Proof of equation (8) in Kolari and Pynnonen (2010), *Review of Financial Studies* 23, 3996–4025.

Given $E[A_i] = \mu$, $\texttt{var}[A_i] = \sigma_A^2$, $\texttt{cor}[A_i, A_j] = \rho_{ij}$, $\texttt{cov}[A_i, A_j] = \sigma_{ij} = \sigma_A^2 \rho_{ij}$, and

$$\bar{\rho} = \frac{1}{n(n-1)} \sum_{i \neq j} \rho_{ij},$$

we have

$$\texttt{cov}[A_i, \bar{A}] = \frac{1}{n} \sum_{j=1}^{n} \texttt{cov}[A_i, A_j] = \frac{\sigma_A^2}{n} \sum_{j=1}^{n} \rho_{ij}, \tag{1}$$

such that

$$\sum_{i=1}^{n} \texttt{cov}[A_i, \bar{A}] = \sigma_A^2 (1 + (n-1)\bar{\rho}). \tag{2}$$

Similarly

$$\texttt{var}[\bar{A}] = \frac{1}{n^2} \texttt{var}\left[\sum_{i=1}^{n} A_i\right] = \frac{\sigma_A^2}{n} \left(1 + (n-1)\bar{\rho}\right). \tag{3}$$

Thus, writing $A_i - \bar{A} = (A_i - \mu) - (\bar{A} - \mu)$ we have

$$E(A_i - \bar{A})^2 = \texttt{var}[A_i] - 2\texttt{cov}[A_i, \bar{A}] + \texttt{var}[\bar{A}],$$

such that

$$E[s^2] = \frac{1}{n-1} \sum_{i=1}^{n} E(A_i - \bar{A})^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(\texttt{var}[A_i] - 2\texttt{cov}[A_i, \bar{A}] + \texttt{var}[\bar{A}]\right). \tag{4}$$

Utilizing equations (1)–(3) in equation (4) and collecting terms, gives equation (8) in Kolari and Pynnonen (2010), i.e.,

$$E[s^2] = (1 - \bar{\rho})\sigma_A^2. \tag{5}$$