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Dynamic equilibrium correction modelling of credit spreads. The case of yen Eurobonds

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Dedicated to Ilkka Virtanen on the occasion of his 60th birthday

Abstract

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This study specifies an equilibrium correction model of the credit spreads on quality Japanese yen Eurobonds. In an important paper by Longstaff and Schwartz (1995) the authors derive a closed form solution of the arbitrage-free value on risky debt in continuous time. However, in discrete time real world data series it is common that many non-stationary economic and financial time series are cointegrated. Nevertheless, until now there is no theory for continuous time cointegration. In addition, the existence of cointegration in prices leads to incomplete markets so that the arbitrage-free valuation should not apply. Instead one must rely on equilibrium pricing, where the markets clear in the equilibrium via a potentially complicated adjusting process. In this paper the important factors driving the credit spreads are introduced into the equilibrium relation and the adjusting process are investigated. The results indicate that the corporate bond yields are cointegrated with the otherwise equivalent Japanese Government Bond (JGB) yields, with the spread defining the cointegration relation. Furthermore, the results indicate that the equilibrium correction term is highly statistically significant in modelling spread changes. The other important factor is the risk-free interest rate with the negative sign as predicted by the Longstaff and Schwartz (1995). On the other hand there is little evidence of the contribution of the asset return to the spread behaviour. The estimated coefficient of the equilibrium correction term indicates that the adjustment process is fairly slow, which indicates that the clearing process in the markets takes time.

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1. Introduction

While the Japanese Government generally does not issue securities in Eurobond, or foreign bond markets, Japanese Yen denominated Eurobond issues by the Japanese corporate sector are now the second largest in terms of new issues and outstandings after issues in U.S. dollars, and comprise nearly US$508 billion worth of outstanding bonds. From the viewpoint of corporate issuers in yen and international portfolio managers holding yen securities, the growth in the yen bond market provides important opportunities for diversification of funding and risk. However, in spite of the importance of the markets, little is known of the behaviour of yen denominated securities generally, and there are few empirical studies that investigate the relation between risky and riskless yen bonds, or specifically, the credit spreads which represent the difference in yield between these two risk classes of security. Batten, Hogan, and Pynnonen (2003a) investigate the time series relationships of Yen Eurobond spread changes to the most important factors predicted by Longstaff and Schwartz (1995) model and some additional factors found important mainly on U.S. markets. Batten, Hogan, and Pynnonen (2003b) further investigates the credit spread behaviour, and find that the equilibrium correction, time varying volatility and correlation factors are potentially important factors affecting the spread behaviour.

Recent theoretical developments on the valuation of risky debt proposed by Longstaff & Schwartz (1995), Das and Tufano (1996) and Duffie and Singleton (1999), predict a negative correlation between changes in default-free interest rates, the return on risky assets and changes in credit spreads. The empirical evidence in support of this relation is mixed. Originally, in U.S. bond markets Longstaff and Schwartz (1995) found evidence of a negative relation for both interest rate and asset changes. A weak but significant negative relation between changes in credit spreads and interest rates was also found by Duffee (1998) and Collin-Dufresne, Goldstein, and Martin (2001), while Neal, Morris, and Rolph (2000) identified a negative short-term relationship with credit spreads that reversed to positive in the long-run. For non-U.S. markets, Kamin and von Kleist (1999) find little evidence of a short-term relation between industrial country interest rates and emerging market bond spreads.

While the above papers focus directly on the modelling under the restriction of static equilibrium relation, the potential cross dynamics between the series demands a far more versatile approach. The strategy in this paper is to start off with an unrestricted setup using vector autoregression (VAR) of the important factors found in the earlier papers, particularly in Batten, Hogan, and Pynnonen (2003a,b), and proceed to identify the cross-dynamics between the series. The obvious advantage of this approach is that we can identify the important feedback relations between the credit spread and related main factors. This allows us to determine the time lags in the adjustment process to the equilibrium once shocks have driven the yields out of the equilibrium.

There is surprisingly much evidence that the credit spreads, measured in terms of yield differences over the corresponding government bonds, per se are non-stationary (for example Pedrosa and Roll 1998, Mansi & Maxwell 2000). In spite of these, it is fairly implausible that the yields could wander without bounds apart from each other in the long run. This was also confirmed in Batten et al. (2003b), where strong evidence was found for stationarity of the AA and AAA rated Yen Eurobonds.

Batten et al. (2003b) find that the most important factor driving the credit spread changes, as predicted by Longstaff and Schwartz (1995), is the changes in the risk-free rate. On the other hand the second important factor, firm asset return, implied by the Longstaff-Schwartz model did not show as being statistically significant. The reason for this may be that the stock market general index returns, that are usually utilized as proxies for the firm asset returns, do not properly reflect the true asset position, particularly when restricted to AA and AAA rated bonds. The other important factor found in Batten et al. (2003b) was the change in the slope of the term structure of Japanese government bonds proxied by the change in the yield spread of 20 year and 2 year bonds. Two other factors were the cointegration relation (spread), and conditional volatility of the spread changes. The importance of the conditional volatility as an explanatory variable in the mean equation of credit spread changes is again a factor that is not predicted by the theoretical model by Longstaff and Schwartz (1995). However, Engle, Lillien, and Robins (1987) find that the excess yield of the long bond depends on the conditional variance rather than being a constant. The empirical results of Batten et al. (2003b) indicate also that the conditional volatility is potentially an important factor in determining the credit spreads. The Longstaff and Schwartz (1995) model predicts that asset return volatility should be an important determinant of the credit spread. In the case of the corporate bonds it may well be that the conditional volatility of the credit spread replaces the more traditional stock return volatility as a surrogate for the firm asset return volatility.

Utilizing these findings we focus on dynamic modelling of the equilibrium relation of credit spreads on the yen Eurobonds. The paper contributes to the existing literature by specifying the dynamic structure of the equilibrium correction and identifying the important feedback links between the factors related to the credit spreads.

2. Methodology

Let $\mathbf{y}_t = (y_{1,t}, \ldots, y_{p,t})'$ denote a column vector of p time series, where the prime denotes transposition. Denote the general unstructured vector auto regression VAR model as

(1) $$\mathbf{y}_t = \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{\Phi}_k \mathbf{y}_{t-k} + \mathbf{B}\mathbf{x}_t + \boldsymbol{\varepsilon}_t = \mathbf{\Phi}(L)\mathbf{y}_t + \mathbf{B}\mathbf{x}_t + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{\Phi}_j$, $j = 1, \ldots, k$ are $p \times p$ autoregressive coefficient matrices, $\mathbf{\Phi}(L) = \mathbf{\Phi}_1 L + \mathbf{\Phi}_2 L^2 + \cdots + \mathbf{\Phi}_k L^k$ is the matrix lag-polynomial, $\mathbf{x}_t$ is a q-vector of predetermined variables including the intercept term unit vector, possible trends, seasonal terms etc., $\mathbf{B}$ is the $p \times q$ coefficient matrix of the predetermined variables, and $\boldsymbol{\varepsilon}_t$ is independent normally distributed error term with contemporaneous correlation matrix $\mathbf{\Sigma}$. Partition $\mathbf{y}_t = (\mathbf{y}'_{1,t}, \mathbf{y}'_{2,t})'$ such that $\mathbf{y}_{1,t}$ is a p_1-vector ($1 \le p_1 \le p$) where the series are $I(1)$, and $\mathbf{y}_{2,t}$ is a $p - p_1$ vector of stationary variables. Using the partition, write (1) as

(2) $$\begin{pmatrix} \mathbf{y}_{1,t} \\ \mathbf{y}_{2,t} \end{pmatrix} = \begin{pmatrix} \mathbf{\Phi}_{11,1} & \mathbf{\Phi}_{12,1} \\ \mathbf{\Phi}_{12,1} & \mathbf{\Phi}_{22,1} \end{pmatrix} \begin{pmatrix} \mathbf{y}_{1,t-1} \\ \mathbf{y}_{2,t-1} \end{pmatrix} + \cdots + \begin{pmatrix} \mathbf{\Phi}_{11,k} & \mathbf{\Phi}_{12,k} \\ \mathbf{\Phi}_{12,k} & \mathbf{\Phi}_{22,k} \end{pmatrix} \begin{pmatrix} \mathbf{y}_{1,t-k} \\ \mathbf{y}_{2,t-k} \end{pmatrix} + \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{pmatrix} \mathbf{x}_t + \begin{pmatrix} \boldsymbol{\varepsilon}_{1,t} \\ \boldsymbol{\varepsilon}_{2,t} \end{pmatrix}.$$

Because all the I(1) variables are in $\mathbf{y}_1$, the equilibrium correction (ECM) representation can be written as

(3a)

$$\Delta\mathbf{y}_{1,t} = \mathbf{\Pi}_1\mathbf{y}_{1,t-1} + \mathbf{\Gamma}_{1,1}\Delta\mathbf{y}_{1,t-1} + \cdots + \mathbf{\Gamma}_{1,k-1}\Delta\mathbf{y}_{1,t-k+1} + \mathbf{\Phi}_{12,1}\mathbf{y}_{2,t-1} + \cdots + \mathbf{\Phi}_{12,k}\mathbf{y}_{2,t-k} + \mathbf{B}_1\mathbf{x}_t + \boldsymbol{\varepsilon}_{1,t}$$

and

(3b)

$$\mathbf{y}_{2,t} = \mathbf{\Pi}_2\mathbf{y}_{1,t-1} + \mathbf{\Gamma}_{2,1}\Delta\mathbf{y}_{1,t-1} + \cdots + \mathbf{\Gamma}_{2,k-1}\Delta\mathbf{y}_{1,t-k+1} + \mathbf{\Phi}_{21,1}\mathbf{y}_{2,t-1} + \cdots + \mathbf{\Phi}_{21,k}\mathbf{y}_{2,t-k} + \mathbf{B}_2\mathbf{x}_t + \boldsymbol{\varepsilon}_{1,t},$$

where $\mathbf{\Pi}_1 = \mathbf{\Phi}_{11,1} + \cdots + \mathbf{\Phi}_{11,k} - \mathbf{I}$, $\mathbf{\Gamma}_{1,j} = -\sum_{l=j}^{k}\mathbf{\Phi}_{11,l}$, $\mathbf{\Pi}_2 = \mathbf{\Phi}_{21,1} + \cdots + \mathbf{\Phi}_{21,k}$, and $\mathbf{\Gamma}_{2,j} = -\sum_{l=j}^{k}\mathbf{\Phi}_{21,l}$, $j = 2,\ldots,k$.

Now if the I(1) series are cointegrated, the rank, r, of $\mathbf{\Pi}_1$ is less than p_1, and there exists a decomposition $\mathbf{\Pi}_1 = \boldsymbol{\alpha}_1\boldsymbol{\beta}_1'$, where $\boldsymbol{\alpha}_1$ and $\boldsymbol{\beta}_1$ are $p_1 \times r$ full rank matrices. The matrix $\boldsymbol{\beta}_2$ contains the stationary cointegration relationships of the I(1) variables, and $\boldsymbol{\alpha}_1$ contains the short term adjustment coefficients of discrepancies from the long term equilibrium determined by the cointegration vectors. In order for (3b) to be balanced in the sense that both sides of the equation are stationary there must exist a $(p - p_1) \times k$ matrix $\boldsymbol{\alpha}_2$ (that can be a zero matrix) such that $\mathbf{\Pi}_2 = \boldsymbol{\alpha}_2\boldsymbol{\beta}_1'$. Thus $\boldsymbol{\alpha}_2$ contains the response coefficients of the stationary variables to the discrepancies of the I(1) variables from the long term equilibrium. In most cases this, however, may not be plausible.

In any case, the important point here is that the rank of $\mathbf{\Pi}_1$ determines the dimension of the cointegration space. As discussed above, for identification of the equilibrium relationships, it may, however, be important to include besides the predetermined terms given in $\mathbf{x}_t$, also the stationary variables of the system in to the cointegration relation. Johansen (1995: 80–84) is a useful reference for discussion concerning the deterministic terms in the cointegration relations. Here, we focus purely on the inclusion of the endogenous stationary series. This approach is particularly useful when the cointegration vectors can be fixed on the basis of prior knowledge, like in our case of credit spreads. For the purpose write the whole system (2) in the ECM form

(4) $$\Delta \mathbf{y}_t = \boldsymbol{\alpha\beta}'\mathbf{y}_{t-1} + \boldsymbol{\Gamma}_1 \Delta \mathbf{y}_{t-1} + \cdots + \boldsymbol{\Gamma}_{k-1}\Delta \mathbf{y}_{t-k+1} + \mathbf{B}\mathbf{x}_t + \boldsymbol{\varepsilon}_t ,$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times r$ matrices with $\boldsymbol{\beta}' = (\boldsymbol{\beta}_1', \boldsymbol{\beta}_2')$ of rank r, the cointegration rank of the system. The matrix $\boldsymbol{\beta}_1$ determines the cointegration relations of the I(1) variables in the system and $\boldsymbol{\beta}_2$ includes the coefficients of the stationary variables in the equilibrium relations. In principle the contribution of the stationary variables into the equilibrium relation can be estimated at the same time as the I(1) variables. Nevertheless, it is more efficient to solve the problem separately first for the integrated variables and then map the cointegration VAR into I(0) space, where one can purely deal with stationary variables and utilize the advanced estimation techniques developed therein.

Let $\mathbf{ci}_t = \boldsymbol{\beta}'\mathbf{y}_t = \boldsymbol{\beta}_1'\mathbf{y}_{1,t} + \boldsymbol{\beta}_2'\mathbf{y}_{2,t}$, then denoting $\mathbf{s}_t = \boldsymbol{\beta}_1'\mathbf{y}_{1,t}$, multiplying (4) from the left by $\boldsymbol{\beta}'$, and collecting terms, we get

(5) $$\Delta \mathbf{s}_t = \tilde{\boldsymbol{\alpha}}\mathbf{s}_{t-1} - \boldsymbol{\beta}_2'\Delta \mathbf{y}_{2,t} + \boldsymbol{\eta}\mathbf{y}_{2,t-1} + \tilde{\Gamma}_1 \Delta \mathbf{y}_{t-1} + \cdots + \tilde{\Gamma}_{k-1}\Delta \mathbf{y}_{t-k+1} + \tilde{\mathbf{B}}\mathbf{x}_t + \boldsymbol{\upsilon}_t ,$$

where $\tilde{\boldsymbol{\alpha}} = \boldsymbol{\beta}'\boldsymbol{\alpha}$, $\tilde{\Gamma}_i = \boldsymbol{\beta}'\Gamma_i$, $\tilde{\mathbf{B}} = \boldsymbol{\beta}'\mathbf{B}$, and $\boldsymbol{\upsilon}_t = \boldsymbol{\beta}'\boldsymbol{\varepsilon}_t$. Regarding parameter $\boldsymbol{\eta}$, it should be noted that, because $\mathbf{y}_{2,t}$ is stationary, it is possible that in the same manner as the exogenous variables, $\mathbf{x}_t$, the stationary variables may play in two roles; in the one hand inside and in the other hand outside the equilibrium correction relation. What we have in formula (5) is the aggregate contribution. If $\mathbf{y}_{2,t}$ solely contributes to the equilibrium correction term, then coefficient vector $\boldsymbol{\eta}$ obeys the restriction $\boldsymbol{\eta} = \tilde{\boldsymbol{\alpha}}\boldsymbol{\beta}_2'$. Consequently the portion $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta} - \tilde{\boldsymbol{\alpha}}\boldsymbol{\beta}_2'$ reflects the contribution of $\mathbf{y}_2$ to the spread, outside of the equilibrium correction relation.

In order to further interpret the parameters, let us assume that the system is in the equilibrium, which is achieved when the variables are in their means. That is, when $E(\mathbf{s}_t) = \boldsymbol{\mu}_s$, $E(\mathbf{y}_t) = \boldsymbol{\mu}_y$, $E(\mathbf{x}_t) = \boldsymbol{\mu}_x$, and $E(\boldsymbol{\upsilon}_t) = 0$, in which case $E(\Delta\mathbf{s}_{1,t}) = 0$, $E(\Delta\mathbf{y}_{2,t}) = 0$, and $E(\Delta\mathbf{y}_{t-j}) = 0$ for all $j = 1,\ldots,k-1$, and we get $0 = \tilde{\boldsymbol{\alpha}}\boldsymbol{\mu}_s + \boldsymbol{\eta}\boldsymbol{\mu}_{y_2} + \tilde{\mathbf{B}}\boldsymbol{\mu}_x$. Now $\tilde{\boldsymbol{\alpha}}$ is a $p_1 \times p_1$ full rank matrix so that the inverse exists, an because in our case $\mathbf{s}$ represents the spreads, we find that the average spread is determined as

(6) $$\boldsymbol{\mu}_s = -\tilde{\boldsymbol{\alpha}}^{-1}(\boldsymbol{\eta}\boldsymbol{\mu}_{y_2} + \tilde{\mathbf{B}}\boldsymbol{\mu}_x).$$

Thus, particularly in our case, where $\mathbf{s}_t$ is the credit spread, we find that parameter $\boldsymbol{\eta}$ relates the long run average spread to the long run averages of the stationary variables.

3. Empirical results

The data consists of AA2, AA5, AA10, AAA2, AAA5, and AAA10 Japanese corporate bonds and corresponding maturity Japanese Government bonds (JGBs). The sample period is from January 2, 1995 to October 21, 1998. Following Longstaff and Schwartz (1995), we adopt the long yield of the 20-year JGB to represent the risk-free rate. The asset factor is measured in terms of the Nikkei return. It is common in the empirical finance literature to work with continuously compounded variables, which leads to log-returns. Particularly, because we have daily observations, we follow this practice and define the yields as

(7) $$y_t = 100 \times \log(1 + Y_t),$$

where Y_t is the yield to maturity on a bond. The credit spreads of maturity n are then simply defined as

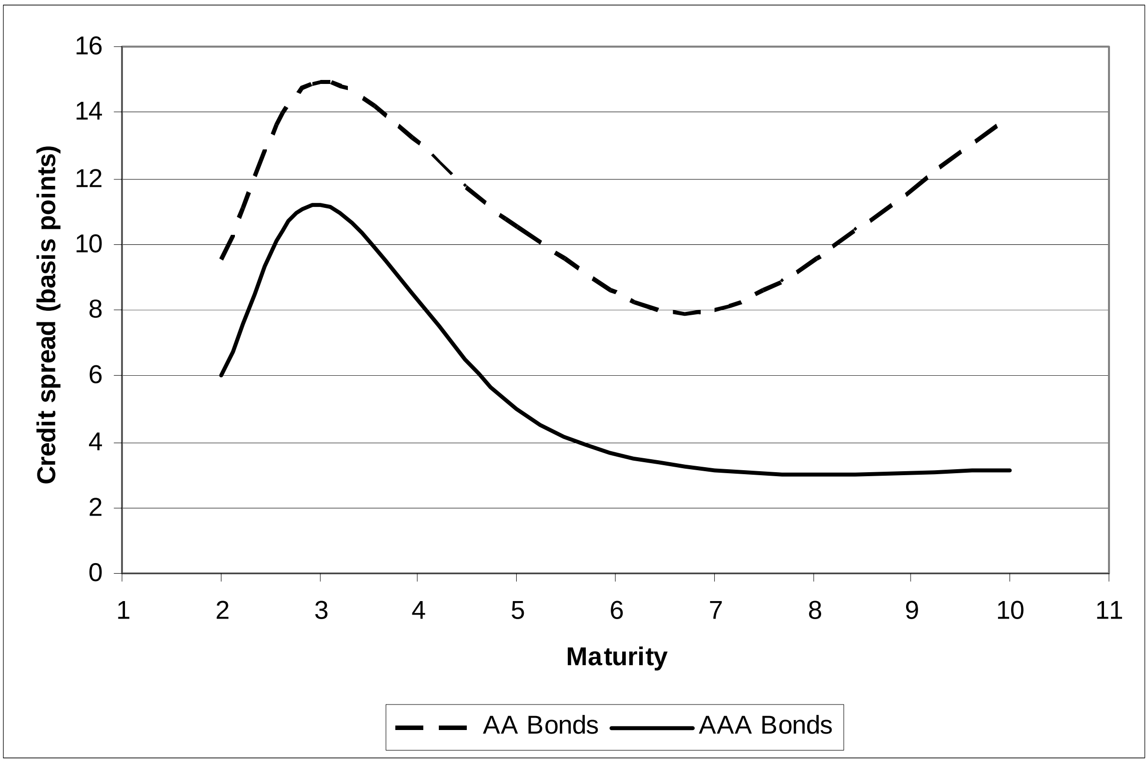

Figure 1. Log credit spread term structure of Japanese AA and AAA rated Eurobonds with maturities 2, 3, 5, 7 and 10 years, estimated from daily observations with sample period January 2, 1995 to October 21, 1998.

(8) $$s_{n,t} = y_{CBn,t} - y_{JGn,t} ,$$

where $y_{CBn,t}$ is the yield of a corporate bond with maturity *n*, and $y_{JGn,t}$ is the corresponding maturity Japan Government Bond yield.

Figure 1 depicts the term structure of the (log) credit spreads of the AA and AAA rated Japanese Eurobonds calculated as averages of the daily observations over the sample period for maturities 2, 3, 5, 7, 10 and 20 years.

The term structure is humped shaped with maximum spread occurring for the 3-year corporate bonds in both credit classes. While the spread is decreasing for the AAA rated longer maturity bonds the spread again increases for the AA rated 10-year corporate bonds.

Table 1. Summary statistics for log credit spreads of Japanese AA and AAA rated Eurobonds.

	AA2 (bp)	*AA5 (bp)*	*AA10 (bp)*	*AAA2 (bp)*	*AAA5 (bp)*	*AAA10 (bp)*	*ΔJGB20 (%)*	*Nikkei ret (%)*
Mean	9.50	10.50	13.82	6.03	4.98	3.15	-0.003	-0.033
Std	7.78	6.85	7.95	7.25	7.43	7.85	0.045	1.450
Kurtosis	0.82	1.33	5.01	0.05	1.39	6.82	5.750	2.654
Skewness	0.29	-0.31	0.59	0.05	-0.46	-0.15	-0.356	0.083
Minimum	-20.93	-18.61	-19.36	-21.93	-28.37	-32.99	-0.317	-5.957
Maximum	42.19	40.10	77.15	30.43	36.24	68.50	0.230	7.660
N	993	993	993	993	993	993	993	993

The sample period is daily observations from January 2, 1995 to October 21, 1998. The yield spreads are computed as $sp_t = y_{c,t} - y_{g,t}$, where y_c is the corporate daily yield and y_g is the corresponding government bond yield as defined in formula (7). The spreads are measured in basis points (1 bp = 0.01%).

Summary statistics for log credit spreads of the yen denominated Eurobonds, Nikkei Daily returns and JGB 20-year log yield changes are presented in Table 1. The mean credit spreads indicate numerically the information presented in Figure 1 for maturities investigated in this paper. The standard deviations are between 6.85 and 7.85 basis points, so within the range of one basis point. This indicates that the volatility is about the same over the maturities. The negative minima of the spreads indicate that there are periods where the yields of the corporate bonds are less than those of the otherwise equivalent government bonds.

From an econometric modelling point of view, adoption of the 20-year JGB yield as representative of the risk-free rate could be worked out through including the yield (which is an I(1) series) into the cointegration relation. Theoretically, this should be justified, because the expectation hypothesis or the liquidity premium hypothesis predicts that the term spread should be stationary. That is, short and long term JGB yields should be cointegrated (see, e.g., Hall, Anderson, and Granger 1992). The second column of Table 2 reports the Augmented Dickey-Fuller (ADF, Dickey and Fuller 1978, 1981) unit root tests for the yields. The null hypothesis of unit root is accepted for all yield series, but strongly rejected for the first differences of the series (fourth and fifth columns). Thus the empirical results support that the series are I(1), i.e., integrated of order on.

Table 2. I(1) tests for the log yields and spreads, and cointegration tests for the terms spreads of various maturity Japanese government bonds (JGBs) with the for the 20-year JGB.

Series	ADF[1)] (level)	p-val	ADF (1. diff)	p-val	Spread	ADF	p-val
AA2	-3.16	0.093	-9.97	0.000	AA2-JGB2	-6.45	0.000
AA5	-3.13	0.100	-8.46	0.000	AA5-JGB5	-6.11	0.000
AA10	-2.10	0.546	-12.65	0.000	AA10-JGB10	-8.89	0.000
AAA2	-3.13	0.100	-12.18	0.000	AAA2-JGB2	-7.66	0.000
AAA5	-3.29	0.068	-8.26	0.000	AAA5-JGB5	-5.68	0.000
AAA10	-2.32	0.421	-35.48	0.000	AAA10-JGB10	-7.80	0.000
JGB2	-2.89	0.165	-14.18	0.000	JGB2-JGB20	-2.02	0.590
JGB5	-2.73	0.224	-34.61	0.000	JGB5-JGB20	-1.77	0.721
JGB10	-2.33	0.418	-15.55	0.000	JGB10-JGB20	-4.04	0.008
JGB20	-2.55	0.304	-6.36	0.000	-	-	-

Johansen (1991, 1995) Trace and Max Eigenvalue test for cointegration of JGB yields

	Trace statistic	Max eigenvalue statistic
JGB2-JGB20	16.43	11.97
JGB5-JGB20	12.52	9.37
JGB10-JGB20	25.76*	22.30*
5% critical value	25.32	18.96
1% critical value	30.45	23.65

* = significant at the 5% level

[1)]The augmented Dickey & Fuller (1979, 1981) (ADF) test is based on the regression $\Delta y_t = \mu + \gamma y_{t-1} + \delta t + \phi_1 \Delta y_{t-1} + \cdots + \phi_m \Delta y_{t-m} + \varepsilon_t$ with null hypothesis that the series are I(1), which implies the to testing that $\gamma = 0$. The lag-length *m* of the differences is determined by Akaike's (1978) information criterion. The trend and intercept terms are allowed to eliminate their possible effect from the series.

Columns seven and eight report the unit root test results for the credit spreads of the corporate bonds and term spreads of the government bonds against the long maturity (20 years) bond. In all cases of the term spreads, the null hypothesis of a unit root is strongly rejected. These results, along with the above unit root findings, suggest that the individual series are best modelled as I(1), and so it is possible to infer that the corporate bonds and otherwise equivalent government bonds are cointegrated with the spread defining the cointegration relation. On the other hand in the case of term spreads, the unit root tests, reported in three last lines in columns seven and eight, indicate that JGB2 and JGB5 spreads with respect to JGB20 are non-stationary. The JGB10 is an exception for which the test results support stationarity. Even if we allow a more general cointegration relation than the term spread, the hypothesis of cointegration is rejected for the 2 and 5-year bonds as can be seen from the lower panel of Table 2, where the Johansen (1995, 1998) cointegration test results are reported. Again with the 10-year bond the null hypothesis of cointegration is accepted, supporting the above unit root test result of the spread.

Cointegration of the 10-year JGB with the 20-year JGB with the spread as the cointegration relation has an implication for the modelling of the 10-year corporate bonds. For example, because the spreads define the cointegration relation for both the AAA and JGB 10-year bonds and JGB 20 and 10-year bonds, the dimension of the cointegration space of the yields of AAA10, JGB10 and JGB20 bonds has the rank equal to two, where the spreads identify the cointegration vectors. The implication for modelling of the credit spread then is that the term spread of JGB 20-year and 10-year may have an effect on the dynamics of the credit spreads of AA and AAA 10-year bonds. However, this finding was not confirmed in the subsequent regressions and so it is ignored in the final model of the 10-year spreads.

Using the Longstaff and Schwartz (1995) and the empirical results found in Batten et al. (2003b), discussed in Section 1, we use in regression (5) besides the lagged spread, the JGB 20-year bond yield change, Nikkei return, r_t, the term structure slope change measured in terms of the JGB20 and JGB2 spread change, $\Delta(y_{JG20} - y_{JG2})_t$, where y_{JG20}, and y_{JG2} are the log-yields of the 20-year and 2-year JGBs. In addition to account for autocorrelation in the residuals we allow for ARMA structure in the residuals, and, furthermore, to model the possible conditional heteroscedasticity, we use GARCH specification, and allow its possible effect on the mean equation as well. These are the major advantages to estimate the possible contribution of the stationary series to the equilibrium relation of the yields, and hence to the spreads. Thus augmenting (5) with these factors the estimated models for the spread changes are of the form

$$
\begin{aligned}
\Delta s_t &= \delta_0 + \delta_1\sqrt{h_t} + \alpha s_{t-1} + \beta_1\Delta r_t + \beta_2\Delta^2 y_{JG20,t} + \beta_2\Delta^2(y_{JG20} - y_{JG2})_t \\
&\quad + \eta_1 r_{t-1} + \eta_2\Delta y_{JG20,t-1} + \eta_3\Delta(y_{JG20} - y_{JG2})_{t-1} + u_t \\
u_t &= \phi u_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t \\
h_t &= \omega + \gamma_1\varepsilon_{t-1}^2 + \gamma_2 h_{t-1}
\end{aligned}
\tag{9}
$$

where, we have dropped for the sake of simplicity the maturity index from the subscript, and ε_t is the residual term with a GARCH(1,1) variance process. There are of course other parameterisations of (9), but we find the above choice the most convenient, where we directly identify the equilibrium correction coefficients of the stationary series, and then find the long-run mean spread using formula (6).

The Longstaff and Schwartz (1995) model predicts that the signs of β_1 and β_2 should be negative, which imply the increased asset return should decrease the spread and an increase in risk-free return should also decrease the credit spread. The former is intuitive since improved asset return drifts the company asset value up and improves the company's solvency. The negative impact of the risk-free return on the spread is explained in Longstaff and Schwartz (1995) in terms of the risk-neutral pricing process implied by the no-arbitrage pricing mechanism of the bonds and consequently yields. Because the risk-free return serves as the drift in the risk neutral asset value process, the increase of the risk-free return drives the risk neutral asset process away from the default boundary, and hence decreases the (risk-neutral) probability of default. Consequently the spread over the risk-free rate should decrease. The slope of the yield curve is related to state of the economy. Collin-Dufrense, Goldstein, and Martin (2001) argue that as the economy moves into recession, the steepness of the yield curve declines. In such an economic phase the asset returns are expected to decrease and hence the firm values decline closer to the default boundary, and therefore increase the default risk. Consequently, the credit spread can be expected to increase, which should shows up in (9) with a negative β_3.

Estimation results and related diagnostic statistics for the selected maturity credit spreads are reported in Appendix 1 and 2. From the tables we find that the cointegration term, the spread s_{t-1}, is highly statistically significant in all cases with a negative coefficient. In the AA case the coefficient ranges from -0.14 to -0.095, which indicates a fairly slow adjustment in the spread towards the equilibrium. In terms of these estimates the model predicts that a shock of 100 basis points deviation from the equilibrium results to a correction of 9.5 to 14 basis points the next day in the spread. In the AAA case the coefficient estimates are bit lower, ranging from -0.111 to -0.048.

The other important findings from the regression results are that the contribution of the risk-free interest rate to the equilibrium can be obviously inferred to be negative (estimate of coefficient β_2), exactly as predicted by the Longstaff and Schwartz (1995) model. On the other hand the contribution of the asset factor to the equilibrium does not show up in the estimation results. Its regression coefficient (β_1) estimates close to borderline statistically significant at the 10 percent level for AAA5 and AAA10 spreads, and even in these cases the signs are opposite to that predicted by the Longstaff and Schwartz (1995) model. However, the significance of the estimate of η_1, which is the parameter measuring the asset factor's mean contribution to the average spread is highly statistically significant. Again the sign is opposite to what could be expected from the Longstaff and Schwartz (1995) model. A partial explanation to this may be that the sample period included a rather exceptional time episode in Japanese economy; the negative average daily stock return of -0.033% (about -8.3% p.a.). This implies that the ultimate contribution of the asset return to the average spread becomes negative as predicted by the Longstaff and Schwartz (1995) theory. More generally, we can write equation (6) as

$$\mu_s = \mu_{s,c} + \mu_{s,a} + \mu_{s,r} + \mu_{s,y} = -(\mu_c + \eta_1\mu_a + \eta_2\mu_r + \eta_3\mu_y)/\alpha, \tag{10}$$

where $\mu_{s,c} = -\mu_c/\alpha$, $\mu_{s,a} = -\eta_1\mu_a/\alpha$, $\mu_{s,r} = -\eta_2\mu_r/\alpha$, and $\mu_{s,y} = -\eta_3\mu_y/\alpha$ are the asset, risk-free rate, and yield curve contributions to the mean spread with μ_c the constant term in the regression and μ_a, μ_r, and μ_y asset return, risk-free interest rate and yield curve change long run averages, respectively. Thus, for example, in the case of the AAA rated 10-year bond, we get an estimate for the asset factor equal to $\hat{\mu}_{s,a} = -0.0020 \times (-0.033)/(-0.111) = -0.0006$, or -0.06 basis points. This is obviously small and economically non-insignificant when compared to the total average spread of 4.10 basis points. Thus, in summary, the asset factor's impact cannot be identified clearly in the Japanese markets as a determinant of the credit spread.

The estimate of the asset factor showed up as being weaker than the interest rate factor in Longstaff and Schwartz (1995) and Collin-Dufresne et al. (2001), but was clearly statistically significant and negative. In both of these studies monthly data were used. It may well be that the daily data used in our study is too noisy to measure accurately the asset factor. In any case the role of this factor as a proxy for the economic state in the model remains unclear.

The third important factor is the change in the slope of the yield curve. The estimates are highly significant, but again of opposite sign to what is predicted by Collin-Dufresne et al. (2001). Again this may be due the sample period covering an exceptional period in Japanese economy. The average yield curve change, as measured in terms of the spread change of 20-year and 2-year Japan Government bonds, has been slightly negative.

In the GARCH volatility process all the estimates, except the constant term, are highly significant. Thus there is obvious conditional heteroscedasticity in the credit spread. The sum of the GARCH parameter estimates of γ_1 and γ_2 is in most cases close to one, indicating that the volatility process is close to being integrated with a weight 0.05 for the latest shock and 0.95 for the persistence. With these weights the volatility process is in any case fairly smooth. The changing volatility, however, does not show up in the mean equation.

4. Summary and Conclusions

In an important paper Longstaff and Schwartz (1995) derive a closed form solution for the price of risky bond under the arbitrage-free assumption. Their model predicts that the yield spreads should be a negative function of both the firm asset return and the risk-free interest rates. The model is an equilibrium solution, where price adjustments are assumed to take place immediately. Nevertheless, in real discrete time trading there are delays and frictions that constitute feedbacks between integrated price series. This implies that a the non-stationary series may become cointegrated. That is, a linear combination of the series is stationary. This study shows that there is strong empirical evidence that the Japanese Yen Eurobond yields are cointegrated with the equivalent maturity Japanese Government Bonds (JGBs) with the spread defining the cointegration relation.

Because cointegration leads to incomplete markets, we must give up arbitrage-free pricing and rely on equilibrium pricing, where the markets clear via a potentially complicated adjustement process. Taking the spread as the core of the equilibrium cointegration relation we derive a model, where the contribution of stationary series, like the asset return and the change in the risk-free rates, to the equilibrium relation can be estimated and tested. The equilibrium correction term of the cointegration relation is an important determinant in the adjustment process of the spread to the equilibrium in each of the investigated series. Furthermore, the results suggest that the adjustment process is fairly slow. Of the stationary series the most important factor with the predicted sign by Longstaff and Schwartz (1995) is the risk-free interest rate. The asset return, on the other hand, does not show up as a significant factor in the equilibrium. The slope of the yield curve of Government bonds, which is intended to reflect the phase of the economy, turned out to also be statistically significant, but with the opposite sign to what would be expected. Thus the real role of this variable remains unclear.

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Appendix 1. Estimates of the parameters of model (9) for AA rated Japanese Eurobond corporate bond spreads.

	Δs2			Δs5			Δs10		
	Coeff	std err	p-val	Coeff	std err	p-val	Coeff	std err	p-val
Constant [δ_0]	0.0064	0.0017	0.000	0.0110	0.0028	0.000	0.0163	0.0038	0.000
Sqrt(h_t) [δ_1]	-	-	-	-	-	-	-	-	-
s_{t-1} [α]	-0.095	0.017	0.000	-0.111	0.024	0.000	-0.140	0.030	0.000
Δ^2log(Nikkei)$_t$ [β_1]	-	-	-	-	-	-	-	-	-
$\Delta^2 y_{JG20,,t}$ [β_2]	-0.723	0.053	0.000	-0.665	0.050	0.000	-0.684	0.047	0.000
$\Delta^2(y_{JG20} - y_{JG2})_t$ [β_3]	0.800	0.045	0.000	0.411	0.052	0.000	0.078	0.036	0.031
Δlog(Nikkei)$_{t-1}$ [η_1]	0.0021	0.0007	0.002	-	-	-	0.0015	0.0008	0.072
$\Delta y_{JG20,t-1}$ [η_2]	-0.665	0.062	0.000	-0.559	0.054	0.000	-0.599	0.058	0.000
$\Delta(y_{JG20} - y_{JG2})_{t-1}$ [η_3]	0.738	0.057	0.000	0.358	0.056	0.000	-	-	-
Residual equation									
ar(1) [ϕ]	-	-	-	-		-	-		-
ma(1) [θ]	-	-	-	-0.217	0.045	0.000	-0.180	0.065	0.006
Variance Equation									
Constant (x 1 000) [ω]	0.008	0.007	0.218	0.013	0.010	0.174	0.067	0.033	0.042
ε^2_{t-1} [γ_1]	0.055	0.017	0.002	0.060	0.015	0.000	0.139	0.039	0.000
log(h_{t-1}) [γ_2]	0.943	0.017	0.000	0.935	0.016	0.000	0.845	0.038	0.000
Diagnostics									
N of outliers removed	1	na	na	0	na	na	1	na	na
Observations	991	na	na	991	na	na	990	na	na
Adj. R^2	0.500	na	na	0.439	na	na	0.480	na	na
s(e)	0.039	na	na	0.046	na	na	0.049	na	na
Skew z	-0.03	p-val	0.737	-0.09	p-val	0.224	-0.09	p-val	0.259
Kurt z	6.70	p-val	0.000	5.22	p-val	0.000	6.88	p-val	0.000
Jarque-Bera	566.0	p-val	0.000	205.1	p-val	0.000	623.1	p-val	0.000
Q(2) z	3.52	p-val	0.172	-0.02	p-val	0.350	0.17	p-val	0.684
Q(5) z	4.16	p-val	0.527	-0.01	p-val	0.912	3.10	p-val	0.541
Q(10) z	9.16	p-val	0.517	0.02	p-val	0.133	16.35	p-val	0.060
Q(2) z^2	1.60	p-val	0.449	-0.04	p-val	0.125	0.47	p-val	0.492
Q(5) z^2	2.47	p-val	0.780	0.01	p-val	0.607	3.20	p-val	0.525
Q(10) z^2	6.16	p-val	0.801	0.02	p-val	0.813	5.52	p-val	0.787
Estimates of the long run mean spreads [see formulas (6) and (10)]									
μ_S (Basis points)	8.42			11.34			13.01		

Dashes indicate variables whose p-values were more than 0.15, and were removed from the final model. Outliers with large residuals were removed. Because the sample size is large the outliers do not materially change the regression results, but they may potentially disturb the ARMA-GARCH residual structure estimation. In the GARCH variance process we have retained the constant term even tough the p-value is larger than 0.15. The standard errors are Bollerslev and Wooldridge (1992) heteroscedasticity corrected.

Appendix 2. Estimates of the parameters of model (9) for AAA rated Japanese Eurobond corporate bond spreads.

		Δs2			Δs5			Δs10	
	Coeff	std err	p-val	Coeff	std err	p-val	Coeff	std err	p-val
Constant [δ_0]	-	-	-	0.0034	0.0013	0.011	0.0029	0.0011	0.009
Sqrt(h_t) [δ_1]	-	-	-	-	-	-	-	-	-
s_{t-1} [α]	-0.048	0.011	0.000	-0.075	0.019	0.000	-0.111	0.024	0.000
$\Delta^2\log(\text{Nikkei})_t$ [β_1]	-	-	-	0.001	0.001	0.092	0.001	0.001	0.122
$\Delta^2 y_{JG20,t}$ [β_2]	-0.732	0.041	0.000	-0.640	0.049	0.000	-0.637	0.050	0.000
$\Delta^2(y_{JG20} - y_{JG2})_t$ [β_3]	0.768	0.041	0.000	0.387	0.050	0.000	0.095	0.036	0.009
$\Delta\log(\text{Nikkei})_{t-1}$ [η_1]	0.0023	0.0007	0.001	0.002	0.001	0.007	0.0020	0.0011	0.064
$\Delta y_{JG20,t-1}$ [η_2]	-0.666	0.050	0.000	-0.519	0.052	0.000	-0.489	0.055	0.000
$\Delta(y_{JG20} - y_{JG2})_{t-1}$ [η_3]	0.688	0.057	0.000	0.303	0.054	0.000	-	-	-
Residual equation									
ar(1) [ϕ]	-	-	-	-		-	-		-
ma(1) [θ]	-0.105	0.042	0.013	-0.270	0.043	0.000	-0.278	0.044	0.000
Variance Equation									
Constant (x 1 000) [ω]	0.010	0.008	0.221	0.010	0.009	0.250	0.013	0.010	0.213
ε^2_{t-1} [γ_1]	0.050	0.019	0.007	0.057	0.014	0.000	0.046	0.013	0.001
$\log(h_{t-1})$ [γ_2]	0.945	0.022	0.000	0.939	0.016	0.000	0.948	0.016	0.000
Diagnostics									
N of outliers removed	0	na	na	0	na	na	4	na	na
Observations	991	na	na	991	na	na	897	na	na
Adj. R^2	0.495	na	na	0.425	na	na	0.571	na	na
s(e)	0.039	na	na	0.047	na	na	0.045	na	na
Skew z	-0.19	p-val	0.015	0.00	p-val	0.958	-0.42	p-val	0.000
Kurt z	7.71	p-val	0.000	4.65	p-val	0.000	5.65	p-val	0.000
Jarque-Bera	921.8	p-val	0.000	112.6	p-val	0.000	318.2	p-val	0.000
Q(2) z	0.21	p-val	0.649	0.78	p-val	0.377	0.80	p-val	0.371
Q(5) z	6.61	p-val	0.158	2.21	p-val	0.697	2.47	p-val	0.651
Q(10) z	11.82	p-val	0.224	6.08	p-val	0.732	10.40	p-val	0.319
Q(2) z^2	1.12	p-val	0.289	1.55	p-val	0.212	0.96	p-val	0.327
Q(5) z^2	1.48	p-val	0.829	2.67	p-val	0.615	6.50	p-val	0.164
Q(10) z^2	5.50	p-val	0.789	8.93	p-val	0.443	9.85	p-val	0.362
Estimates of the long run mean spreads [see formula (6) and (10)]									
μ_s (Basis points)	3.31			6.41			4.10		

Dashes indicate variables whose p-values were more than 0.15, and were removed from the final model. Outliers with large residuals were removed. Because the sample size is large the outliers do not materially change the regression results, but they may potentially disturb the ARMA-GARCH residual structure estimation. In the GARCH variance process we have retained the constant term even tough the p-value is larger than 0.15.