
Given $E[A_i] = \mu$, $\text{var}[A_i] = \sigma_A^2$, $\text{cor}[A_i, A_j] = \rho_{ij}$, $\text{cov}[A_i, A_j] = \sigma_{ij} = \sigma_A^2 \rho_{ij}$, and

$$\hat{\rho} = \frac{1}{n(n-1)} \sum_{i \neq j} \rho_{ij},$$

we have

$$\text{cov}[A_i, \bar{A}] = \frac{1}{n} \sum_{j=1}^{n} \text{cov}[A_i, A_j] = \frac{\sigma_A^2}{n} \sum_{j=1}^{n} \rho_{ij},$$

such that

$$\sum_{i=1}^{n} \text{cov}[A_i, \bar{A}] = \sigma_A^2(1 + (n-1)\hat{\rho}).$$

(2)

Similarly

$$\text{var}[\bar{A}] = \frac{1}{n^2} \text{var} \left[ \sum_{i=1}^{n} A_i \right] = \frac{\sigma_A^2}{n} (1 + (n-1)\hat{\rho}).$$

(3)

Thus, writing $A_i - \bar{A} = (A_i - \mu) - (\bar{A} - \mu)$ we have

$$E(A_i - \bar{A})^2 = \text{var}[A_i] - 2\text{cov}[A_i, \bar{A}] + \text{var}[\bar{A}],$$

such that

$$E[s^2] = \frac{1}{n-1} \sum_{i=1}^{n} E(A_i - \bar{A})^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left( \text{var}[A_i] - 2\text{cov}[A_i, \bar{A}] + \text{var}[\bar{A}] \right).$$

(4)

Utilizing equations (1)–(3) in equation (4) and collecting terms, gives equation (8) in Kolari and Pynnonen (2010), i.e.,

$$E[s^2] = (1 - \hat{\rho})\sigma_A^2.$$ 

(5)