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On regression based event study

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Dedicated to Timo Salmi on the occasion of his 60th birthday

Abstract

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This paper revisits event-study methodology based on regression estimation of abnormal returns. The paper reviews the traditional event study and gives a more detailed discussion of the regression based approach with quantitative event variables. The paper discusses also briefly the the dummy variable regression which is a special case of the quantitative case. Use of GARCH to predict event period volatility is suggested as a partial remedy for the event induced volatility. Furthermore, it is demonstrated that in order to optimally estimate the average (cumulative) abnormal returns, maximum likelihood (ML) or weighted least squares (WLS) type estimators should be used. As an empirical illustration the impact of inflation shocks on stock market index returns is examined. Particularly it is demonstrated that although the data suggest a volatility increase around the inflation release dates, modeling the residuals with GARCH adequately captures the changing volatility, and no actual level shift in volatility can be empirically detected.

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1. Introduction

Although event studies have a long history (e.g. Dolley 1933, Myers and Bakay 1948, Baker 1956, 1957, 1958, Ashley 1962, Ball and Brown 1968), Brown and Warner (1980), and Fama, Fisher, Jensen, and Roll (1969) are considered the papers that introduced the event study methodology as it is in use and known today. Since then the method has become a widely used standard to examine the impact of firm-specific and economy wide events on the value of a firm. Typical firm-specific events are earnings or other accounting announcements (e.g. Binder 1985), and typical economy wide events are regulatory changes (Schwert 1981, Schipper and Thompson 1983, Brockett, Chen, and Graven 1999) or macroeconomic announcements (Schwert 1981b, Sharpe 2001, Knif, Kolari, and Pynnonen 2005).

This paper briefly reviews the traditional event study methodology introduced by Fama et al. (1969), and gives a detailed discussion on the regression based event study approach both from dummy event variable and quantitative event variable point of view. In the dummy variable case the regression coefficient indicates the abnormal return while in the quantitative case the regression coefficient indicates the marginal effect on the return as the variable behind the event changes by one unit. The advantages of the regression approach over the traditional event study are discussed. The approach is return responses to inflation news.

The rest of the paper is organized as follows. In the next section the traditional event study is briefly recapped. Section 3 discusses the regression based event-study and its variants. In section 4 the regression based approach is illustrated with bank-merger data, and Section 5 concludes.

2. Traditional event-study

There are several useful references that cover the traditional event study set forward by Fama et al. (1969). Besides the original Fama et al. (1969) paper, Brown and Warner (1980) and particularly MacKinlay (1997) (see also Campbell, Lo, and MackKinlay 1997, Ch. 4) give in depth presentation to the traditional event study analysis. Binder (1989) present a useful overview of event study methodology. Our aim here is only to give brief overview to recap the main features, and we refer to the above papers for a more detailed exposure.

General steps of an event-study

Although there is no universally accepted unique structure of an event study, there is a general setup of the analysis. The first task is to define the event of interest and the period—the *event window*—over which the security prices of relevant firms will be examined. The next task is to select the *firms* to be included to the study. After these initial steps follows a operational stage to appraise the impact of the event. This is done by defining the *abnormal return* (AR), which generally is the residual between the observed and predicted return,

$$AR_{i,t} = r_{i,t} - \mathrm{E}[r_{i,t}|X_t], \tag{1}$$

where AR is the abnormal return of firm i at time t, r_{it} is the actual return, and $\mathrm{E}[r_{it}|X_t]$ is the (conditional) expected return, given information X_t, under normal conditions. There are several practices to estimate the normal return. The simplest is the constant mean, where $\mathrm{E}[r_{i,t}]$ is estimated by the sample mean. Another popular measure is the market model, $\mathrm{E}[r_{i,t}|R_{m,t}] = \alpha_i + \beta_i r_{m,t}$, where $r_{m,t}$ is the market return. Also multi-index models with $\mathrm{E}[r_{i,t}|X_t] = \alpha + \beta_1 I_{1,t} + \cdots + \beta_p I_{p,t}$ are sometimes used, where $I_{j,t}$ are index returns, like industry portfolios, $j = 1, \ldots, p$. After the operationalization stage follows the estimation and testing stages. A common approach is to select an estimation period—estimation window—which is commonly selected as the period prior to the event window. In daily data a typical choice is at least 120 trading days prior to the event window. From this sample the market model alpha and beta are estimated and used in the mean equation against which the abnormal returns are calculated.

Statistical properties of the abnormal returns

Let $\tau = 0$ denote the event day, let $t_1 <= \tau <= t_2$ denote the beginning and end of the event window, and let $t_0 < t_1$ be the beginning of the estimation period. Thus the end of the estimation period is $t_1 - 1$, and the number of days in the estimation period is $m_1 = t_1 - t_0$. Denote furthermore the m_1-vector of returns from the estimation period as $\mathbf{r}_i = (r_{i,t_0}, r_{i,t_0+1}, \ldots, r_{i,t_1-1})'$, and the $m_2 = t_2 - t_1 + 1$ vector of returns from the event period as $\mathbf{r}_i^e = (r_{i,t_1}, r_{i,t_1+1}, \ldots, r_{i,t_2})'$, where the prime denotes the transposition.

Let $\mathbf{X}_i = (\iota, \mathbf{r}_i)$ denote the $m_1 \times 2$ data matrix from the sample period of firm i. Then the ordinary least squares (OLS) estimates of the parameters $\mathbf{b}_i = (\alpha_i, \beta_i)'$ and $\sigma_i^2 = \mathrm{Var}(u_{i,t})$ are

$$\hat{\mathbf{b}} = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{r}_i \tag{2}$$

and

$$\hat{\sigma}_i^2 = \frac{1}{m_1 - 2}\sum_{s=t_0}^{t_1-1} \hat{u}_{i,s}^2 = \frac{1}{m_1 - 2}\hat{\mathbf{u}}_i'\hat{\mathbf{u}}_i \tag{3}$$

where $u_{i,t} = r_{i,t} - \alpha_i - \beta_i r_{m,t}$ is the residual of the market model, $\hat{u}_{i,s} = r_{i,s} - \hat{\alpha}_i - \hat{\beta}_i r_{m,s}$ is the OLS residual, and $\hat{\mathbf{u}}_i = (\hat{u}_{i,t_0}, \hat{u}_{i,t_0+1}, \ldots, \hat{u}_{i,t_1-1})'$.

In order to evaluate the statistical properties of the abnormal returns we need to make an assumption about the distribution of the returns. The standard assumption is that the returns are serially independent and normally distributed. Under this assumption the abnormal returns $\widehat{AR}_{i,\tau} = r_{i,\tau}^e - \hat{\alpha}_i - \hat{\beta}_i r_{m,\tau}^e$, $\tau = t_1, t_1+1, \ldots, t_2$, are again normally distributed. Nevertheless, they are strictly speaking not independent, because the market model parameters α_i and β_i are estimated, and hence contain estimation error. The expected values depend on the effect of the event. In the most general case each day in the event window has a separate effect, such that the market model becomes

$$r_{i,\tau}^e = \alpha_i + \gamma_{i,\tau} + \beta_i r_{m,\tau}^e + u_{i,\tau}^e, \tag{4}$$

where $\gamma_{i,\tau}$ are the true return effects due to the event on day τ within the event window. Consequently the abnormal returns are

$$\begin{aligned} \widehat{AR}_{i,\tau} &= r_{i,\tau}^e - (\hat{\alpha}_i + \hat{\beta}_i r_{m,\tau}^e) \\ &= \alpha_i + \gamma_{i,\tau} + \beta_{i,\tau} r_{m,\tau}^e + u_{i,\tau}^e - (\hat{\alpha}_i + \hat{\beta}_i r_{m,\tau}^e) \\ &= (\alpha_i - \hat{\alpha}_i) + (\beta_i - \hat{\beta}_i) r_{m,\tau}^e + \gamma_{i,\tau} + u_{i,\tau}^e. \end{aligned} \tag{5}$$

By the properties of the OLS estimators, $\hat{\alpha}_i$ and $\hat{\beta}_i$ are unbiased estimators of α_i and β_i, i.e., $\mathrm{E}[\hat{\alpha}_i] = \alpha_i$ and $\mathrm{E}[\hat{\beta}_i] = \beta_i$, so that the expected value of the abnormal returns are

$$\mathrm{E}[\widehat{AR}_{i,\tau} | r_{m,\tau}^e] = \gamma_{i,\tau}, \tag{6}$$

i.e., they are unbiased estimates of the true return effects of the events. Let us write (5) in the matrix form

$$\widehat{\mathbf{AR}}_i = \mathbf{X}_i^e(\mathbf{b}_i - \hat{\mathbf{b}}_i) + \mathbf{g}_i + \mathbf{u}_i^e, \tag{7}$$

where $\widehat{\mathbf{AR}}_i = (\widehat{AR}_{i,t_1}, \widehat{AR}_{t_1+1}, \ldots, \widehat{AR}_{t_2})'$ is the vector of abnormal returns, and $\mathbf{g}_i = (\gamma_{i,t_1}, \gamma_{i,t_1+1}, \ldots, \gamma_{i,t_2})'$ is the vector of event effects within the event window. Because $\mathrm{E}[\mathbf{u}_i'\mathbf{X}_i^e|\mathbf{X}_i^e] = 0$ and $\mathrm{E}\left[(\hat{\mathbf{b}}_i - \mathbf{b}_i)(\hat{\mathbf{b}}_i - \mathbf{b}_i)'|X_i^e\right] = (\mathbf{X}'\mathbf{X})^{-1}\sigma_i^2$, using (7), the covariance matrix of the abnormal returns is

$$\begin{aligned}
\mathbf{V}_i &= \mathrm{E}\left[\left(\widehat{\mathbf{AR}}_i - \mathrm{E}\left[\widehat{\mathbf{AR}}_i\right]\right)\left(\widehat{\mathbf{AR}}_i - \mathrm{E}\left[\widehat{\mathbf{AR}}_i\right]\right)'|\mathbf{X}_i^e\right] \\
&= \mathrm{E}\left[\left(\mathbf{X}_i^e(\mathbf{b}_i - \hat{\mathbf{b}}_i) + \mathbf{u}_i^e\right)\left(\mathbf{X}_i^e(\mathbf{b}_i - \hat{\mathbf{b}}_i) + \mathbf{u}_i^e\right)'|\mathbf{X}_i^e\right] \\
&= \mathrm{E}\left[\mathbf{X}_i^e(\hat{\mathbf{b}}_i - \mathbf{b}_i)(\hat{\mathbf{b}}_i - \mathbf{b}_i)'\mathbf{X}_i^{e'}|\mathbf{X}_i^e\right] + \mathrm{E}\left[\mathbf{u}_i\mathbf{u}_i'|\mathbf{X}_i^e\right] \\
&= \mathbf{X}_i^e(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i^{e'}\sigma_i^2 + \sigma_i^2\mathbf{I}_{m_2},
\end{aligned} \tag{8}$$

where $\mathbf{I}_{m_2}$ is the $m_2 \times m_2$ identity matrix (c.f. Campbell, Lo and McKinlay 1997, Ch. 4). Thus under the normality the abnormal returns are distributed as

$$\widehat{AR}_{i,\tau} \sim N\left(\gamma_{i\tau}, \sigma^2(\widehat{AR}_{i\tau})\right), \tag{9}$$

where $\sigma^2(\widehat{AR}_{i,\tau})$ is the τth diagonal element of $\mathbf{V}_i$. It may be noted that under usual regularity conditions, particularly that $(\mathbf{X}'\mathbf{X})/m_1 \to \mathbf{Q}$, a positive definite matrix, as $m_1 \to \infty$, the first term on the last line of (8) vanishes and $V_i \to \sigma_i^2\mathbf{I}_{m_2}$ as $m_1 \to \infty$, so that the abnormal returns are approximately independent if the sample period is large enough. Consequently, in practice this assumption is used and the variances of the abnormal returns are just approximated by $\sigma^2(AR_{i\tau}) \approx \sigma_i^2$.

Referring to regression (4) it may be noted that the whole event estimation can be run in one step by estimating regression (4) over the combined sample of the sample window and event window. Essentially one just implements m_2 dummy variables into the regression, one for each day within the event window, and estimates the regression. The result will be exactly the same as the two step approach. We will discuss this and more elaborated models in the next section. Before that we consider the cumulative abnormal returns and the related statistical inference.

Cumulative abnormal returns

Abnormal returns are rarely per se useful in drawing general inferences about the event effects. The abnormal returns must be first aggregated resulting to cumulative abnormal returns (CARs). For individual stocks the CAR is defined as

$$\widehat{CAR}_{i,\tau} = \sum_{s=t_1}^{\tau} \widehat{AR}_{i,s}, \tag{10}$$

$\tau = t_1, t_1 + 1, \ldots, t_2$. These are again normally distributed with mean

$$\mu_{i,\tau} = \mathrm{E}[\widehat{CAR}_{i,\tau}] = \sum_{s=t_1}^{\tau} \gamma_{i,\tau} \tag{11}$$

and variance

$$\sigma^2_{i,\tau} = \mathrm{Var}[\widehat{CAR}_{i,\tau}] = \sigma^2_i \left((\tau - t_1 + 1) + 2 \sum_{s=t_1+1}^{\tau} \sum_{u=t_1}^{s-1} w_{s,u} \right), \tag{12}$$

where $w_{s,u}$ is the (s,u)th element of the matrix $\mathbf{X}^e_i(\mathbf{X}'_i\mathbf{X}_i)^{-1}\mathbf{X}^{e\prime}_i$, i.e., the matrix of the first term on the last line of equation (8). Again in large sample this term is approximately zero, and the variance simplifies to $\sigma^2_{i,\tau} \approx (\tau - t_1 + 1)\sigma^2_i$. Thus

$$\widehat{CAR}_{i,\tau} \sim N(\mu_{i,\tau}, \sigma^2_{i,\tau}). \tag{13}$$

The null hypothesis in event study is no event effect, which in terms of the parameterized model means that $\gamma_{i,\tau} = 0$ for all $\tau = t_1, t_1 + 1, \ldots, t_2$, and consequently, $\mu_{i,\tau} = 0$. Hence, under the null hypothesis

$$\widehat{CAR}_{i,\tau} \sim N(0, \sigma^2_{i,\tau}). \tag{14}$$

A test statistic for this hypothesis is the standard t-statistic, which in event studies is usually referred as the standardized CAR (SCAR)

$$S\widehat{CAR}_{i,\tau} = \frac{S\widehat{CAR}_{i,\tau}}{\hat{\sigma}_{i,\tau}}, \tag{15}$$

where $\hat{\sigma}_{i,\tau}$ is an estimate of $\sigma_{i,\tau}$, obtained by replacing σ_i:s in (8) by the estimates from the sample period. Under the null hypothesis of no event effect (14) has t-distribution with $m_1 - 2$ degrees of freedom (the degrees of freedom is $m_1 - 2$ because two parameters,

α and β, are estimated in the mean equation). Again in practice, because the sample period is usually large enough ($m_1 > 30$), the standard normal distribution, $N(0, 1)$, can be used as an accurate enough approximation.

Cumulative abnormal returns of individual companies are, however, usually pretty noisy which may deteriorate reliable inference. A common practice is to aggregate the returns over individual firms and deal with averages instead. Assuming there are n firms, the average abnormal returns are

$$\widehat{AR}_\tau = \frac{1}{n}\sum_{i=1}^{n}\widehat{AR}_{i,\tau}, \tag{16}$$

$\tau = t_1, t_1 + 1, \ldots, t_2$, and the average CARs are obtained aggregating average ARs over time

$$\widehat{CAR}_\tau = \sum_{s=t_0}^{\tau}\widehat{AR}_s, \tag{17}$$

$\tau = t_1, t_1 + 1, \ldots, t_2$. The expected values of the CAR is

$$\mu_\tau = \mathrm{E}[\widehat{CAR}_\tau] = \frac{1}{n}\sum_{i=1}^{n}\sum_{s=t_1}^{\tau}\gamma_{i,s}. \tag{18}$$

Under the null hypothesis of no event effect, again this mean is equal to zero. In order to compute the variance, the traditional assumption is that the abnormal returns are independent over firms, which is the case if the event days are different for the firms. Otherwise, if the event days are overlapping, the contemporaneous return correlations should be taken into account in the computations.

Under the independence assumption the variance of $\widehat{AR}_\tau$, defined by (16), is

$$\sigma^2(\widehat{AR}_\tau) = \mathrm{Var}[\widehat{AR}_\tau] = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2(\widehat{AR}_{i,\tau}) \tag{19}$$

Using (12), the variance of $\widehat{CAR}_\tau$ is

$$\sigma_\tau^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sigma_{i,\tau}^2 = \left((\tau - t_1 + 1) + 2\sum_{s=t_1+1}^{\tau}\sum_{u=t_1}^{s-1} w_{s,u}\right)\left(\frac{1}{n^2}\sum_{i=1}^{n}\sigma_i^2\right), \tag{20}$$

$\tau = t_1, t_1 + 1, \ldots, t_2$.

Again under the normality assumption the distribution of $\widehat{CAR}_\tau$ is normal

$$\widehat{CAR}_\tau \sim N(\mu_\tau, \sigma_\tau^2) \tag{21}$$

with μ_τ and σ_τ^2 given in (18) and (19), respectively. Under the null hypothesis of no event effect $\mu_\tau = 0$ for all $\tau = t_1, t_1 + 1, \ldots, t_2$. These individual null hypothesis can be tested with the traditional t-test

$$t_1(\tau) = \frac{\widehat{CAR}_\tau}{\hat{\sigma}_\tau}, \tag{22}$$

where $\hat{\sigma}_\tau$ is the square root of (20) after replacing individual σ_i^2s with their sample estimates. Under the null hypothesis $t_1(\tau)$ is approximately $N(0, 1)$ distributed.

Another test statistic, suggested by Patell (1976), is based on average the $SCAR$s of (15)

$$\widehat{SCAR}_\tau = \frac{1}{n}\sum_{i=1}^{n} \widehat{SCAR}_{i,\tau}. \tag{23}$$

As noted above, the individual $\widehat{SCAR}_{i,\tau}$s are under the null hypothesis t-distributed with $m_1 - 2$ degrees of freedom. Thus the variance $\widehat{SCAR}_{i,\tau}$ is $(m_1 - 2)/(m_1 - 4)$, and hence the variance of $\widehat{SCAR}_\tau$ is

$$\mathrm{Var}\left[\widehat{SCAR}_\tau\right] = \frac{1}{n}\frac{m_1 - 2}{m_1 - 4}, \tag{24}$$

and one gets a test statistic

$$t_2(\tau) = \sqrt{\frac{n(m_1 - 4)}{m_1 - 2}}\,\widehat{SCAR}_\tau \tag{25}$$

which is asymptotically $N(0, 1)$ distributed under null hypothesis of no event effects.

The above tests assume that the event affects only on mean return if any. However, Brown and Warner (1980, 1985) and Brown, Harlow, and Tinic (1988) find that many events may cause changes both in mean and variance. In such instances Boehmer, Musumeci, and Poulsen (1991) find that commonly used methods, like above, reject the null hypothesis of zero mean effect too frequently. Consequently the tests may have severe size problems. To alleviate this Boehmer et al. suggest use $SCAR$s given in (15), and estimate their variance over the firms on the event days τ, such that

$$\hat{\sigma}^2_{\mathrm{SCAR}_\tau} = \frac{1}{n-1}\sum_{i=1}^{n}\left(\widehat{SCAR}_{i,\tau} - \widehat{SCAR}_\tau\right)^2, \tag{26}$$

where $\widehat{SCAR}_\tau$ is the sample mean defined in (23). Consequently the standard error of $\widehat{SCAR}_\tau$ is

$$\mathrm{std}(\widehat{SCAR}_\tau) = \frac{\hat{\sigma}_{\mathrm{SCAR}_\tau}}{\sqrt{n}}, \tag{27}$$

and the related t-statistic

$$t_3(\tau) = \frac{\widehat{SCAR}_\tau\sqrt{n}}{\hat{\sigma}_{\mathrm{SCAR}_\tau}}, \tag{28}$$

which again is asymptotically $N(0,1)$ distributed under the null hypothesis. Boehmer et al. (1991) demonstrate via simulation studies that this modified test statistic preserves the power and adjusts the size problem of the traditional tests. Brockett, Chen, and Garven (1999) utilize GARCH and time varying market model beta to account for the temporal changes in the return process during the event period.

3. Regression based event methodology

As discussed in connection of equation (4), the traditional event study with non-overlapping event windows is equivalent to estimating dummy variable regressions over the combined sample and event windows, where the event window residuals become dummied out with the zero-one variables. The dummy variable coefficients correspond to the abnormal returns. If the event windows are overlapping or the same, the abnormal returns are contemporaneously correlated, which may cause serious bias in the standard error estimates. An easy ad hock way to circumvent this problem is to construct equally weighted portfolios and work out the event study with the portfolio returns. That is, one estimates a regression

$$r_{p,t} = \alpha_p + \beta_p r_{m,t} + \sum_{\tau=t_1}^{t_2}\gamma_{p,\tau}D_{\tau,t} + u_{p,t} \tag{29}$$

where $r_{p,t} = \frac{1}{n}\sum_{i=1}^{n} r_{i,t}$, $\alpha_p = \frac{1}{n}\sum_{i=1}^{n}\alpha_i$, $\beta_p = \frac{1}{n}\sum_{i=1}^{n}$, $\gamma_{p,\tau} = \frac{1}{n}\sum_{i=1}^{n}\gamma_{i,\tau}$, $u_{p,\tau} = \frac{1}{n}\sum_{i=1}^{n} u_{i,\tau}$, and $D_{\tau,t}$ are dummy variables assuming value one on event day $t = \tau$ and zero otherwise, $\tau = t_1 + 1, \ldots, t_2$. Essentially this portfolio method has been suggested by Jaffe (1974), Brown and Warner (1980, 1985), and is widely used event studies with contemporaneous events (e.g. O'Hara and Shaw 1990, Chandra and Balachandran 1990).

In principle this approach could also be utilized in the non-overlapping case. The regression (29) becomes then

$$\bar{r}_t = \alpha_p + \beta_p^* \bar{r}_{m,t} + \sum_{\tau=t_1}^{t_2} \gamma_{p,\tau} D_{\tau,t} + \bar{u}_{p,t}, \tag{30}$$

where the bar indicates averaging over the n firms. The major difference here is that we are not really working with an equally weighted portfolio, because the averaging is not over the same time points. Nevertheless, the regression works perfectly, where α_p is the average alpha as in (29), and β_p^* is approximately the average beta of (29). This is because, if we average over the market models, the exact form is $\bar{r}_t = \alpha_p + (1/n)\sum_{i=1}^n \beta_i r_{m,t_i} + \bar{u}_t$, where r_{m,t_i} denotes the market return in firm i estimation (or event) period at day t. However, it turns out that $(1/n)\sum_{i=1}^n \beta_i r_{m,t_i} \approx \bar{\beta}_p \bar{r}_{m,t}$. This can be seen as follows. Using the covariance formula $\sum(x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - n\bar{x}\bar{y}$, we can write $(1/n)\sum x_i y_i = \sum(x_i - \bar{x})(y_i - \bar{y}) + \bar{x}\bar{y}$. It is appropriate to assume that firm betas are not related to specific day market returns. Thus the covariation between them can be assumed to be approximately zero, which implies that $(1/n)\sum \beta_i r_{m,t_i} = (1/n)\sum_{i=1}^n(\beta_i - \beta_b)(r_{m,t_i} - \bar{r}_{m,t}) + \beta_p \bar{r}_{m,t} \approx \beta_p \bar{r}_{m,t}$. To emphasize the approximation β_p^* is used in place of β_p in (30).

Perhaps the most important advantage of the regression based approach is that one can analyze the effect of quantitative events. We call this *quantitative event study.* For example in the case of earnings surprise one can use in regression (29) instead of the dummy variables the actual surprise measured as the difference between the actual and predicted earnings growths. Thus, let $x_{i,\tau,t}$ be a variable with a value equal to the amount of surprise (e.g., the difference of the actual and expected earnings growth) if $t = \tau$ (the event day), and zero otherwise, then (29) generalized for an individual firm to

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \sum_{\tau=t_1}^{t_2} \gamma_{i,\tau} x_{i,\tau,t} + u_{i,t}. \tag{31}$$

The γ-coefficients measure the abnormal marginal return (AMR). That is, for example if the earnings surprise is one percent, the return effect is expected to be γ percents (c.f. Schwert 1981, Knif, Kolari, and Pynnonen, 2005). In the same manner the CARs in the dummy regression become cumulative abnormal marginal returns (CAMR) that measure cumulative abnormal return per unit in the event variable. Consequently, the abnormal returns are $\widehat{AR}_{i,\tau} = \hat{\gamma}_{i,\tau} x_{i,\tau} = r_{i,\tau} - \hat{\alpha}_i - \hat{\beta}_i r_{m,\tau}$, where $x_{i,\tau} = x_{i,\tau,\tau}$ is the event day τ value of the event variable. The aggregated analysis can be performed by replacing the event variables with their averages over the firms. Thus because the traditional event study is a special case of the quantitative event study, we deal only with the later one in what follows.

The estimation and testing for the AMRs and CAMRs can be easily worked out with standard regression packages once the series are lined up and averaged over the firms. The null hypothesis in either case of no event effect amounts to testing for hypotheses

$$H_{0,\tau} : \gamma_{t_1} + \gamma_{t_1+1} + \cdots + \gamma_\tau = 0, \tag{32}$$

$\tau = t_1, t_1 + 1, \ldots, t_2$. Most statistical packages, like EViews or SAS, have ready made procedures to test this hypothesis. In order to derive formally the needed test statistic, let $\mathbf{y} = (r_{t_0}, r_{t_0+1}, \ldots, r_{t_1-1}, r_{t_1}, r_{t_1+1}, \ldots, r_{t_2})'$ be the $m_1 + m_2$ vector of average returns over the firms from the combined sample and event periods, $\mathbf{X} = (\mathbf{1}, \mathbf{r}_m, \mathbf{x}_{t_1}, \mathbf{x}_{t_1+1}, \ldots, \mathbf{x}_{t_2})$ be the $(m_1 + m_2) \times (m_2 + 2)$ data matrix , where $\mathbf{1}$ is an $m_1 + m_2$ vector of ones, $\mathbf{r}_m$ is the $m_1 + m_2$ vector market returns, and $\mathbf{x}_\tau$ are the $m_1 + m_2$ vectors having the value of the event variable in position τ, and zero elsewhere, $\tau = t_1, t_1 + 1, \ldots, t_2$, then the regression can be written in the standard form

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{u}, \tag{33}$$

where $\mathbf{b} = (\alpha, \beta, \gamma_{t_1}, \gamma_{t_1+1}, \ldots, \gamma_{t_2})'$ the $m_2 + 2$ parameter vector, and $\mathbf{u}$ is the $m_1 + m_2$ residual vector with $\mathrm{E}[\mathbf{u}] = 0$, and $\mathrm{Cov}(\mathbf{u}) = \sigma_u^2 \mathbf{I}$. Then the standard OLS estimates are $\hat{\mathbf{b}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$, and $\hat{\sigma}_u^2 = \sum_{t=t_0}^{t_2} \hat{u}_t^2/(m_1 - 2)$, and the standard errors of the estimates are obtained as the square roots of the diagonal elements of $\hat{\sigma}_u^2(\mathbf{X}'\mathbf{X})^{-1}$. Introducing vectors ι_τ having ones in positions $t_1, t_1 + 1, \ldots, \tau$, and zero elsewhere, $\tau = t_1, t_1 + 1, \ldots, t_2$, the t-statistic for testing the null hypothesis (32) is simply

$$t_\tau = \frac{\iota_\tau' \hat{\mathbf{b}}}{\hat{\sigma}_u \sqrt{\iota_\tau'(\mathbf{X}'\mathbf{X})^{-1}\iota_\tau}}, \tag{34}$$

which is asymptotically $N(0, 1)$ distributed under the null hypothesis.

Note further, that with the regression approach the GARCH effect in the residuals can also be easily accounted for in the estimation and testing by simply estimating (33) with GARCH residuals. However, as will be suggest below, it is better to use predicted volatilities in the case of single events because in both quantitative and traditional event study the residuals will be 'dummied out' resulting to zero shocks purely on technical basis in the GARCH. This biases the true volatility down. In the case of multiple similar events per firm the GARCH volatility can be used straightforwardly. The other advantage of the regression approach is that additional explanatory variables can be also introduced to the regression to control their effects.

Above we have discussed mainly the aggregated analysis. Being straightforward and convenient, it potentially has a serious drawback in terms of efficacy of estimation and sacrificed power of statistical tests. As will be seen below, using disaggregated returns with maximum likelihood or weighted least squares leads at least theoretically to optimal estimates, and hence more powerful statistical testing. Firm level analysis may also be desirable for a number of other reasons. First, if there are multiple event windows per firm, the aggregated analysis cannot be applied unless the event windows are the same for all firms. Second, if one is interested in testing firm specific event effects, then a disaggregated analysis with multivariate regression needs to be used, see Binder (1985) or Malatesta (1986). In the latter two a general framework for disaggregated (dummy variable) regression event study are considered whether the events occur at the same time or non-contemporaneously for the firms. In most cases, however, investigators are interested in the average effect for which the aggregated analysis is appropriate. Third, in the case of single events, the aggregated analysis does not allow for accounting the event's possible variance effect, which may bias inference based on cross sectional averages (Fama et al. 1969, Boehmer et al. 1991).

As a remedy to this problem, the Boehmer et al. (1991) t-test has become popular in the traditional event study. Harrington and Schnider (2002) deals in more detail the cross-sectional heteroscedasticity problem in the traditional event study and suggest that WLS with robust standard errors and maximum likelihood estimation with non-proportional heteroscedasticity may be useful supplements to OLS with robust standard errors. By the non-proportional heteroscedasticity they mean that the event induced variance is not proportional to the firms residual variance.

Below we consider the possible event induced volatility problem within the quantitative event regression framework for which the traditional results are special cases. First we deal with the multiple event case, and find that GARCH is a promising method to solve the problem. In the single event case we need to introduce additional structure to the modeling in order to derive estimation results.

Multiple events

If there are multiple events and the events occur at the same time for all firms and one is interested only on the average effect, then, as discussed above, using average returns over the firms is an appropriate way to analyze event effects. If there is an event induced volatility effect, a plausible assumption is that the level of volatility shifts on the event day or within the event window. Often GARCH(1,1) has been found to capture adequately stock return volatility. A temporary shift can be easily introduced into this model by introducing a dummy variable equaling one within the whole event window and zero otherwise Thus in the earlier notations, if the whole observation period is from t_0 to t_2, introducing a dummy variable D_t that is one if t is an event day zero elsewhere, the GARCH(1,1) specification is

$$\mathrm{Var}_t[u_{t+1}] = h_{t+1} = \omega_0 + \omega_{0,1}D_t + \omega_1 u_t^2 + \delta h_t, \tag{35}$$

where $\mathrm{Var}_t[\cdot]$ is the conditional variance given information at time t. It may be noted that this approach can be equally well applied in disaggregated data to investigate firm level event induced volatility. However, in both cases (aggregated and firm level), if the event window is several days it is not feasible to model different volatility levels for individual days due to the fact that a reliable time varying volatility estimation requires pretty large number of observations. Furthermore, because time varying volatility itself is designed to capture temporal increases in volatility, no additional mean shift parameters are usually needed. This point is illustrated below in the empirical part of the study.

Single events

In the case of single event windows we do not have repeated time points to evaluate volatility effects. However, GARCH volatility can be still a potential alternative because it can be used to predict the volatility. Thus given an estimated GARCH(1,1) for the ith firm,

$$h_{i,t+1} = \hat{\omega}_{i,0} + \omega_{i,1}\hat{u}_{i,t}^2 + \delta_i h_{i,t}, \tag{36}$$

from the estimation period, $t = t_0, t_0+1, \ldots, t_1-1$, the predicted volatility can be obtained from the prediction equation (see e.g. Engle 1995)

$$h_{i,\tau|t_1-1} = \mathrm{E}_{t_1-1}[u_{i,\tau}^2] \tag{37}$$

$\tau = t_1, t_1+1, \ldots, t_2$, where $\mathrm{E}_{t_1-1}[\cdot]$ is the conditional expectation given information at time point $t_1 - 1$ (the end of the estimation period). Applying (37) with the law of iterated expectations (see e.g. Engle 1995), the predicted volatility for the abnormal return $AR_{i,\tau}$ for firm i is

$$h_{i,\tau|t_1-1} = \omega_{i,0}\left(\frac{1-(\omega_{i,1}+\delta_i)^{\tau-t_1}}{1-(\omega_{i,1}+\delta_i)}\right) + (\omega_{i,1}+\delta_i)^{\tau-t_1}h_{t_1}, \tag{38}$$

$\tau = t_1, t_1+1, \ldots, t_2$. Given $0 < \omega_{i,1} + \delta_i < 1$, we find from (38) that $h_{i,\tau|t_1-1} \to \omega_{0,i}/(1-\omega_{i,1}-\delta_i)$, the unconditional variance, the further away τ is from the last observation from the sample period.

Estimating the parameters $\omega_{i,0}$, $\omega_{i,1}$, and δ_i one can utilize (38) to predict the standard errors and test the significance of the abnormal returns with the t-statistic

$$t_{\mathrm{AR}_{i,\tau}} = \frac{\widehat{AR}_{i,\tau}}{\sqrt{\hat{h}_{i,\tau|t_1-1}}}, \tag{39}$$

which are approximately $N(0,1)$ distributed under the null hypothesis of zero abnormal returns. In (39) $\widehat{AR}_{i,\tau} = \hat{\gamma}_{i,\tau} x_{i,\tau}$ is the (total) abnormal return with $\hat{\gamma}_{i,\tau}$ the estimate of $\gamma_{i,\tau}$-coefficient of regression (31), $\tau = t_1, t_1+1, \ldots, t_2$ and $\hat{h}_{i,\tau|t_1-1}$ indicates that the parameters in (38) are replaced by the estimates.

Assuming independence of the abnormal returns, the variance of the cumulative abnormal returns, CARs, are the sums of the abnormal return variances

$$\mathrm{Var}[CAR_{i,\tau}] = \sum_{s=t_1}^{\tau} h_{i,s|t_1-1}. \tag{40}$$

Consequently the t-statistic to test the null hypothesis of $H_0 : CAR_{i,\tau} = 0$ is

$$t_{\mathrm{CAR}_{i,\tau}} = \frac{\sum_{s=t_1}^{\tau} \hat{\gamma}_{i,s} x_{i,s}}{\sqrt{\widehat{\mathrm{Var}}(CAR_{i,\tau})}} \tag{41}$$

Aggregating ARs over firms: Aggregating the individual marginal abnormal returns over the firms allows us to evaluate the event induced volatility. As will be seen below, even if there is no volatility effect due to the event, one should take into account the firm residual variance in order to optimally estimate the average AMRs or ARs (special cases of AMRs) over the firms (c.f. Harrington and Schnider 2002).

There are several possible alternatives to model the possible additional volatility due to the event in the regression (31). One reasonable alternative is to include additional error terms $v_{i,\tau}$ to the regression. Then for an event day $\tau = t_1, t_1+1, \ldots, t_2$ regression (31) becomes

$$r_{i,\tau} = \alpha_i + \beta_i r_{m,\tau} + \gamma_{i,\tau} x_{i,\tau} + v_{i,\tau} + u_{i,\tau}, \tag{42}$$

$\tau = t_1, t_1+1, \ldots, t_2$, where $v_{i,\tau}$ and $u_{i,\tau}$ are independent normal distributed error terms with $\mathrm{E}[v_{i,\tau}] = \mathrm{E}[u_{i,\tau}] = 0$, $\mathrm{Var}[v_{i,\tau}] = \sigma^2_{i,\tau}$ and $\mathrm{Var}[u_{i,\tau}] = \sigma^2_i$.

Even though the regression parameters are assumed as known (estimated form the sample period), additional structure must be introduced to the model before the remaining parameters can be estimated from the cross sectional data on event day τ. First, we assume that the marginal response $\gamma_{i,\tau}$, i.e., the $AMR_{i,\tau}$, is the same for all firms in day τ, such that $\gamma_{i,\tau} = \gamma_\tau$. Note that abnormal returns $AR_{i,\tau} = \gamma_\tau x_{i,\tau}$ are still firm specific because of the firm specific shocks $x_{i,\tau}$. Second we need to assume something about the event shock variances $\mathrm{Var}[v_{i,\tau}]$.

In the traditional event study, Harrington and Snider (2002) consider cases where variance, $\sigma^2_{i,\tau}$, of the abnormal returns are linear functions of the residual variance, i.e.,

$$\sigma^2_{i,\tau} = \sigma^2_\tau + \kappa_\tau \sigma^2_i. \tag{43}$$

There are three interesting special cases. The first is the traditional assumption, where the events do not have volatility effects. In terms of (43) this means that $\sigma^2_{i,\tau} = 0$, or $\sigma^2_\tau = 0$ and $\kappa_\tau = 0$, so that $v_{i,\tau}$ disappears form regression (42). The second case is where $\sigma^2_\tau = 0$ and $\kappa_\tau > 0$, which implies that the event variance is proportional to the residual variance. Boehmer, et al. (1991) use this kind of specification. A third special case is where $\kappa_\tau = 0$, which implies that the event induced variance is common to all firms. We consider only the first case here in more detail, and show that even in this restricted case the quantitative event study potentially may serve as a remedy increased event day abnormal return volatility in certain cases The other two cases are only reviewed briefly.

Case $\sigma^2_{i,\tau} = 0$ [i.e., in (43) $\sigma^2_\tau = \kappa_\tau = 0$]: Assuming that the event windows do not overlap, and that the parameters α_i, β_i, and σ_i^2 are known, which in practice means that we replace them with the estimates $\hat{\alpha}_i$, $\hat{\beta}_i$ and $\hat{\sigma}_i^2$ from the estimation period, the log-likelihood for γ_τ becomes

$$\ell(\gamma_\tau) = \sum_{i=1}^{n} \ell_{i,\tau}(\gamma_\tau; \hat{\alpha}_i, \hat{\beta}_i, \hat{\sigma}_i^2), \tag{44}$$

$\tau = t_1, t_1 + 1 \ldots, t_2$. Under the normality assumption and dropping out constant terms from the log-likelihood, we get

$$\ell_{i,\tau}(\gamma_\tau; \hat{\alpha}_i, \hat{\beta}_i, \hat{\sigma}_i^2) = -\frac{1}{2}\log(\hat{\sigma}_i^2) - \frac{1}{2}(\hat{u}_{i\tau} - \gamma_\tau x_{i,\tau})^2/\hat{\sigma}_i^2, \tag{45}$$

where $\hat{u}_{i,\tau} = r_{i,\tau} - \hat{\alpha}_i - \hat{\beta}_i r_{m_i,\tau}$ are the residual returns of firm i on the event period before adjusting for the event effects, $\tau = t_1, t_1 + 1, \ldots, t_2$.

The first order conditions for the maximum of (44) with respect to γ_τ are found by finding the zeros of the partial derivatives. Denote the solutions as $\hat{\gamma}_\tau$, so that

$$\sum_{i=1}^{n} x_{i,\tau}(\hat{u}_{i,\tau} - \hat{\gamma}_\tau x_{i,\tau})/\hat{\sigma}_i^2 = 0, \tag{46}$$

$\tau = t_1, \ldots, t_2$. Solving (46) for $\hat{\gamma}_\tau$ gives the estimators for the abnormal marginal returns

$$\widehat{AMR}_\tau = \hat{\gamma}_\tau = \sum_{i=1}^{n} w_{i,\tau}\hat{u}_{i,\tau}, \tag{47}$$

where

$$w_{i,\tau} = \frac{x_{i,\tau}/\hat{\sigma}_i^2}{\sum_{j=1}^{n} x_{j,\tau}^2/\hat{\sigma}_j^2}. \tag{48}$$

In terms of regression, estimator (47) is the weighted least squares (WLS) estimator of the regression parameter γ_τ.

By the central limit theorem and straightforward calculations, asymptotically

$$\widehat{AMR}_\tau \sim N(\gamma_\tau, \sigma_{\hat{\gamma}_\tau}^2) \tag{49}$$

where

$$\sigma_{\hat{\gamma}_\tau}^2 = \frac{1}{\sum_{i=1}^{n} x_{i,\tau}^2/\hat{\sigma}_i^2}. \tag{50}$$

Consequently the t-statistic for testing the null hypothesis $H_0 : \gamma_\tau = 0$ is

$$t = \frac{\hat{\gamma}_\tau}{\hat{\sigma}_{\hat{\gamma}_\tau}} = \hat{\gamma}_\tau \sqrt{\sum_{i=1}^{n} \frac{x_{i,\tau}^2}{\hat{\sigma}_i^2}}, \tag{51}$$

which is asymptotically $N(0,1)$ distributed under the null hypothesis.

The *CAMR*s are obtained simply by summing the individual *AMR*s, i.e.,

$$\widehat{CAMR}_\tau = \sum_{s=t_1}^{\tau} \widehat{AMR}_s \tag{52}$$

with asymptotic distributions

$$\widehat{CAMR}_\tau \sim N\left(\sum_{s=t_1}^{\tau} \gamma_s, \sum_{s=t_1}^{\tau} \sigma_{\hat{\gamma}_s}^2\right), \tag{53}$$

$\tau = t_1, t_1 + 1, \ldots, t_2$. Finally, the t-statistic to test for no cumulative effect, $H_0 : AMR_\tau = 0$, is

$$t = \frac{C\hat{A}MR_\tau}{\sqrt{\sum_{s=t_1}^{\tau} \hat{\sigma}_{\hat{\gamma}_s}^2}} \tag{54}$$

which is asymptotically $N(0,1)$ distributed under the null hypothesis.

Discussion

We observe that with $x_{i,\tau} = 1$ the problem reduces to the traditional event study, where the weights in (47) are the reciprocals of the variances. The important implication is that in order to estimate efficiently the average abnormal return in the traditional event study one should use the WLS solution (47) in place of the traditional sample average. The intuition is also pretty clear, because in the WLS observations are essentially weighted by the inverse of the variance, so that more volatile, i.e., more noisy abnormal returns get less weight in the average than the less volatile, i.e., more reliable ones. Consequently this approach should lead to more accurate end results. It is notable that even if we do not assume additional event induced volatility, the heteroscedasticity across firms may deteriorate the estimation results if not properly accounted for via the WLS estimation. Even using the WLS in the traditional event, it may not capture properly the variability on the event day because the dummy variable only measures the occurences and not the size. Lack of measuring the size of a surprise is likely shown up as a larger dummy variable coefficient, which in the study seems as if the event induces extra variability. The quantitative event study, introduced here, captures this problem via the event variable. Consequently the increased variability in abnormal returns, $AR_{i,\tau} = \gamma_\tau x_{i,\tau}$, can be explained by a higher value of the event variable $x_{i,\tau}$ rather than by increased noise.

Case $\sigma_\tau^2 = 0$ [i.e., in (43) $\sigma_{i,\tau}^2 = \kappa_\tau \sigma_i^2$]: Assuming normality the likelihood for single observation given $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{\sigma}_i^2$ is

$$\ell(\gamma_\tau, \kappa_\tau; \hat{\alpha}_i, \hat{\beta}_i) = -\frac{1}{2}\log((1+\kappa_\tau)\hat{\sigma}_i^2) - \frac{1}{2}(\hat{u}_{i,\tau} - \gamma_\tau x_{i,\tau})^2/((1+\kappa_\tau)\hat{\sigma}_i^2) \tag{55}$$

Given the sample observation, straightforward calculations show that maximizing the likelihood function produces in this case the ML-estimator for γ_τs identical to (47). The maximum likelihood estimator for the variance proportion $1+\kappa_\tau$ becomes

$$\widehat{1+\kappa_\tau} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\hat{u}_{i,\tau} - \hat{\gamma}_\tau x_{i,\tau}}{\hat{\sigma}_i}\right)^2 \tag{56}$$

which is the variance of the standardized abnormal returns over the firms. If this is close to one, it indicates that events do not have impact on the volatilities. This can be easily tested, because under the null hypothesis $H_0 : (1+\kappa_\tau) = 1$, i.e. $\kappa_\tau = 0$, asymptotically $n(\widehat{1+\kappa}) \sim \chi^2$ with $n-1$ degrees of freedom.

Furthermore, again asymptotically

$$AMR_\tau \sim N(\gamma_\tau, \nu_{\hat{\gamma}_\tau}^2) \tag{57}$$

where

$$\nu_{\hat{\gamma}_\tau}^2 = \frac{1+\kappa_\tau}{\sum_{i=1}^{n}\frac{x_{i,\tau}^2}{\hat{\sigma}_i^2}}. \tag{58}$$

Consequently, by substituting the parameters with the estimators, the needed t-statistics analogous to (51) and (54) for testing the abnormal returns can be calculated straightforwardly.

Case $\kappa_\tau = 1$ [i.e., in (43) $\sigma^2_{i,\tau} = \sigma^2_\tau + \sigma^2_i$]: The likelihood of a single observation given $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{\sigma}^2_i$ is in this case under the normality assumption

$$\ell(\gamma_\tau, \sigma^2_\tau; \hat{\alpha}_i, \hat{\beta}_i, \hat{\sigma}^2_i) = -\frac{1}{2}\log(\sigma^2_\tau + \hat{\sigma}^2_i) - \frac{1}{2}\frac{(\hat{u}_{i,\tau} - \gamma_\tau)^2}{\sigma^2_\tau + \hat{\sigma}^2_i}. \tag{59}$$

In this case the ML-estimates for the parameters γ_τ and σ^2_τ must be solved numerically. We do not, however, elaborate this case further here.

4. Inflation shocks and stock returns

In the empirical example we use data from Knif, Kolari, and Pynnonen (2005), and investigate the SP500 return responses to inflation shocks to illustrate the methodology discussed above. A more detailed analysis is given in the original paper Knif et al. (2005). The sample period is January 1980 to September 2004. The Monday October 19, 1987 is excluded from the analysis. The inflation shocks are measured in terms as the difference between actual and median predicted CPI (Consumer Price Index) changes. The median predictions are compiled by MMS International (Subsidiary of Standard and Poors). For a more detailed description of the data, see Knif et al. (2005). The event day is the day the CPI change is released by the U.S. Bureau of Labor Statistics (BLS). Survey forecasts are normally released four or five days before the BLS release.

In Knif et al. (2005) an event window of ± 10 was used to cover also the release of the survey results. Furthermore the economic condition was taken into account. Here we, however, do not condition on the economic state but estimate the effect of shock over the whole sample, and demonstrate modeling and testing the shock effect on returns and volatility.

Table 1. Sample statistics of actual and predicted CPI changes.

	Monthly CPI Changes Actual	Predicted	CPI Shock
Mean [%]	0.31	0.32	-0.01
t (mean = 0) [p-value]	0.00	0.00	0.27
Standard Deviation [%]	0.28	0.22	0.15
Excess kurtosis	3.07	4.63	1.60
Skewness	1.16	1.58	0.08
Minimum [%]	-0.40	-0.30	-0.60
Maximum [%]	1.40	1.40	0.50
Observations (months)	296	296	296

Table 1 reports sample statistics of the monthly actual and predicted CPI changes. The average monthly CPI change in the sample period was 0.31%, while the average prediction was 0.32%, i.e., about the same, which implies that there is no indication of systematic bias in the predictions. Accordingly, the average shock is about zero.

Table 2. Non-event window and event window sample statistics.

Panel A. Sample statistics.

	Mean	Std	Skew.	Kurt.	Obs.
Non event	0.049	0.991	-0.201	6.917	4876
Event	0.016	1.084	-0.057	11.233	1400
All	0.042	1.011	-0.166	8.174	6276

Panel B. Equality tests.

	Means	Variances
Test stat	1.029	1.196
p-value	0.303	0.000

Table 2 presents sample statistics of the S&P500 daily returns over the non-event days, event window days, and the whole sample. The overall daily mean return is 0.042% (4.2 basis points). In the non-event days the average return is 4.9 basis points while in the event window days 1.5 basis points. The difference, however is not statistically significant. The non-event window volatility estimate is 0.991% while the estimate of the event window volatility is 1.084, i.e., more than 9% higher. The difference also is statistically significant, which suggests that the inflation shock may also have a volatility effect.

In order to investigate more detailed the inflation shock effect on the stock return and volatility, define the event regression

$$\begin{aligned} r_t &= \mu + \gamma_{-3}x_{t+3} + \gamma_{-2}x_{t+2} + \gamma_{-1}x_{t+1} + \gamma_0 x_t \\ &\quad + \gamma_1 x_{t-1} + \gamma_2 x_{t-2} + \gamma_3 x_{t-3} + u_t \end{aligned} \tag{60}$$

where $x_t = \Delta CPI_t^{act} - CPI_t^{pred}$ if t is an event day and 0 otherwise, and u_t is the regression residual. Consequently, for example γ_{-3} indicates the event's return effect (abnormal marginal return, AMR) three days before the event (-3 days from the event, which in this case is the beginning of the event window), while γ_3 gives the event's effect three days after the event, which here is the end of the event window. As a further illustration, suppose t_0 is an event day, then in day $t = t_0 - 3$, $x_{t+3} = x_{t_0}$, which is nonzero, while $x_{t+2} = x_{t+1} = \cdots = x_{t-3} = 0$, and the regression reduces to $r_{t_0-3} = \mu + \gamma_{-3}x_{t_0} + u_{t_0-3}$.

Panel A of Table 3 reports the OLS estimation results. The CAMRs are the sums of the individual AMRs, and standard errors of CAMRs are simply $\sqrt{j} \times \text{std}(AMR_j)$, $j = 1, \ldots, 7$. From the table we find that all CAMRs are negative and statistically significant from day one before the event day to the end of the event window. The negative sign suggest that a higher than expected inflation, i.e. when $CPI_t^{act} - CPI_t^{pred} > 0$, tend to have a negative return effect. Furthermore, for example the total cumulative effect of -2.966 means that a one percent inflation shock tend to have on the average a total of -2.966% cumulative return effect over the event window.

Panel B of the table reports correlation diagnostics of the residuals in terms of the Ljung-Box Q-statistic (Ljung and Box 1978). After accounting for the autocorrelations at lags 3 and 5, there is no more statistically discernible serial correlation in the residuals. However, the squared residuals are highly autocorrelated, which is an indication of the stylized fact of clustering volatility in the returns. Thus the GARCH-effect should be modeled.

Table 3. OLS estimates of the event regression (60) and cumulative marginal abnormal returns.

Panel A. Coefficient estimates.

Variable	*AMR*	*std* *AMR*)	*t-AMR*	*p*-val	*CAMR*	*std* *CAMR*	*t-AMR*	*p*-val
x_3 $[\gamma_{-3}]$	-0.129	0.403	-0.320	0.749	-0.129	0.403	-0.320	0.749
x_2 $[\gamma_{-2}]$	-0.974	0.403	-2.417	0.016	-1.103	0.571	-1.932	0.053
x_1 $[\gamma_{-1}]$	-0.883	0.403	-2.192	0.028	-1.986	0.699	-2.840	0.005
x_0 $[\gamma_0]$	-1.204	0.403	-2.986	0.003	-3.190	0.808	-3.950	0.000
x_{-1} $[\gamma_1]$	0.258	0.403	0.641	0.521	-2.932	0.903	-3.247	0.001
x_{-2} $[\gamma_2]$	-0.100	0.403	-0.248	0.804	-3.032	0.989	-3.065	0.002
x_{-3} $[\gamma_3]$	0.066	0.403	0.163	0.871	-2.966	1.069	-2.776	0.006
μ	0.040	0.012	3.412	0.001				
$AR(3)$	-0.060	0.013	-4.734	0.000				
$AR(5)$	-0.030	0.013	-2.381	0.017				

Panel B. Diagnostic statistics

	Stat	*p*-value
$Q(3)\ u$	3.209	0.073
$Q(5)\ u$	4.412	0.220
$Q(3)\ u^2$	884.198	0.000
$Q(5)\ u^2$	1198.348	0.000

Panel C. Event period and non-event period volatilities		Obs.
Residual standard deviation (event window)	0.991	4876
Residual standard deviation (non-event window)	1.079	1400
Residual standard deviation (all observations)	1.010	6276
Ratio of non-event and event stds	1.089	
F-test for equlity of event and non-event stds	1.186	
p-value of F-test	0.000	

Furthermore, Panel C of the table shows that the residual volatility is about 9% higher during the event days than the non-event days. The difference is also highly statistically significant ($p = 0.000$).

GARCH(1,1) has been found in most cases to capture adequately the stock return volatility clustering. Consequently, we utilize it, and in order to test the possible volatility shift, suggested by the results of Panel C, we introduce a dummy variable having value 1 within the event windows and zero otherwise into the volatility equation.

The enhanced GARCH(1,1) model is

$$\mathrm{Var}[u_t|u_{t-1}] = h_t = \omega_0 + \omega_1 u_{t-1}^2 + \delta h_{t-1} + \lambda D_t, \tag{61}$$

where $D_t = 1$ if t is an event window day and zero otherwise. In addition we use the Bollerslev and Wooldridge (1992) robust standard errors in the test statistics.

Table 4 reports the estimation results. Qualitatively the results regarding AMRs and CAMRs remain the same as in Table 3. The only exception in the mean equation (Panel A of Table 4) is that using GARCH residuals the first and third lag autoregressions are in the residuals instead of the third and fifth of Table 3.

Panel B of Table 4 shows the variance equation with the event window dummy. The GARCH-parameters are all highly significant, but the dummy coefficient is not statistically significant at any convenient levels. Thus there is no indication of a level shift in the volatility during the event days.

We double checked this by calculating again the standard deviations of the GARCH standardized residuals over the event days and non-event days. The results are reported in Panel D of Table 4. The F-statistic for the ratio of the variances is 1.041 with p-value 0.171, and hence not statistically significant in any standard levels. Thus together with the results of Panel B this strongly suggests that the GARCH fully captures the temporal volatility increase during the event days.

Finally Panel C of Table 4 reports diagnostic statistics regarding the autocorrelations of the GARCH standardized residuals and squared residuals. None of the statistics are statistically significant at the convenient 5% level, and we can consider the model adequate in this respect.

Thus in all, this example demonstrates the flexibility of the regression based event study, and its ability to capture easily various aspects of an event study.

Table 4. ML-estimates of the event egression (60) with GARC(1,1) residuals.

Panel A. Mean equation.

Variable	*AMR*	*std* *AMR*	*t-AMR*	*p*-val	*CAMR*	*std* *CAMR*	*t-AMR*	*p*-val
x_3 [γ_{-3}]	-0.111	0.335	-0.331	0.741	-0.111	0.335	-0.331	0.741
x_2 [γ_{-2}]	-0.612	0.380	-1.609	0.108	-0.723	0.483	-1.496	0.135
x_1 [γ_{-1}]	-0.825	0.381	-2.163	0.031	-1.548	0.611	-2.532	0.011
x_0 [γ_0]	-0.985	0.327	-3.013	0.003	-2.533	0.695	-3.645	0.000
x_{-1} [γ_1]	-0.192	0.351	-0.549	0.583	-2.725	0.772	-3.532	0.000
x_{-2} [γ_2]	-0.360	0.314	-1.146	0.252	-3.085	0.832	-3.708	0.000
x_{-3} [γ_3]	0.352	0.333	1.056	0.291	-2.733	0.893	-3.061	0.002
μ	0.050	0.011	4.706	0.000				
$AR(1)$	0.036	0.013	2.714	0.007				
$AR(3)$	-0.037	0.014	-2.737	0.006				

Panel B. Variance equation.

	Coeff	std	*t*-val	*p*-val
Const [ω_0]	0.007	0.003	2.718	0.007
ARCH [ω_1]	0.049	0.007	6.650	0.000
GARCH [δ]	0.945	0.007	132.171	0.000
Dummy [λ]	0.001	0.008	0.189	0.850

Panel C. Diagnostic statistics

	Stat	*p*-value
$Q(3)\ z$	0.583	0.445
$Q(5)\ z$	5.682	0.128
$Q(3)\ z^2$	3.071	0.080
$Q(5)\ z^2$	3.594	0.309

Panel D. Event period and non-event period volatilities		Obs.
Residual standard deviation (event window)	0.995	4876
Residual standard deviation (non-event window)	1.015	1400
Residual standard deviation (all observations)	1.000	6530
Ratio of non-event and event stds	1.020	
F-test for equlity of event and non-event stds	1.041	
p-value of F-test	0.171	

5. Concluding remarks

This paper reviewed the standard event study methodology and discussed in more detail regression based event study with quantitative event variables. The traditional event study is a special case of the quantitative approach with event dummies in place of the quantitative event variables. When aggregating abnormal returns over firms, the results of the paper suggest that due to the firm specific return variances, the maximum likelihood estimation (ML) should be used in place of the simple arithmetic mean in order to obtain optimal estimates for the mean abnormal returns. Also the associated t-statistics should be based with the standard errors of the ML-estimators. In the paper utilizing the GARCH model was also considered to estimate event day volatilities.

As an empirical example the paper considered the effects of inflation shocks on stock returns. Using the regression event study approach, the result indicated that there is a statistically discernible negative return effect due to the inflation surprises. Furthermore, although the preliminary analysis suggested that stock return volatility might also have been shifted in the event days, there was no evidence of that after the residuals were modeled with a GARCH process. Thus the possible temporal increase in the volatility could be fully captured by the GARCH-model.

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