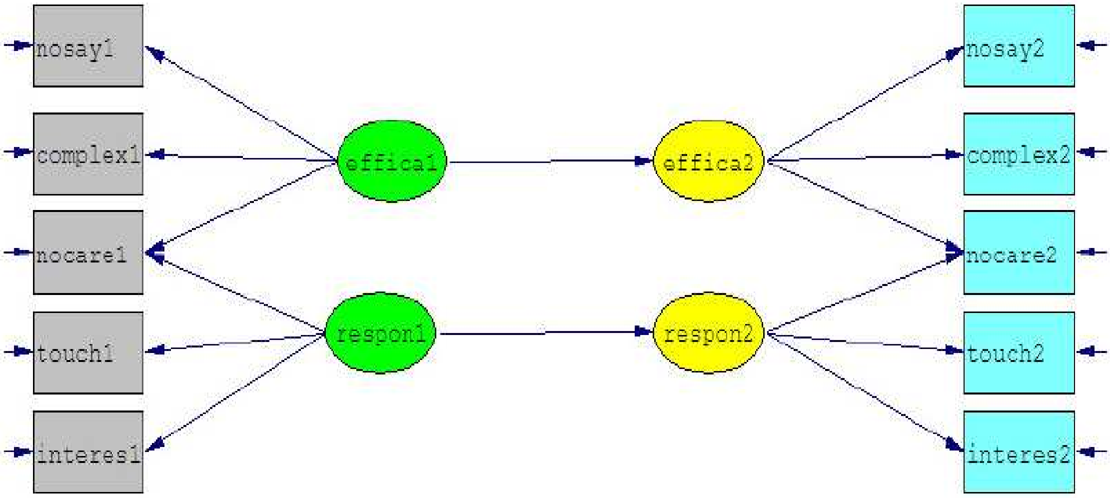
	nosay1	voting1	complex1	nocare1	touch1	interes1	nosay2	voting2	complex2	nocare2	touch2	interes2
1	2.000	2.000	1.000	1.000	1.000	1.000	9.000	2.000	2.000	9.000	2.000	2.000
2	2.000	3.000	3.000	3.000	2.000	3.000	2.000	3.000	2.000	2.000	2.000	2.000
3	3.000	2.000	2.000	3.000	3.000	3.000	3.000	2.000	2.000	2.000	2.000	2.000
4	2.000	2.000	1.000	1.000	2.000	1.000	2.000	2.000	2.000	2.000	1.000	2.000
5	3.000	2.000	2.000	3.000	3.000	3.000	3.000	2.000	2.000	3.000	2.000	2.000
6	2.000	2.000	2.000	2.000	1.000	2.000	3.000	2.000	1.000	3.000	2.000	2.000
7	3.000	1.000	2.000	2.000	2.000	2.000	2.000	2.000	3.000	3.000	2.000	2.000
8	2.000	1.000	2.000	2.000	1.000	1.000	3.000	3.000	1.000	2.000	9.000	3.000
9	3.000	3.000	2.000	2.000	3.000	3.000	3.000	3.000	1.000	3.000	2.000	2.000
10	2.000	2.000	3.000	1.000	1.000	1.000	2.000	2.000	2.000	2.000	1.000	2.000
11	3.000	2.000	1.000	1.000	2.000	2.000	3.000	2.000	2.000	2.000	2.000	2.000
12	1.000	1.000	1.000	1.000	1.000	1.000	3.000	3.000	2.000	3.000	2.000	2.000
13	2.000	2.000	2.000	1.000	2.000	2.000	1.000	1.000	1.000	2.000	2.000	2.000
14	3.000	3.000	2.000	3.000	2.000	2.000	3.000	2.000	2.000	3.000	3.000	3.000

From menu `Data` → `Define Variables...` check that the variables are ordinal scale and define the missing values.

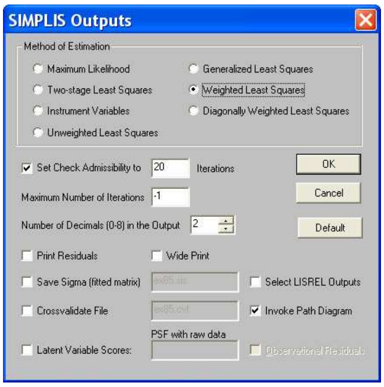
Form menu `Statistics` → `Output Options...` select



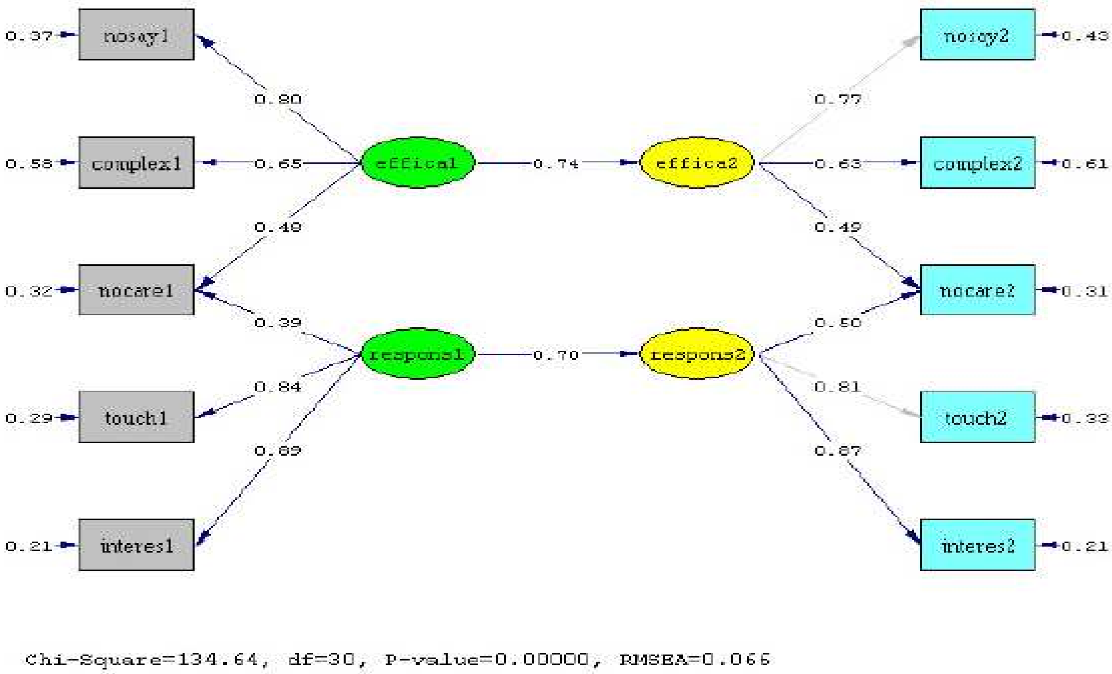
Conceptual model:



Select from menu `Output` → `SIMPLIS outputs` and check `Weighted Least Square`



Estimation results are:



The χ^2 indicates that the model does not fit.

Checking the modification indices suggest that the error covariances should be freed

8.4 Econometric Models

Econometric models with directly observable variables are a special case of latent variable SEM.

These are usually again non-recursive.

An additional feature of econometric models is that they usually contain *identities*, which are by definition exact relationships (accounting identities) with no disturbance terms or parameters to be estimated.

We demonstrate here only the simple case, which is an "ordinary" econometric SEM.

The only additional feature to the SEMs dicussed in this case is the estimation of the intercept terms.

Example 8.5: Demand and supply of food

$$q(\texttt{demand}) = \alpha_1 + \beta_1 p + \gamma_1 d + \zeta_1 \tag{12}$$

$$q(\texttt{supply}) = \alpha_2 + \beta_2 p + \gamma_2 \Delta p + \gamma_3 t + \zeta_2. \tag{13}$$

where q is food consumption, p is food price, d is the disposable income, Δp is the price change, and t is time.

All price variables are log-prices, implying that the coefficients are elasticities.

In order to estimate these in SEM, they must be transformed into the form

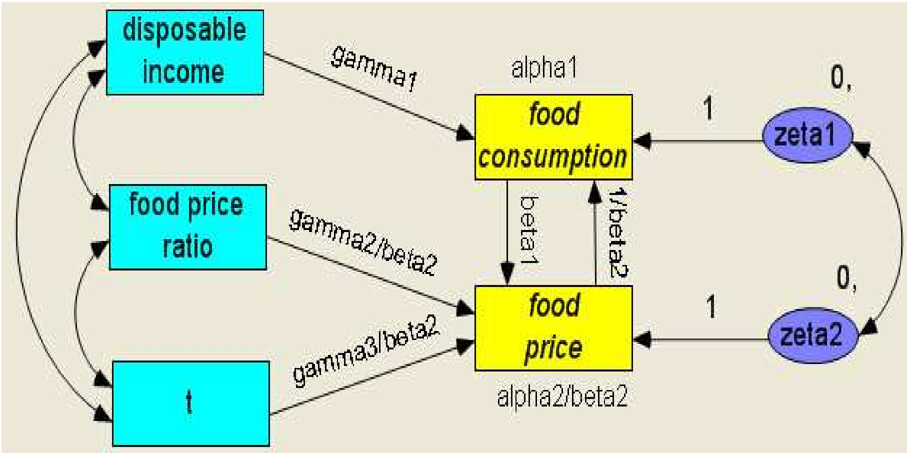
$$(I - B)\boldsymbol{\eta} = \boldsymbol{\alpha} + \Gamma\boldsymbol{\xi} + \boldsymbol{\zeta}.$$

This can be done by solving first (13) with respect to p.

Thus,

$$\begin{aligned} q - \beta_1 p &= \alpha_1 + \gamma_1 d + \zeta_1 \\ p - \tfrac{1}{\beta_2} q &= -\tfrac{\alpha}{\beta_2} - \tfrac{\gamma_2}{\beta_2}\Delta p - \tfrac{\gamma_3}{\beta_2} t + \zeta_2^* \end{aligned} \tag{14}$$

The path diagram is



with estimates:

```
Stability index  0.967

Chi-square = 3.046
Degrees of freedom = 1
Probability level = .081

Regression Weights:
          Estimate   S.E.   t-val     P     Label
q <--- d     0.310  0.045   6.919    ***   gamma1
p <--- t    -1.558  0.665  -2.343   0.019  gamma3/beta2
p <--- f    -0.931  0.396  -2.350   0.019  gamma2/beta2
p <--- q     4.214  1.754   2.403   0.016  1/beta2
q <--- p    -0.230  0.092  -2.486   0.013  beta1

Intercepts:
     Estimate    S.E.    C.R.    P      Label
q      93.619   7.574  12.360   ***   alpha1
p    -218.892 137.697  -1.590  0.112  alpha2
```

Thus, the estimates of the parameters in (12) and (13) are

```
===================
Parameter  Estimate
-------------------
beta1      -0.230
beta2       0.218
gamma1      0.310
gamma2      0.212
gamma3      0.00349
alpha1     93.6
alpha2     47.7
==================
```

Because the variables are logarithms, the parameters are *elasticities*.

Thus, in particular this model predicts that a one percent increase in price implies an estimated decrease in demand by about 0.230 percent and an increase in supply by 0.218 percent.

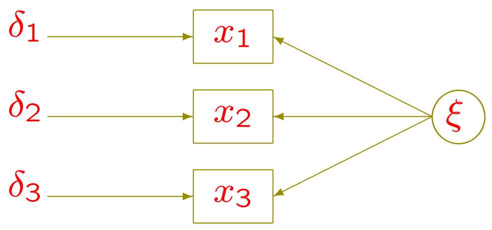
A one percent increase in disposable income implies an estimated increase in demand by 0.310 percent.

8.5 Formative Measurements

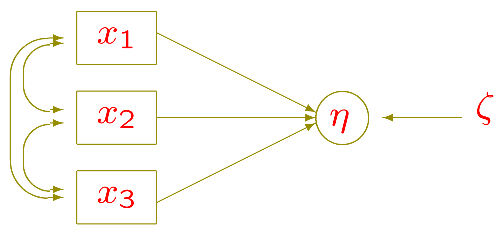
In *reflective* measurement the causality runs from the construct to the indicators (changes in construct are expected to be manifest in changes in respective indicators).

In *formative* measurement models (emergent variable systems) changes in indicators cause variation in the construct rather than the other way round.

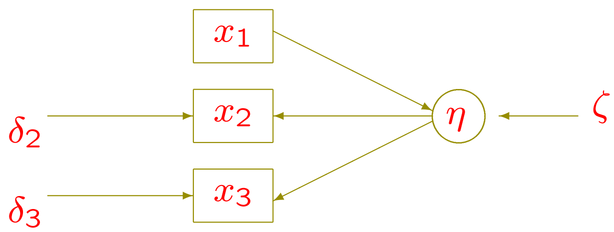
In *feedback loops* some variables are specified as causes and effects of for each other (see, job satisfaction–performance example).



(a) Reflective Measurement Model



(b) Formative Measurement Model



(b) Measurement Model with MIMIC Factor

Mathematical representations

(a) Reflective

(15) $$x_i = \lambda_i \xi + \delta_i,$$

$i = 1, 2, 3$

(b) Formative

(16) $$\eta = \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + \zeta.$$

(c) MIMIC

(17) $$\begin{aligned} \eta &= \gamma x_1 + \zeta \\ x_2 &= \lambda_1 \eta + \delta_2 \\ x_3 &= \lambda_3 \eta + \delta_3 \end{aligned}$$

Formative measurement models (Cause indicators)

Formative indicators affect (cause) the latent variable (Figure b).

Formative latent variable is a composite index.

In particular, if $\zeta = 0$ a formative (latent) variable is just a weighted sum of the indicators (like principal component).

Remark 8.2: The disturbance term ζ in the latent variable of model in (16) reflects measurement errors in the cause indicators and thus can be considered measuring reliability of the cause indicators.

Unlike in reflective model, a change in any of the formative indicators may affect the latent variable even if other indicators did not change.

Conversely, a change in formative latent factor does not necessarily imply a change in all indicators.

Unlike reflective (effect) indicators, formative indicators are generally not interchangeable, because removal of an indicator may change the content of the construct.

Internal consistency (within factor correlations of indicators are higher than between factor correlations of indicators) is *not* generally required for a set of formative indicators (cause indicators are exogenous).

Formative indicator model in Figure b is not identified (must be placed in a lager model with specific pattern, like the MIMIC in Fig. c).

Identification Issues

Two necessary rules for all SEM:

(1) The number of free parameters cannot exceed the number of variances and covariances (plus means if they are analyzed).

(2) Every latent variable must be assigned a scale (ULI, UVI)

An additional requirement for identification for the disturbance variance of a latent composite is that the composite must have at least two direct effects on other endogenous factors measured with reflective (effect) indicators (see the first part of Example 8.6 below).*

*MacCallum, R.C. and M.V. Browne (1993). The use of causal indicators in covariance structure models. *Psychological Bulletin* 114, 533–541.

One way to remedy many of the identification problems is to add effect indicators for latent composites represented in the original model as measured with cause indicators only (see Example 8.6 below).

This results to a MIMIC model (Fig. c).

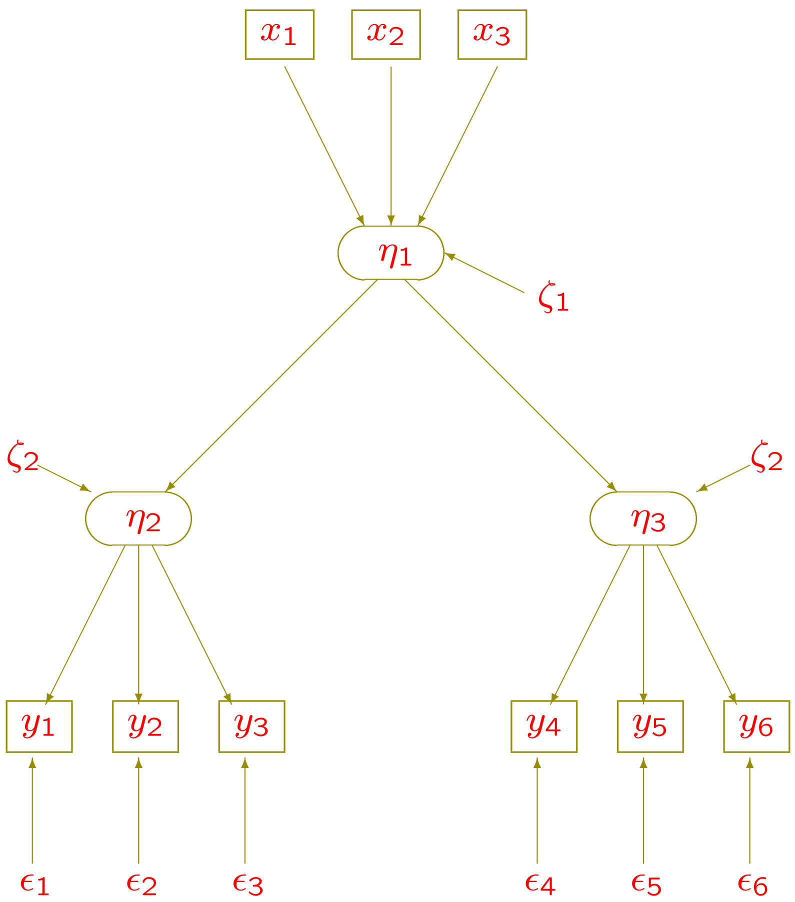
Example 8.6:* Cognitive and achievement status of adolescents ($n = 158$).

Control/additional variables: Classroom adjustment indicators (teacher reports), family SES, and degree of parental psychiatric disturbance.

η_1: Risk, high risk associated with any combination of low SES, high degree of parental psychopathology, and low verbal IQ (adolescent).

*Worland, J., G.G. Weeks, C.L. James, and B.D. Stock (1984). Intelligence, classroom behavior, and academic achievement in children at high and low risk for psychopathology: A structural equation analysis. *Journal of Abnormal Child Psychology* 12, 437–454.

See also: MacCallum, R.C. and M.W. Browne (1993). The use of causal indicators in covariance structure models. Some practical issues. *Psychological Bulletin* 114, 533–541.


x_1
x_2
x_3
η_1
ζ_1
ζ_2
η_2
η_3
ζ_2
y_1
y_2
y_3
y_4
y_5
y_6
ϵ_1
ϵ_2
ϵ_3
ϵ_4
ϵ_5
ϵ_6

Risk indicators (eta1)
x1: Parental psychiatric disturbance
x2: Family SES
x3: Verbal IQ

Achievement indicators (eta2)
y1: Reading
y2: Arithmetic
y3: Spelling

Classroom adjustment indicators (eta3)
y4: Motivation
y5: harmony
y6: Stability

Correlation matrix

	x1	x2	x3	y1	y2	y3	y4	y5	y6
x1	1								
x2	-.42	1							
x3	-.43	-.50	1						
y1	-.39	-.43	.78	1					
y2	-.24	-.37	.69	.73	1				
y3	-.31	-.33	.63	.87	.72	1			
y4	-.25	-.25	.49	.53	.60	.59	1		
y5	-.25	-.26	.42	.42	.44	.45	.77	1	
y6	-.16	-.18	.33	.36	.38	.38	.59	.58	1
std	13.00	13.50	13.10	12.50	13.50	14.20	9.50	11.10	8.70

Step 1: Specification of the measurement model: CFA for achievemnet and adjustment

```
! Example 8.6: Cognitive and achievement status of adolescents
Observed Variables:
ses psych verbiq read math spell motiv harmon stabil
Correlation Matrix:
 1
 -.42  1
 -.43  -.50   1
 -.39  -.43   .78   1
 -.24  -.37   .69   .73   1
 -.31  -.33   .63   .87   .72   1
 -.25  -.25   .49   .53   .60   .59   1
 -.25  -.26   .42   .42   .44   .45   .77   1
 -.16  -.18   .33   .36   .38   .38   .59   .58  1
Standard Deviations:
13.00 13.50 13.10 12.50 13.50 14.20  9.50 11.10  8.70
sample Size: 158
Latent Variables: risk achiev adj
Relationships
read = 1*achiev
math spell = achiev
motiv = 1*adj
harmon stabil = adj
Path Diagram
Print Residuals
End of Problem
```

Standardized residuals confirm this ($> |2.58|$ is considered bad fit).

Standardized Residuals

	read	math	spell	motiv	harmon	stabil
read	- -					
math	0.42	- -				
spell	2.89	-2.49	- -			
motiv	-1.55	3.42	1.54	- -		
harmon	-2.36	0.54	-1.63	-0.19	- -	
stabil	-0.54	0.97	-0.22	-2.02	2.15	- -

```
 Summary Statistics for Standardized Residuals

 Smallest Standardized Residual =   -2.49
   Median Standardized Residual =    0.00
  Largest Standardized Residual =    3.42
```

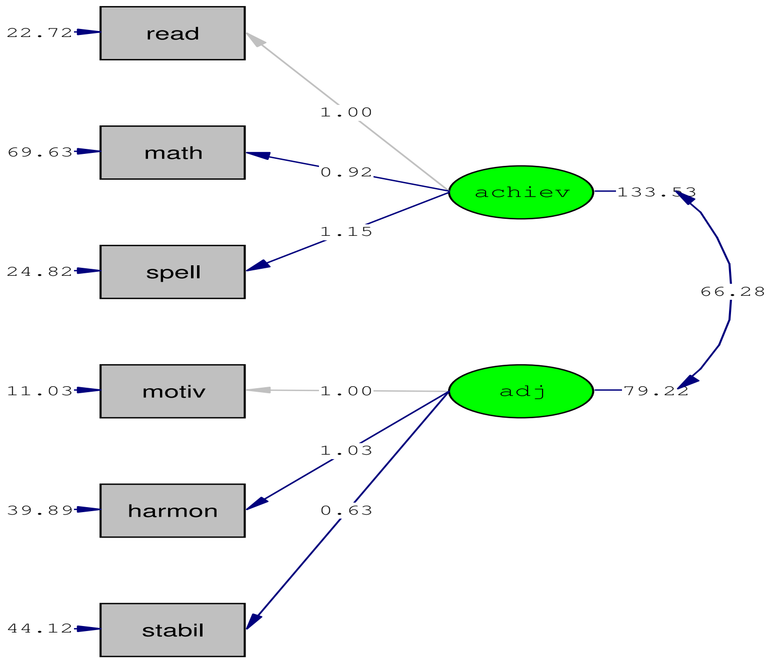
Remark 8.3: Standardized residuals are calculated under the assumption of multivariate normality, and thus will be biased if the normality does not hold.

The interpretation typically is:

(1) If a subset of variables has large standardized positive residuals (over fitting) with other variables and negative standardized residuals (under fitting) among themselves, this suggest that the subset constitutes a separate factor.

(2) An item that is related to a wrong factor has usually large negative residuals with the other items on that factor and large positive residuals with items on the "correct" factor.

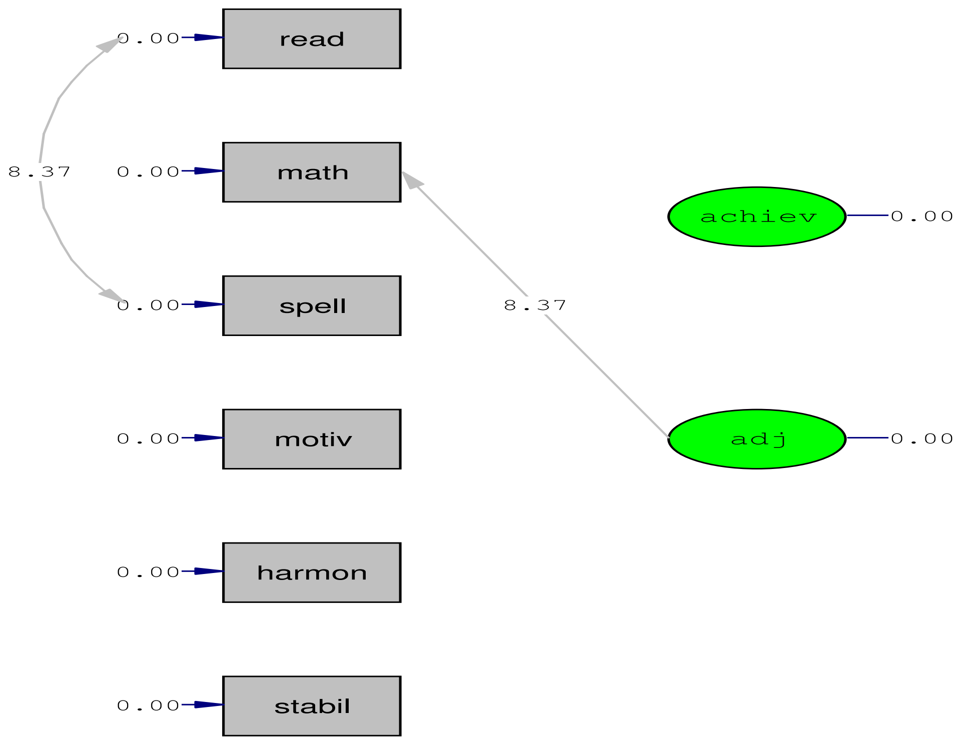
(3) If an item has many large residuals but no clear pattern exist, the item should be considered to be deleted.



Chi-Square=18.36, df=8, P-value=0.01866, RMSEA=0.091

The fit of the measurement model is not satisfactory.

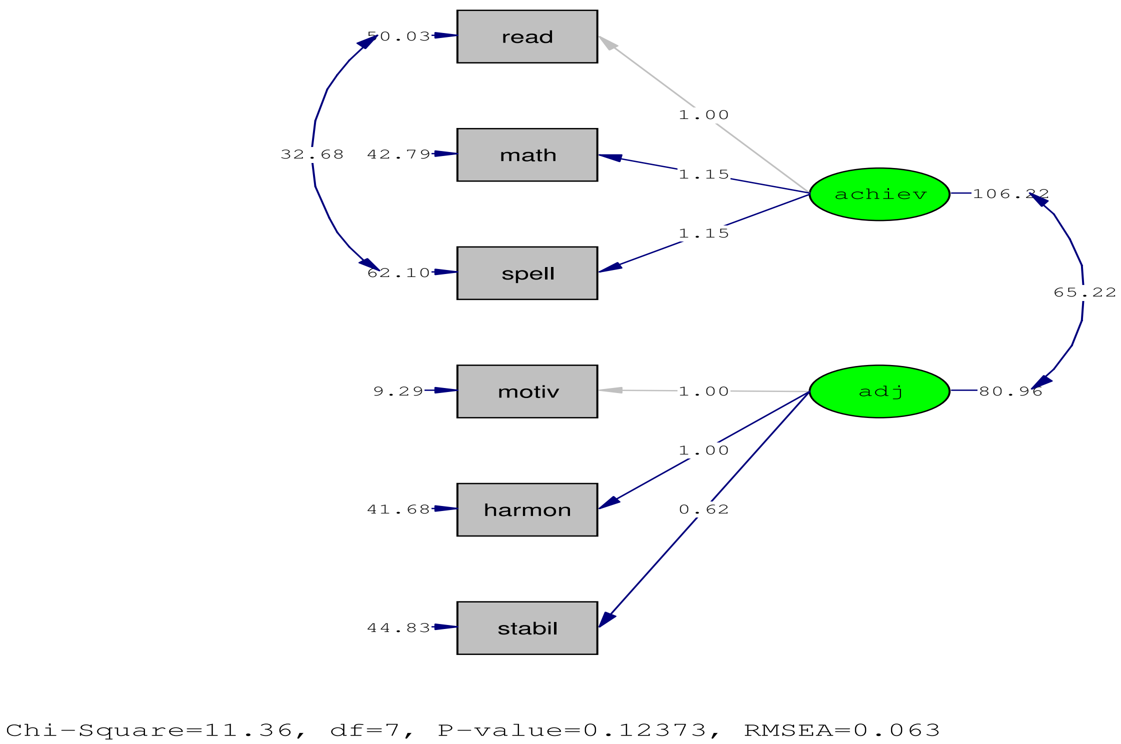
Modification indices:



Modification indices suggest correlation between the residuals of **read** and **spell** or that **math** measures also **adjustment**.

Allowing correlation between residuals of **read** and **spell** might be justified.

Allowing the residual correlation improves the goodness of fit.



Standardized residuals of the modified model

```
        Standardized Residuals
======================================================
           read    math   spell   motiv  harmon  stabil
------------------------------------------------------
  read      - -
  math     1.85     - -
 spell      - -   -1.85     - -
 motiv    -1.19    2.41    2.45     - -
harmon    -1.65   -2.27   -0.85   -0.19     - -
stabil    -0.22   -0.29    0.14   -2.25    2.41     - -
======================================================

 Summary Statistics for Standardized Residuals

 Smallest Standardized Residual =   -2.27
   Median Standardized Residual =    0.00
  Largest Standardized Residual =    2.45
```

All loadings are statistically significant ($|t| > 2$)

```
      Factor loadings

=============================
           achiev        adj
-----------------------------
  read       1.00        - -
  math       1.15        - -
           (0.12)
             9.88
 spell       1.15        - -
           (0.06)
            18.24
 motiv        - -       1.00
harmon        - -       1.00
                      (0.08)
                       12.32
stabil        - -       0.62   <--- loading
                      (0.07)   <--- standard error
                        8.90   <--- t-value
=============================
```

R-squares of the individual items indicate that `stabil` is most prone to error.

There are no "gold standards" of individual coefficients, but generally ≥ 0.90 are considered "excellent", ≥ 0.80 "very good", and ≥ 0.70 "adequate".

```
Squared Multiple Correlations for X - Variables
=========================================
  read   math  spell  motiv harmon stabil
-----------------------------------------
  0.68   0.77   0.69   0.90   0.66   0.41
=========================================
```

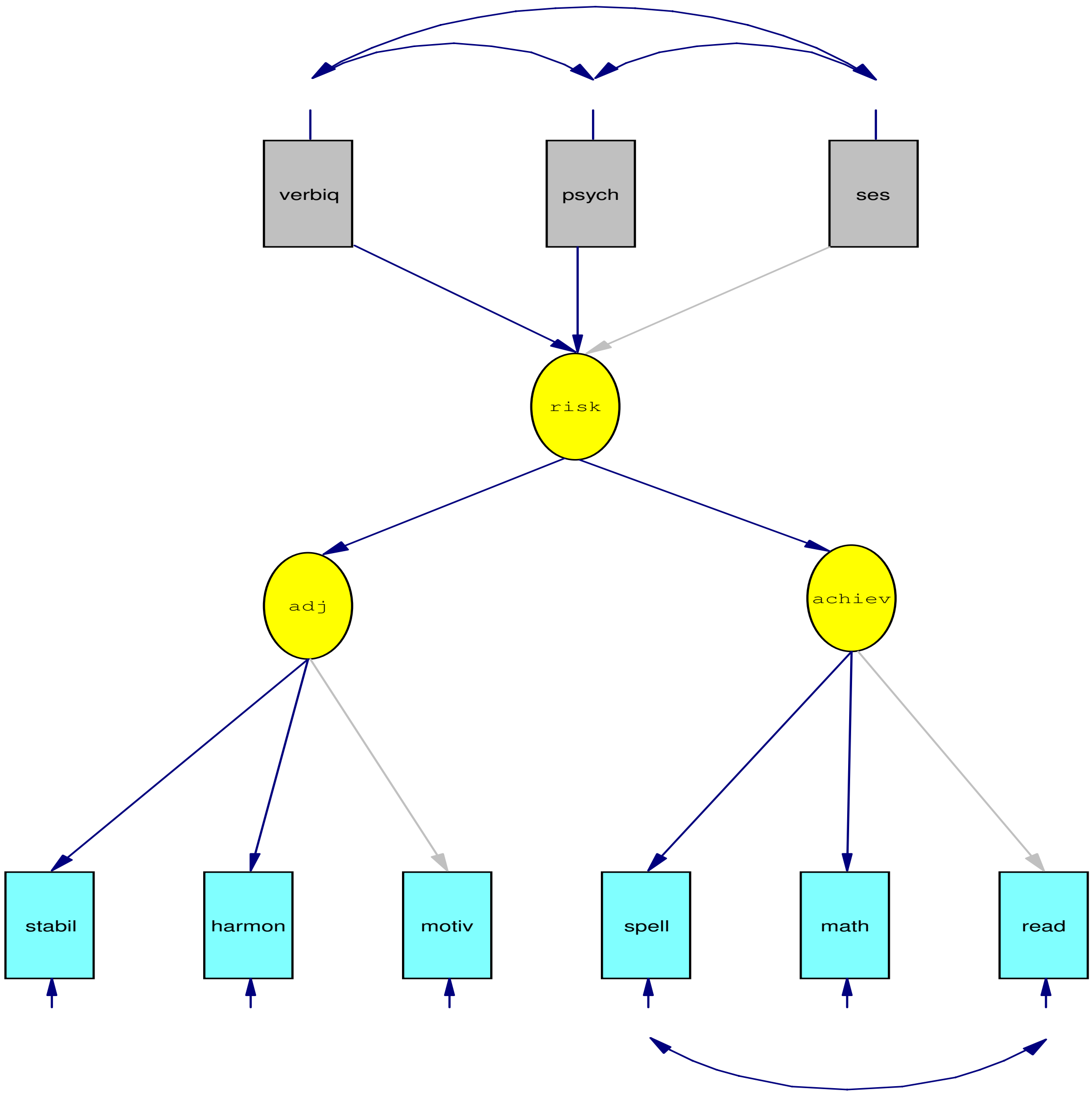
Internal consistencies of measures x_i, $i = 1, \ldots, p$ are typically measure in terms of Cronbach's alpha

$$\alpha = \frac{p}{p-1}\left(1 - \frac{\sum_{i=1}^{p} \mathbb{V}\mathrm{ar}[x_i]}{\mathbb{V}\mathrm{ar}[x]}\right), \tag{18}$$

where $x = x_1 + x_2 + \cdots + x_p$.

Cronbach's alpha for `read`, `math`, and `spell` is 0.91 and for `motiv`, `harmon`, and `stabil` is 0.84. Thus, at least "excellent" and "very good".

Fit of the model



yields $\chi^2(21) = 75.48$ with p-value 0.0000 and RMSEA 0.129 which indicate poor overall fit.

Remark 8.6: In SEM estimation covariance matrix should be used!

Use of sample correlation matrix tend to bias standard errors and thus the the related test statistics.

CFA is probably as sensitive to this bias in particular (in the cases where the measurements are kind of scale invariant). However, even in this case it seems that use of covariance matrix seems to be recommended.

Modification indices do unfortunately not help much here to improve the model.

Residual correlations:

```
         Standardized Residuals
=============================================================
        read  math spell motiv harmon stabil ses psych verbiq
-------------------------------------------------------------
  read   - -
  math -3.27   - -
 spell   - -  3.27  - -
--------------------------+
 motiv -1.94  3.17  2.78 |  - -
harmon -2.76 -0.35 -0.01 |-0.19   - -
stabil -0.69  0.49  0.52 |-1.87  2.00   - -
-------------------------------------------+
   ses -2.28  2.24  0.15 -0.47 -0.85 -0.06   - -
 psych -0.78  0.27  1.40  0.34 -0.39  0.09   - -   - -
verbiq  2.41 -0.55 -4.35 -0.16 -0.52 -0.25   - -   - -    - -
=============================================================
```

Intercorrelations between {`read`, `math`, `spell`} and {`motiv`, `harmon`, `stabil`} are under or over estimated, which indicates a need for relationship between constructs `adj` and `achiev`.

MacCallum and Browne suggest a causal relation `achiev` $\rightarrow$ `adj`.

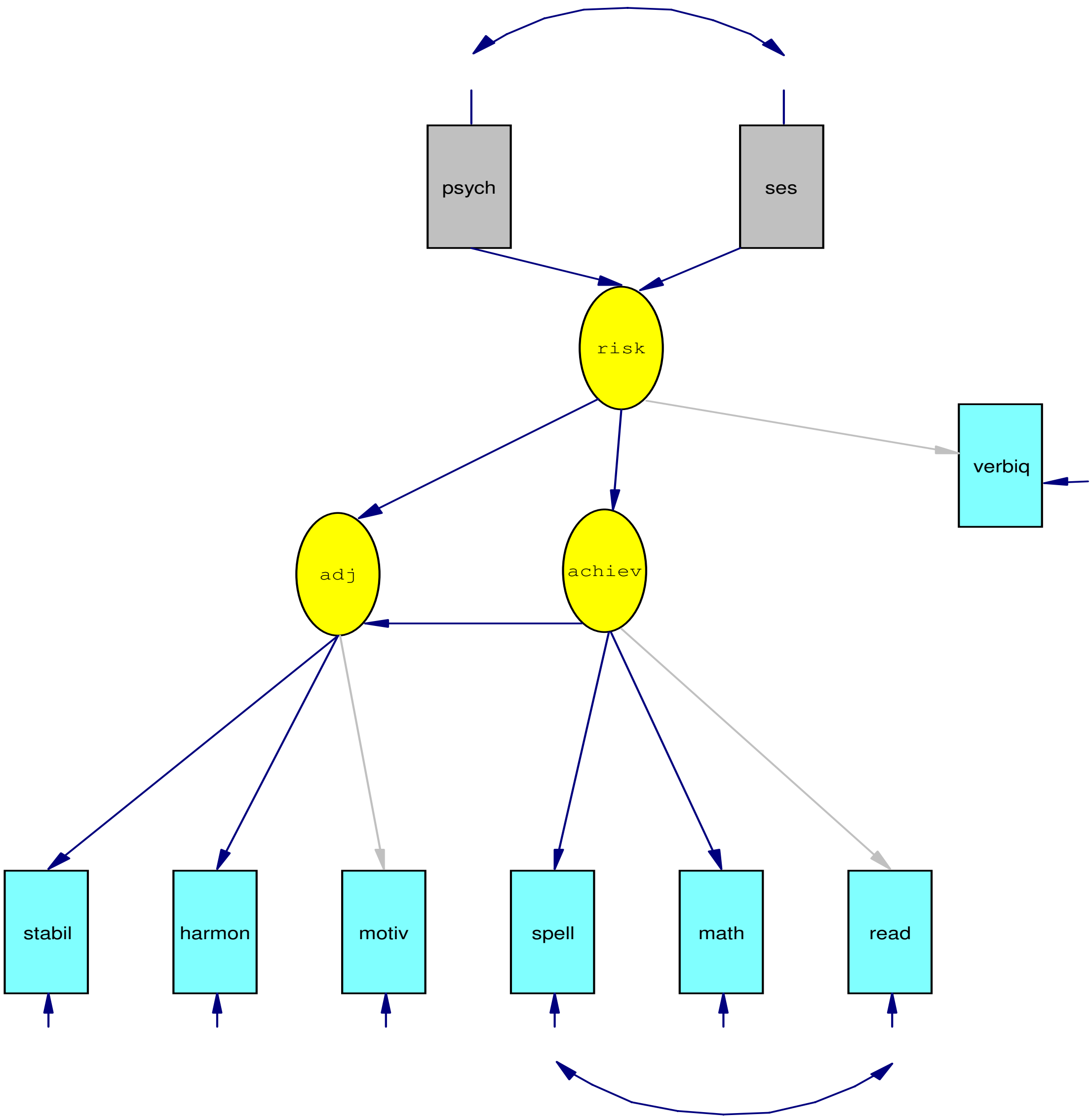
Adding this link, however makes the model unidentified!

This can be resolved if the risk composite is respecified as MIMIC factor with at least one effect indicator.

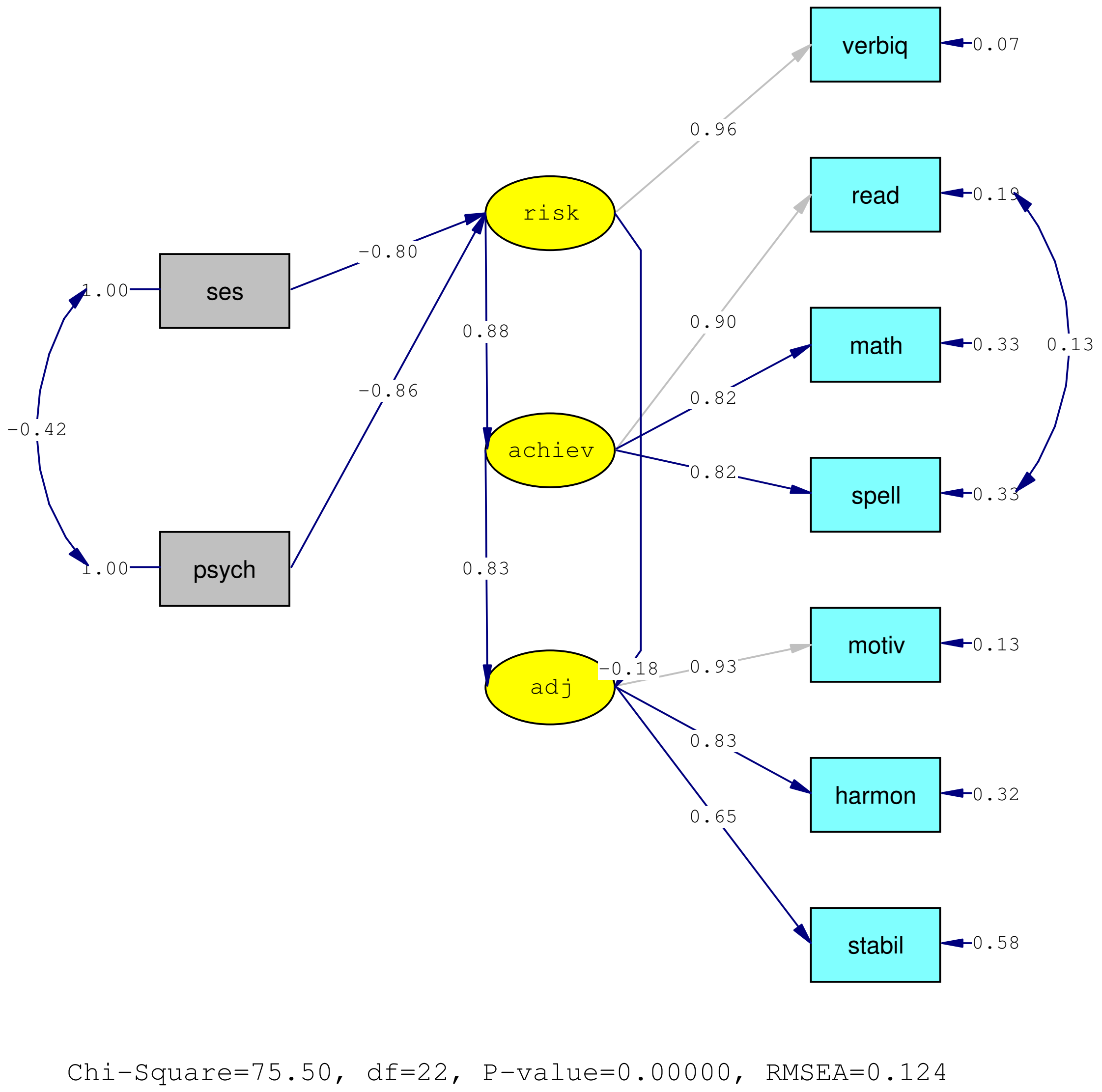
Suppose that verbal IQ is affected by family SES and parental psychopathology through the latent risk component.

```
! Example 8.6: Cognitive and achievement status of adolescents
!  MIMIC with an effect indicator
Observed Variables:
ses psych verbiq read math spell motiv harmon stabil
Correlation Matrix:
 1
 -.42  1
 -.43  -.50   1
 -.39  -.43   .78   1
 -.24  -.37   .69   .73   1
 -.31  -.33   .63   .87   .72   1
 -.25  -.25   .49   .53   .60   .59   1
 -.25  -.26   .42   .42   .44   .45   .77   1
 -.16  -.18   .33   .36   .38   .38   .59   .58  1
Standard Deviations:
13.00 13.50 13.10 12.50 13.50 14.20  9.50 11.10  8.70
sample Size: 158
Latent Variables: risk achiev adj
Relationships
verbiq = 1*risk         ! Effect indicator of risk factor
read = 1*achiev
math spell = achiev
motiv = 1*adj
harmon stabil = adj
risk = ses psych ! verbiq
achiev adj = risk
adj = achiev
Let the residuals of read and spell correlate
Lisrel output: mi ss rs
! options ad = off
Path Diagram
End of Problem
```

MIMIC with one effect indicator:



Estimation yields (standardized solution)



Goodness of Fit Index (GFI) = 0.90
Adjusted Goodness of Fit Index (AGFI) = 0.80

The overall fit is still poor.

The model could be improved by adding some theoretically sensible measurement error correlations.

The point of this example, however, is that there may be great flexibility in hypothesis testing when a MIMIC factor with both effect and cause indicators is substituted for a latent composite with cause only indicators (if it is sensible to do so).

Interaction effects

Moderation occurs when size (or direction) of the effect of X on Y varies with the levels of (moderating) variable M.

Example 8.7: Consider consumption C and income Y.

$$C = \alpha + \beta Y. \tag{19}$$

If the marginal propensity to consume (MPC), b, depend on, say level of wealth (W), we would have $\beta = \beta(W)$. If the relation is linear, then $\beta = \beta_1 + \beta_2 W$ and equation (19) becomes

$$C = \alpha + (\beta_1 + \beta_2 W)Y = \alpha + \beta_1 Y + \beta_2 WY, \tag{20}$$

where β_2 measures the change of MPC when the wealth W changes by one unit.

We can say that W has an moderating effect on the MPC (the relationship between consumption C and income Y).

Generally moderating effects are modeled via interaction terms.

Interaction of latent variables

In general form fairly complicated.

Special cases

Classifying moderating variable (e.g. gender):

Multiple group analysis can be used and testing equality of parameters of interest.

Continuous moderating variable:

Several approaches exist*

*Schumacker, R.E. and G.A. Marcoulides (Eds.) (1998). *Interaction and nonlinear effects in structural equation modeling*. Elbaum.

See also March, H.W., Z. Wen, and K.-T. Hau (2006). Structural equation models of latent interaction and quadratic effects, In *Structural Equation Modeling: Second Course*, G.R. hancock and R.O. Mueller (eds.), IAP.

(a) Using factor scores

The procedure has two steps:

(1) Estimate the measurement model and generate factor scores for each latent variable.

(2) The structural equation model is estimated as if the latent variables were observed.

Typically used for only preliminary purposes because measurement error structure cannot be properly accounted in SEM analysis (ignores measurement errors).

Example 8.8: Head Start Summer program.[*]

Head Start summer program of $n_1 = 148$ children with a control group of $n_2 = 155$ children for school readiness.

Measurements:

MED: Mother's education,

FED: Father's educations,

FOC: Father's occupation,

FAMINC: Family income,

MRT: Score on the Metropolitan readiness test,

ITPA: Score on the Illinois test of psycholinguistic abilities

[*]Magidson, J. (1977). Toward a causal model for adjusting for pre-existing differences in the non-equivalent control group situation. *Evaluation Quarterly* 1, 399–420.

MED—FAMINC are supposed to measure socio-economic status (ses).

MRT and ITPA are supposed to measure cognitive ability (abil).

Problems:

A: Test whether MED—FAMINC can be regarded as a single construct of ses for both groups. Is the measurement model the same for both groups? Is there a difference in the mean of ses between groups?

B: Estimate the structural equation model

$$\texttt{abil} = \alpha + \gamma \texttt{ses} + \zeta \tag{21}$$

Is γ the same (no moderation of group variable on the slope)? Test the hypothesis $\alpha = 0$.

The measurement model for the $\texttt{ses}$ are of the form

$$x_i^{(g)} = \tau_i + \lambda_i \mathsf{ses}^{(g)} + \delta_i^{(g)}, \tag{22}$$

where we assume τ_i and λ_i are group invariant, while $\mathbb{E}[\mathsf{ses}]$ may differ. Again this means testing

$$H_1 : q = 1 \tag{23}$$

Imposing restrictions means testing series of hypotheses:

$$\text{Given } H_1,\ H_2 : \Theta^{(1)} = \Theta^{(2)}. \tag{24}$$

$$\text{Given } H_2,\ H_{3.2} : \phi^{(1)} = \phi^{(2)}. \tag{25}$$

Alternatively

$$\text{Given } H_1,\ H_{3.1} : \phi^{(1)} = \phi^{(2)}. \tag{26}$$

Note that, $\mathbb{V}\mathsf{ar}[\mathsf{ses}^{(g)}] = \phi^{(g)}$, $g = 1, 2$.

```
Group Control       ! Example 8.8: Problem A: Initial model
Observed Variables:
MED FED FOC FAMINC MRT ITPA
Correlation matrix
1
.484 1
.224 .342 1
.268 .215 .387 1
.230 .215 .196 .115 1
.265 .297 .234 .162 .635 1
Standard Deviations
1.360 1.195 1.193 3.239 3.900 2.719
Means
3.839 3.290 2.600 6.435 20.415 10.070
Sample Size: 148
Latent Variables: ses abil
Relationships:
MED = 1*ses
FED FOC FAMINC = ses

Group Head Start
Correlation Matrix
1
.441 1
.220 .203 1
.304 .182 .377 1
.274 .265 .208 .084 1
.270 .122 .251 .198 .664 1
Standard Deviations
1.221 1.281 1.075 2.648 3.764 2.677
Means
3.520 3.081 2.088 5.358 19.672 9.562
Sample Size: 155
Relationships
 ses = CONST
 set error variances of MED - FAMINC free
 set variance of ses free
Path Diagram
End of Problem
```

For the initial model $\chi^2(10) = 37.13$, p-val 0.00005, and RMSEA $= 0.134$ indicates a poor fit.

Modification indices suggest correlation between MED and FED errors.

Allowing this by including the line

```
let the errors of MED and FED correlate
```

gives $\chi^2(8) = 5.99$, p-val 0.648, and RMSEA $= 0.000$, indicating acceptable fit.

Testing the series of above hypotheses gives:

Hypothesis	Chisq	df	Chisq diff	df diff	p-val	Decision
H1	5.99	8	na	na	0.648	Accept
H2	14.86	13	8.87	5	0.114	Accept
H3.2	17.96	14	3.10	1	0.078	Accept (borderline)
H3.1	7.24	9	1.25	1	0.273	Accept

The data supports hypothesis even $H_{3.2}$ which implies equal covariance matrices. However, in subsequent analyses we allow different measurement error variances and correlations and ses variances, i.e., adopt H_1 as our basic model.

In (22)

$$\mathbb{E}[x_i^{(g)}] = \tau_i + \lambda\kappa, \tag{27}$$

where $\kappa = \mathbb{E}[\text{ses}]$ in Head Start group and $= 0$ in the control group (latent variables do not have scales or origin, so we fix the origin to the control group).

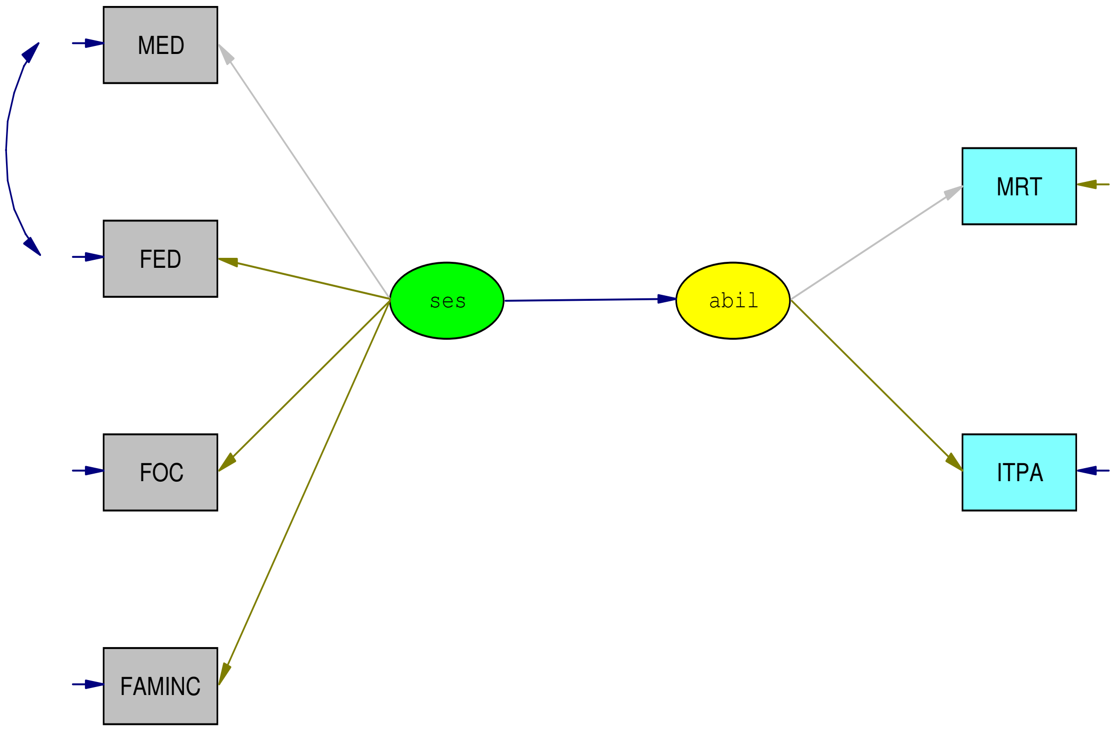
```
Group Control       ! Example 8.8: Problem A: ses mean
Observed Variables:
MED FED FOC FAMINC MRT ITSA
Correlation matrix
.....
Standard Deviations
....
Means
....
Sample Size: 148
Latent Variables: ses abil
Relationships:
 MED = CONST 1*ses
 FED FOC FAMINC = CONST ses
 Let the errors of MED and FED correlate
Path Diagram

Group Head Start
Correlation Matrix
....
Standard Deviations
....
Means
....
Sample Size: 155
Relationships
 ses = CONST
set error variances of MED - FAMINC free
let the errors of MED and FED correlate
set variance of ses free
LISREL OUTPUT
Path Diagram
End of Problem
```

	Control Group	Common Parms	Head Start
MED(tau1)		3.85 (0.09)	
FED(tau2)		3.33 (0.08)	
FOC(tau3)		2.58 (0.09)	
FAMINC(tau4)		6.47 (0.24)	
MED(l1)		1	
FED(l2)		0.88 (0.17)	
FOC(l3)		1.41 (0.29)	
FAMINC(l4)		3.40 (0.70)	
th1	1.56 (0.20)		1.24 (0.16)
th21	0.50 (0.14)		0.49 (0.13)
th2	1.16 (0.15)		1.48 (0.18)
th3	0.79 (0.17)		0.72 (0.13)
th4	6.91 (1.16)		4.29 (0.78)
kappa	0.0		-0.33 (0.09)
phi	0.32 (0.12)		0.23 (0.08)

The mean of the `ses` is -0.33 below in the head start group.

Model for Problem C:



The model is

$$\texttt{abil}^{(g)} = \alpha^{(g)} + \gamma^{(g)}\texttt{ses} + \zeta^{(g)} \tag{28}$$

α is fixed to zero in the control group. Thus, its value in Head Start group indicates the effect of the Head Start program when social status is controlled for.

```
Group Control        ! Example 8.8: Problem B                ! 1
Observed Variables:                                          ! 2
MED FED FOC FAMINC MRT ITPA                                  ! 3
Correlation matrix                                           ! 4
...
Standard Deviations                                          ! 5
...
Means                                                        ! 6
...
Sample Size: 148
Latent Variables: ses abil                                   ! 7
Relationships:                                               ! 8
  MED = CONST 1*ses                                          ! 9
  FED FOC FAMINC = CONST ses                                 ! 10
  MRT = CONST 1*abil                                         ! 11
  ITPA = CONST abil                                          ! 12
  abil = ses                                                 ! 13
Let the errors of MED and FED correlate                      ! 14

Group Head Start                                             ! 15
Correlation Matrix                                           ! 17
...
Standard Deviations                                          ! 18
...
Means                                                        ! 19
...
Sample Size: 155                                             ! 20
Relationships                                                ! 21
 ses = CONST                                                 ! 22
 abil = CONST  ses                                           ! 23
 set error variances of MED - FAMINC free                    ! 24
 set error variances of MRT - ITPA free                      ! 25
 set variance of ses free                                    ! 26
 set variance of abil free                                   ! 27
 Let the errors of MED and FED correlate                     ! 28
LISREL OUTPUT                                                ! 29
Path Diagram                                                 ! 30
End of Problem                                               ! 31
```

Estimating yields $\chi^2(22) = 26.37$, p-val 0.236, which indicates acceptable fit.

$\hat{\gamma}^{(1)} = 2.06\ (0.36)$ and $\hat{\gamma}^{(2)} = 2.40\ (0.76)$.

Imposing equality constraint between the groups (deleting or commenting ses oway from line 23, i.e., changing the line as: abil = CONST ! ses) gives $\chi^2(23) = 26.54$. The difference is

$$26.54 - 26.37 = 0.17$$

with one degree of freedom, thus we can treat the slopes equal. The estimate of the common slope coefficient is $\hat{\gamma} = 2.21(0.57)$.

The estimate of α is $\hat{\alpha} = 0.19\ (0.38)$ which is not significant.

The conclusion is that when controlling for `ses`, the Head Start program does not cause difference in cognitive ability.

The final estimated model is (comment off or delete line 23 and estimate)

	Control Group	Common Parms	Head Start
X-taus (intercepts)			
MED		3.86 (0.09)	
FED		3.34 (0.08)	
FOC		2.57 (0.09)	
FAMINC		6.41 (0.23)	
Y-taus (intercepts)			
MRT		20.43 (0.26)	
ITPA		10.15 (0.18)	
X-measurement model lambdas (factor loadings)			
MED		1	
FED		0.87 (0.15)	
FOC		1.25 (0.23)	
l4		2.86 (0.53)	
Y-measurement model lambdas (factor loadings)			
MRT		1	
ITPA		0.85 (0.14)	
X-measurement error variances and covariances			
MED	1.48 (0.20)		1.18 (0.15)
MED,FED	0.43 (0.13)		0.44 (0.13)
FED	1.09 (0.15)		1.44 (0.18)
FOC	0.83 (0.15)		0.72 (0.13)
FAMINC	7.40 (1.09)		4.60 (0.72)
Y-measurement error variances			
MRT	7.29 (1.62)		6.28 (1.52)
ITPA	1.67 (1.01)		1.50 (0.98)
Structural Equation			
gamma		2.14 (0.54)	
alpha		0	
var(ses)	0.39 (0.10)		0.29 (0.10)
var(abil)	6.26 (1.50)		6.42 (1.47)
kappa	0.0		-0.36 (0.10)

```
Normal Theory Weighted Least Squares Chi-Square = 26.91 (P = 0.31)
          Root Mean Square Error of Approximation (RMSEA) = 0.028
```