The Stability and Volatility of Electricity Prices: An Illustration of (λ, σ)-Analysis

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Abstract

The aim of this paper is to illustrate the usefulness of, what we call, (λ, σ)-analysis, i.e., to distinguish between the volatility of asset returns and the stability of the stochastic dynamic system generating these returns. Specifically, the stability of the stochastic dynamic system is measured by the average of the smooth Lyapunov exponents, while the volatility of asset returns is measured by the conditional variance. It is the lack of a one-to-one correspondence between these two measures that motivates (λ, σ)-analysis. The data set used in an empirical illustration is spot electricity prices from Nord Pool, in which we demonstrate how the stability and volatility of electricity prices have been affected in the integration process in the Nordic power exchange market.

JEL Codes: C12; C14; C22.
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1 Introduction

Certainly, it is important to understand the mechanisms behind the observed variability of asset prices since, to give one reason, variable asset prices are

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associated with risk. For example, having the mean-variance analysis developed by H.M. Markowitz in mind, an asset is considered to be more (less) risky than another asset, if the variance of the distribution of returns for the former asset is higher (lower) than for the latter asset. Thus, R.F. Engle's ARCH model, and its subsequent developments, is an invaluable tool since a common theme for these models is to estimate the conditional moments characterizing the probability distribution of asset returns.

However, as will be argued in this paper, it is necessary to find some complementary measure of the conditional variance of asset returns since the conditional moments of a probability distribution do not really focus on the *mechanisms* behind the observed variability of asset prices. Specifically, since asset returns depend on the underlying economic structure, meaning that economic variables interact with each other through time, and, therefore, constitute a dynamic system, the stability of the system generating asset returns is crucial for the variability of these returns. That is, a less (more) stable dynamic system is associated with more (less) variable asset returns, and, thus, a more risky asset.

To make the argument more distinct, let σ^2 denote the conditional variance of asset returns, and, for reasons that will be obvious below, let λ denote the stability of the stochastic dynamic system generating these returns. Then,

$$\sigma^2 = \sigma^2 (\lambda, \varepsilon), \tag{1}$$

where ε is exogenous shocks to the dynamic system. According to (1), the conditional variance (σ^2) is not only affected by the dynamic system's stability (λ), it is also affected by the shocks to the system (ε). Specifically, the conditional variance of asset returns increases (decreases) when the dynamic system is less (more) stable, but also when the amplitude of the shocks increases (decreases). Thus, because of ε in (1), there is no one-to-one correspondence between the conditional variance of asset returns and the stability of the dynamic system generating these returns. The lack of a one-to-one correspondence is also what motivates the proposed method in this paper.

The origin to the proposed method, hereafter called (λ, σ)-analysis, can be found in Bask and de Luna [2] and [3] in which they argue that the smooth Lyapunov exponents for a stochastic dynamic system may be utilized to construct stability measures of the system. In particular, it is argued in [2] that the largest Lyapunov exponent as well as the average of the exponents may be utilized as stability measures. As an illustration, it is shown in [2] that the decrease in volatility in the exchange rates for the Swedish Krona against the ECU/the Euro, after the launch of the Euro, is due more to a decrease in the amplitude of the exogenous shocks to the economic system than to a stabilization of the exchange rates.

In Bask and de Luna [3], a large-scale analysis of the European monetary integration, with the creation of the EMU, is carried out from the point of view of stability and volatility of foreign exchange. Specifically, changes in the stability and volatility of 16 European currencies, and in the volatility of the shocks to these currencies, are investigated, and the results indicate that when most of the currencies became less (more) stable, a majority of them also became more (less) volatile. For example, following the agreement of the Maastricht Treaty, most currencies became more stable and less volatile, whereas they became less stable and more volatile when the Danish public voted against the treaty.

A short-coming in [2] and [3], however, is the lack of a distributional theory for the average of the smooth Lyapunov exponents. Consequently, one should not be too hasty when making conclusions about the findings in the large-scale analysis in [3] since it only contains point estimates of the Lyapunov exponents. Bask and de Luna [2], however, present a consistent estimator of the largest smooth Lyapunov exponent, based on a parametric model, that is asymptotically normal, and this estimator was used in the aforementioned example with the Swedish Krona against the ECU/the Euro exchange rates.

In this paper, we derive the asymptotic distribution for an estimator of the average of the smooth Lyapunov exponents, based on a (feedforward) neural network, which is a non-parametric model. For our purpose, we use the algorithm developed by Gencay and Dechert [5] to estimate the Lyapunov exponents as well as the asymptotic results for neural network estimation of the these exponents that is found in Shintani and Linton [11]. Even if the derivations in this paper are not difficult to do, after employing the nice results in Shintani and Linton [11], our contribution is still important for two reasons. Firstly, a distributional theory for the more appealing stability measure is derived since this measure is for an "average scenario," and not a "worst case scenario" as when the largest Lyapunov exponent is used.

Secondly, the distributional theory derived allows us to investigate, in a more satisfactory way, the relationship between the stability of the stochastic dynamic system generating asset returns and the conditional volatility of these returns. An empirical analysis is also performed in this paper, with pleasing results, using spot electricity prices from the Nordic power exchange market, Nord Pool. In fact, the main finding is that when the conditional volatility of asset returns decreases (increases), when a new country joins the power market, the stability of the dynamic system generating these returns increases (decreases). However, which motivates the use of (λ, σ)-analysis, there is not a one-to-one correspondence between the stability and volatility of electricity prices.

In the empirical analysis, the conditional volatility is measured by the persistent volatility parameter in the EGARCH model. Of course, other models might be useful as well when measuring the conditional volatility. For this reason, a nice guide to the literature is Poon and Granger [9], who provide a review of the out-of-sample forecasting performance of different volatility models. The same is true when measuring the stability of a stochastic dynamic system. That is, other algorithms than the one proposed in Gencay and Dechert [5] might be useful. However, since the focus is on the stability of stochastic dynamic systems, and not deterministic dynamic systems, it is the smooth Lyapunov exponents that should be estimated. This is because these exponents are defined with respect to an observed realization of a stochastic process.

It should be emphasized that there is a crucial difference between the suggested analysis in this paper and the analysis in Bask and de Luna [2] and [3].

In the latter papers, it is argued that when the volatility of a variable modelled is of interest, one should also consider the stability properties of *the same model*. In these two papers, a parametric model in the form of a polynomial autoregression on a projected space is fitted to the data (see de Luna [4] for details). However, in the present paper, a non-parametric approach is utilized when estimating the stability of a stochastic dynamic system whereas any kind of (good) volatility model may be used when measuring the volatility.

Clearly, the two approaches are complementary and their appropriateness depend on the purpose of the analysis. For instance, when it comes to developing a theoretical model aiming to give an economic interpretation to observed movements in, for example, asset returns, we believe that one should not only evaluate the out-of-sample forecasting performance of the model, but also the stability properties of the same model to match it with the stability properties of asset returns. Thus, the approach suggested in [2], and applied in the large-scale analysis in [3], is more appropriate.

However, when, for example, a successful risk management is in focus, it is necessary to measure the stability of the "true" stochastic dynamic system generating asset returns, and not the stability of the model fitted to these returns. The reason is that there is no guarantee that the smooth Lyapunov exponents for the "true" dynamic system and the model selected to measure the volatility coincide with each other. ...

Finally, before moving on to the main parts of the paper, we should mention impulse-response functions as another tool to measure the stability of stochastic dynamic models. Specifically, Koop et al. [7] and Potter [10] extend, in an appealing way, the linear technique of impulse-response functions to the non-linear case, although they show that there is no unique definition of such a function when non-linear dynamic models are considered. Certainly, impulse-response functions are useful graphical tools in the non-linear case, even if they are less appropriate when inference needs to be performed on a change in the stability of a stochastic dynamic model.

The rest of the paper is organized as follows. The proposed method, (λ, σ)-analysis, is presented and discussed in Section 2, where the focus is on measuring the stability of a stochastic dynamic system generating observed data. The empirical illustration of (λ, σ)-analysis using spot electricity prices is, thereafter, carried out in Section 3, whereas Section 4 concludes the paper with the main findings.

2 Method: (λ, σ)-analysis

Lambda (λ), which is our measure of the stability of a stochastic dynamic system generating asset returns, is in focus in Section 2.1. Thereafter, in Section 2.2, we will shortly discuss sigma (σ^2), which is our measure of the conditional volatility of asset returns. Thus, the focus is on measuring the stability of stochastic dynamic systems, and this is because this topic, so far, has been highly neglected in the literature. Consequently, the theoretical foundation for the proposed method is not well-known.

2.1 Lambda: a measure of stability

The aim of this section is fourfold: (i) to define the smooth Lyapunov exponents of a stochastic dynamic system, which forms the basis of our measure of stability; (ii) to motivate why these exponents provide a measure of the stability of a stochastic dynamic system; (iii) to demonstrate how the smooth Lyapunov exponents can be estimated from time series data like asset returns; and (iv) to demonstrate how a hypothesis test of the change in the stability of a stochastic dynamic system can be constructed.

Definition of lambda As argued in Bask and de Luna [2] and [3], and to be further explained below (see the section on motivation of lambda), the Lyapunov exponents can be used in the determination of the stability of a stochastic dynamic system. Specifically, assume that the stochastic dynamic system, $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$, generating asset returns is

$$S_{t+1} = f\left(S_t\right) + \varepsilon_{t+1}^s, \tag{2}$$

where S_t and ε_t^s are the state of the system and a shock to the system, respectively, both at time $t \in [1, 2, \ldots, \infty]$. For an n-dimensional system as in (2), there are n Lyapunov exponents that are ranked from the largest to the smallest exponent:

$$\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n, \tag{3}$$

and it is these exponents that provide information on the stability properties of the dynamic system f in (2).

Then, how are the Lyapunov exponents in (3) defined? Start by considering how the dynamic system f in (2) amplifies a small difference between the initial states S_0 and S_0':

$$S_j - S_j' = f^j\left(S_0\right) - f^j\left(S_0'\right) \simeq Df^j\left(S_0\right)\left(S_0 - S_0'\right), \tag{4}$$

where $f^j\left(S_0\right) = f\left(\cdots f\left(f\left(S_0\right)\right)\cdots\right)$ denotes j successive iterations of the dynamic system starting at state S_0, and where Df is the Jacobian of the system:

$$Df^j\left(S_0\right) = Df\left(S_{j-1}\right) Df\left(S_{j-2}\right) \cdots Df\left(S_0\right). \tag{5}$$

Then, associated with each Lyapunov exponent, λ_i, $i \in [1, 2, \ldots, n]$, there are nested subspaces $U^i \subset \mathbb{R}^n$ of dimension $n + 1 - i$ with the property that

$$\begin{aligned} \lambda_i &\equiv \lim_{j \rightarrow \infty} \frac{\log_e \left\| Df^j\left(S_0\right) \right\|}{j} \\ &= \lim_{j \rightarrow \infty} \frac{1}{j} \sum_{k=0}^{j-1} \log_e \left\| Df\left(S_k\right) \right\|, \qquad \forall S_0 \in U^i - U^{i+1}. \end{aligned} \tag{6}$$

Due to Oseledec's multiplicative ergodic theorem, the limit in (6) exists and is independent of S_0 almost surely with respect to the measure induced by the stochastic process, $\{S_t\}_{t=1}^{\infty}$ (see Guckenheimer and Holmes [6] for a careful definition of the Lyapunov exponents and their properties). In the literature (see, e.g., Bask and de Luna [2]), the Lyapunov exponents in (6) have been named smooth Lyapunov exponents.

After defining the smooth Lyapunov exponents for a stochastic dynamic system, we will now motivate how these exponents may be utilized to construct measures of the stability of a stochastic dynamic system.

Motivation of lambda[1] The reason why smooth Lyapunov exponents provide a measure of the stability of a stochastic dynamic system may be seen by considering two different starting values of the system, where the difference is an exogenous shock at time $t = 0$. The largest smooth Lyapunov exponent, λ_1, measures the slowest exponential rate of convergence of two trajectories of the dynamic system starting at these two different values at time $t = 0$, but with identical exogenous shocks at times $t > 0$.[2] Indeed, λ_1 measures the convergence of a shock in the direction defined by the eigenvector corresponding to this exponent. If the difference between the two different starting values lies in another direction of $\mathbb{R}^n$, then the convergence is faster. Thus, λ_1 measures the "worst case scenario."

The average of the smooth Lyapunov exponents,

$$\lambda \equiv \frac{1}{n}\sum_{i=1}^{n}\lambda_i, \tag{7}$$

measures the exponential rate of convergence in a geometrical average direction. That is, the convergence of two trajectories of the stochastic dynamic system in this average direction, which, here, is the geometrical average of the directions defined by the eigenvectors corresponding to the different exponents. Thus, λ measures the "average scenario."

We may, therefore, compare the stability of two different stochastic dynamic systems via the smooth Lyapunov exponents since a one-time shock has a smaller effect on the dynamic system with the smaller λ (λ_1), than for the system with the larger λ (λ_1). Which of the two measures, λ or λ_1, is more appropriate is an open question. However, with reference to λ as a measure of the "average scenario," the focus in this paper will be on the average of the smooth Lyapunov exponents.

Let us now turn to the question how the smooth Lyapunov exponents for a stochastic dynamic system generating asset returns may be estimated from data. Certainly, there are several algorithms that may be used for this task, but

[1] An extensive discussion of the smooth Lyapunov exponents as a measure of the stability of a stochastic dynamic system is provided in Bask and de Luna [2].

[2] In particular, when $\lambda_1 > 0$, the two trajectories diverge from each other, and for a bounded stochastic dynamic system, a positive exponent is an operational definition of chaos.

for reasons explained below (see the section on inference of lambda), we will focus on an algorithm proposed in Gencay and Dechert [5].

Estimation of lambda Since the actual functional form of the dynamic system f in (2) is not known, it may seem like an impossible task to determine the stability of the system. However, it is possible to reconstruct the dynamics of the system using only an asset return series, and, then, measure the stability of this reconstructed system. Therefore, associate the dynamic system f in (2) with an observer function, $g : \mathbb{R}^n \to \mathbb{R}$, which generates observed asset returns:

$$s_t = g\left(S_t\right) + \varepsilon_t^m, \tag{8}$$

where $s_t \in S_t$ and ε_t^m are the asset return and a measurement error, respectively, both at time t. Thus, (8) means that the asset return series

$$\left\{s_t\right\}_{t=1}^N, \tag{9}$$

is observed, which is used to reconstruct the dynamics of the system f in (2), where N is the number of consecutive returns in the time series.

Specifically, the observations in a scalar time series, like the asset return series in (9), contain information about unobserved state variables that can be used to define a state in present time. Therefore, let

$$T = \left(T_1, T_2, \ldots, T_M\right)', \tag{10}$$

be the reconstructed trajectory, where T_t is the reconstructed state at time t and M is the number of states on the reconstructed trajectory. Each T_t is given by

$$T_t = \left\{s_t, s_{t+\tau}, \ldots, s_{t+(m-1)\tau}\right\}, \tag{11}$$

where τ and m are the reconstruction delay and the embedding dimension, respectively, and $t \in [1, 2, \ldots, N - m + 1]$. Thus, T is an $M \times m$ matrix and the constants M, m and N are related as $M = N - m + 1$.

Takens [12] proved that the map

$$\Phi\left(S_t\right) = \left\{g\left(f^0\left(S_t\right)\right), g\left(f^\tau\left(S_t\right)\right), \ldots, g\left(f^{(m-1)\tau}\left(S_t\right)\right)\right\}, \tag{12}$$

which maps the n-dimensional state S_t onto the m-dimensional state T_t, is an embedding if $m > 2n$. This means that the map is a smooth map that performs a one-to-one coordinate transformation and has a smooth inverse. A map that is an embedding preserves topological information about the unknown dynamic system, like the Lyapunov exponents, and, in particular, the map induces a function, $h : \mathbb{R}^m \to \mathbb{R}^m$, on the reconstructed trajectory,

$$T_{t+1} = h\left(T_t\right), \tag{13}$$

which is topologically conjugate to the unknown dynamic system f in (2). That is,

$$h^j\left(T_t\right) = \Phi \circ f^j \circ \Phi^{-1}\left(T_t\right). \tag{14}$$

Thus, h in (13) is a reconstructed dynamic system that has the same smooth Lyapunov exponents as the unknown dynamic system f in (2).

Now, in order to estimate the smooth Lyapunov exponents of the dynamic system generating asset returns, one has to estimate h in (13). However, since

$$h : \begin{pmatrix} s_t \\ s_{t+1} \\ \vdots \\ s_{t+m-1} \end{pmatrix} \longrightarrow \begin{pmatrix} s_{t+1} \\ s_{t+2} \\ \vdots \\ v\left(s_t, s_{t+1}, \ldots, s_{t+m-1}\right) \end{pmatrix}, \tag{15}$$

where the reconstruction delay in (11) is $\tau = 1$, the estimation of h reduces to the estimation of v:

$$s_{t+m} = v\left(s_t, s_{t+1}, \ldots, s_{t+m-1}\right). \tag{16}$$

Moreover, note that the Jacobian of h at the reconstructed state T_t, when $\tau = 1$, is

$$Dh\left(T_t\right) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial v}{\partial s_t} & \frac{\partial v}{\partial s_{t+1}} & \frac{\partial v}{\partial s_{t+2}} & \cdots & \frac{\partial v}{\partial s_{t+m-1}} \end{pmatrix}. \tag{17}$$

Thus, the smooth Lyapunov exponents can be estimated from data. ...

For reasons soon to be explained (see the next section), a (feedforward) neural network estimator is used when estimating v in (16) and its derivatives. This is because we easily can derive, by employing the results in Shintani and Linton [11], the asymptotic distribution for the average of the smooth Lyapunov exponents, based on a neural network estimator of the exponents. Thus, we can construct a hypothesis test of a change in the stability of a stochastic dynamic system. Therefore, let us now turn to this subject.

Inference of lambda Shintani and Linton [11] derive the asymptotic distribution of a neural network estimator of the smooth Lyapunov exponents. Specifically, given some technical conditions (see [11] for details), they show that

$$\sqrt{M}\left(\widehat{\lambda}_{iM} - \lambda_i\right) \Longrightarrow \mathbb{N}\left(0, V_i\right), \qquad i \in [1, 2, \ldots, n], \tag{18}$$

where $\widehat{\lambda}_{iM}$ is the estimator of the i:th Lyapunov exponent, based on the M reconstructed states on the trajectory, and V_i is the variance of the i:th Lyapunov exponent. Then, if $\widehat{V}_i$ is a consistent estimator of V_i (see, e.g., Andrews [1]), a test statistic for the null and alternative hypotheses,

$$\mathbf{H}_0 : \lambda_i \geq 0 \quad (\lambda_i \leq 0), \qquad \mathbf{H}_1 : \lambda_i < 0 \quad (\lambda_i > 0), \qquad i \in [1, 2, \ldots, n], \tag{19}$$

is

$$\widehat{t}_i = \frac{\widehat{\lambda}_{iM}}{\sqrt{\frac{\widehat{V}_i}{M}}}, \qquad i \in [1, 2, \ldots, n]. \tag{20}$$

Thus, the null hypothesis is rejected when

$$\widehat{t}_i \leq -z_\alpha \quad \left(\widehat{t}_i \geq z_\alpha\right), \qquad i \in [1, 2, \ldots, n], \tag{21}$$

where $\mathbb{Z}$ is the standard normal random variable, and, most often, the significance level is $\Pr[\mathbb{Z} \geq z_\alpha] = \alpha = 0.01$, $\alpha = 0.05$ or $\alpha = 0.1$.

However, since the focus in this paper is on the stability of a stochastic dynamic system or, to be more specific, whether the stability of the dynamic system has changed over time, it is necessary to construct a test statistic based on a change in the average of the smooth Lyapunov exponents. Therefore, ...

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Now, after having thoroughly presented and discussed lambda (λ) as a measure of stability, we will in the next section turn our focus to sigma (σ^2) as a measure of volatility.

2.2 Sigma: a measure of volatility

As was explained in the introductory discussion in this paper, any kind of (good) volatility model may be used when measuring the conditional variance of asset returns. For example, we employ the EGARCH model and focus on the persistent volatility parameter when obtaining an estimate of sigma (σ^2). In Bask and de Luna [2] and [3], however, a polynomial autoregression on a projected space is identified and fitted to the data (see [4] for details).

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3 Illustration: (λ, σ)-analysis of electricity prices

The aim of this section is to illustrate the usefulness of (λ, σ)-analysis with the help of a data set from the Nordic power exchange market, Nord Pool, which contains spot electricity prices. A short description of the Nordic power market as well as the data set used are given in Section 3.1. Thereafter, the empirical results are found in Section 3.2, and a sensitivity analysis of these results is given in Section 3.3.

3.1 The Nordic power market and data set used

Nord Pool is a multinational exchange for trade in power exchange, joining the Nordic countries. Norway was, in 1991, the first of the Nordic countries to deregulate the power market. Nord Pool ASA was established in 1993, then under the name Statnett Marked AS. Sweden started the deregulation process in 1991, and went step-wise to a deregulated power market. 1 January 1996 was the start-up of the joint Norwegian-Swedish power exchange market, renamed to Nord Pool ASA.

Finland started a power exchange market of its own, EL-EX in August 1996. They joined Nord Pool in 1997. In 1999, Elbas is launched as a separate market for power balance adjustment in Sweden and Finland, giving a fully integrated market between Norway, Sweden and Finland. Denmark Nord Pool Consulting is established in 1998, and Western Denmark (Jutland/Funen) joins the market in 1999 as a Nordic power exchange price area. When Eastern Denmark (Zealand) joins in 2000, the Nordic power exchange market becomes fully integrated. See Table 1 for the specific dates in the integration process.

[Table 1 about here.]

The data set used is spot electricity prices from Nord Pool. Specifically, it is the system price, average daily prices as well as hourly, for the period 1 January 1993 to 25 September 2005. The data are analyzed both as one time series but also split in parts, with the natural breakpoints when a new country is joining the market. Graphs over the system prices and the logarithmic returns of these prices, for the whole time period as well as for all sub-periods, are found in Figures 1-6.

[Figures 1-6 about here.]

Since the system prices are not stationary, we will use the logarithmic returns of the prices in the empirical analysis.[3] [4]

3.2 Empirical results

Firstly, we estimated an EGARCH model for each time series, where our estimate of the conditional volatility is the persistent volatility parameter in the model. See Tables 2-6 for the results.

[Tables 2-6 about here.]

The persistent volatility parameter is significant in all cases, except for the period 1 July 1999 to 30 September 2000 when only Norway, Sweden, Finland and Western Denmark participate in the power exchange market. These parameter estimates can also be found in Table 7 (and Table 9).

[Table 7 about here.]

The general picture is that the integration process in the power market is associated with a decrease in the conditional volatility of prices, except when Eastern Denmark as the last region joins the power exchange market. [Significant changes in volatility?]

Secondly, we estimated the smooth Lyapunov exponents using the neural network algorithm proposed in Gencay and Dechert [5].[5] Specifically, for each

[3] Hereafter, the logarithmic returns of the system prices are simply called prices.

[4] The results of the stationarity tests of the time series can be found in the Appendix. See Table A.1-A.2.

[5] We have used NETLE 3.01, a computer program developed by C.-M. Kuan, T. Liu and R. Gencay, when estimating the Lyapunov exponents. See Gencay and Dechert [5] and Kuan and Liu [8] for details.

time series, we estimated the Lyapunov exponents making use of 4, 8, 12, 16 and 20 inputs, respectively, to the neural network, and calculated λ as in (7). Moreover, the number of hidden units in the neural network in each case runs from 1 unit to 20 units. The specific estimate chosen for each number of inputs is when SIC is minimized. See Figures 7-11 for graphs over the stability of λ over the number of hidden units for the different number of inputs in the neural network.[6]

[Figures 7-11 about here.]

As can be seen in the graphs, λ is rather stable, even if there is a weak tendency that it increases with the number of hidden units. In all cases, however, λ is negative, which also is consistent with the theory of dissipative dynamic systems (i.e., "energy-loosing" systems).[7] In Table 7 (and Table 8), the estimate of λ that minimizes SIC in each sub-period in the integration process is reported. The general picture is that the integration process in the power market is associated with an increase in the stability of the stochastic dynamic system generating prices. [Significant changes in stability?]

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3.3 Sensitivity analysis

When studying the time series, its clear that there are some extreme values, outliers.[8] In order to see their impact on the result, we eliminated the outliers from the time series and performed the same analysis as above in Section 3.2.

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4 Concluding discussion

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[6] Detailed results of the estimations can be found in the Appendix. See Tables A.3-A.27.

[7] In fact, in three cases out of 500, λ is positive. But these exceptions occur for the period 1 July 1999 to 30 September 2000, which is the shortest sub-period in the integration process.

[8] The excluded outliers are 28 February to 2 March 1994, 8 December 1998, 24 January 2000, 5 February 2001, 5 December 2002 to 14 January 2003. In total, 44 outliers are excluded.

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Country	Date for affiliation
Norway	1 January 1993
Sweden	1 January 1996
Finland	29 December 1997
Western Denmark	1 July 1999
Eastern Denmark	1 October 2000

Table 1: The dates in the integration process in the power market.

Variable	Coefficient	Standard error	t statistic	Significance
BO	0,000160	0,00109	0,147	0,883
VC	-0,106	0,0424	-2,51	0,0120
VA	0,975	0,00734	133	0,00000000
VB	0,363	0,0508	7,16	0,00000000
VD	0,308	0,107	2,88	0,00400

Table 2: Estimated EGARCH model (the number of significant figures is 3) for the period 1 January 1993 to 31 December 1995, i.e., when only Norway participates in the power market.

Variable	Coefficient	Standard error	t statistic	Significance
BO	0,00148	0,00194	0,763	0,446
VC	-0,358	0,159	-2,26	0,0240
VA	0,935	0,0269	34,7	0,00000000
VB	0,418	0,0755	5,53	0,00000003
VD	0,330	0,114	2,90	0,00377

Table 3: Estimated EGARCH model (the number of significant figures is 3) for the period 1 January 1996 to 28 December 1997, i.e., when only Norway and Sweden participate in the power market.

Variable	**Coefficient**	**Standard error**	t **statistic**	**Significance**
BO	-0,00480	0,00469	-1,03	0,305
VC	-1,39	0,329	-4,22	0,0000240
VA	0,686	0,0825	8,31	0,00000000
VB	0,653	0,0953	6,85	0,00000000
VD	0,397	0,0670	5,93	0,00000000

Table 4: Estimated EGARCH model (the number of significant figures is 3) for the period 29 December 1997 to 30 June 1999, i.e., when only Norway, Sweden and Finland participate in the power market.

Variable	**Coefficient**	**Standard error**	t **statistic**	**Significance**
BO	-0,00766	0,00418	-1,83	0,0667
VC	-4,07	0,952	-4,28	0,0000189
VA	0,148	0,211	0,702	0,482
VB	0,814	0,191	4,27	0,0000199
VD	-0,345	0,242	-1,43	0,154

Table 5: Estimated EGARCH model (the number of significant figures is 3) for the period 1 July 1999 to 30 September 2000, i.e., when only Norway, Sweden, Finland and Western Denmark participate in the power market.

Variable	**Coefficient**	**Standard error**	t **statistic**	**Significance**
BO	0,0000711	0,0000287	2,48	0,0131
VC	-0,263	0,353	-0,745	0,456
VA	0,943	0,0677	14,0	0,00000000
VB	0,444	0,202	2,20	0,0277
VD	0,469	0,0721	6,50	0,00000000

Table 6: Estimated EGARCH model (the number of significant figures is 3) for the period 1 October 2000 to 25 September 2005, i.e., when Norway, Sweden, Finland and Denmark participate in the power market.

Countries	**Stability (λ)**	**Stability change**	**Volatility**	**Volatility change**
Norway	-0,268 12 inputs 5 hidden units		0,975	
		Increase		Decrease
Norway and Sweden	-0,359 8 inputs 2 hidden units		0,935	
		Decrease		Decrease
Norway, Sweden and Finland	-0,168 12 inputs 3 hidden units		0,686	
		Increase		Decrease
Norway, Sweden, Finland and Western Denmark	-0,273 8 inputs 1 hidden unit		0,148	
		Increase		Increase
Norway, Sweden, Finland and Denmark	-0,317 8 inputs 5 hidden units		0,943	

Table 7: Stability and volatility measures (the number of significant figures is 3) for the different time periods in the integration process in the power market. The stability measure is lambda (λ) and the volatility measure is the persistent volatility parameter in the EGARCH model. The given numbers of inputs and hidden units in the neural network are when SIC is minimized.

Countries	**Stability (λ) with outliers**	**Stability change**	**Stability (λ) without outliers**	**Stability change**
Norway	-0,268 12 inputs 5 hidden units		-0,279 16 inputs 3 hidden units	
		Increase		Increase
Norway and Sweden	-0,359 8 inputs 2 hidden units		-0,359 8 inputs 2 hidden units	
		Decrease		Decrease
Norway, Sweden and Finland	-0,168 12 inputs 3 hidden units		-0,163 12 inputs 3 hidden units	
		Increase		Decrease
Norway, Sweden, Finland and Western Denmark	-0,273 8 inputs 1 hidden unit		-0,0172 20 inputs 19 hidden units	
		Increase		Increase
Norway, Sweden, Finland and Denmark	-0,317 8 inputs 5 hidden units		-0,244 12 inputs 3 hidden units	

Table 8: The stability measure (the number of significant figures is 3) for the different time periods in the integration process in the power market, both when outliers are included as well as excluded in the estimations, where the stability measure is lambda (λ). The given numbers of inputs and hidden units in the neural network are when SIC is minimized.

Countries	**Volatility with outliers**	**Volatility change**	**Volatility without outliers**	**Volatility change**
Norway	0,975		0,976	
		Decrease		Decrease
Norway and Sweden	0.935		0,935	
		Decrease		Decrease
Norway, Sweden and Finland	0,686		0,643	
		Decrease		Decrease
Norway, Sweden, Finland and Western Denmark	0,148		-0,101	
		Increase		Increase
Norway, Sweden, Finland and Denmark	0,943		0,950	

Table 9: The stability measure (the number of significant figures is 3) for the different time periods in the integration process in the power market, both when outliers are included as well as excluded in the estimations, where the volatility measure is the persistent volatility parameter in the EGARCH model.

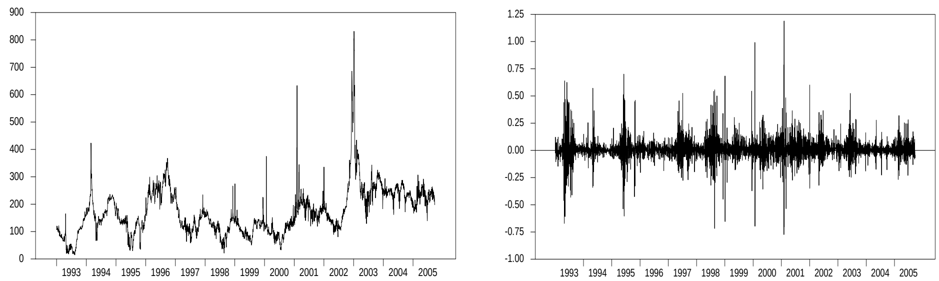

Figure 1: The price and logarithmic returns for the system price at Nord Pool, for the period 1 January 1993 to 25 September 2005, i.e., for the whole period.

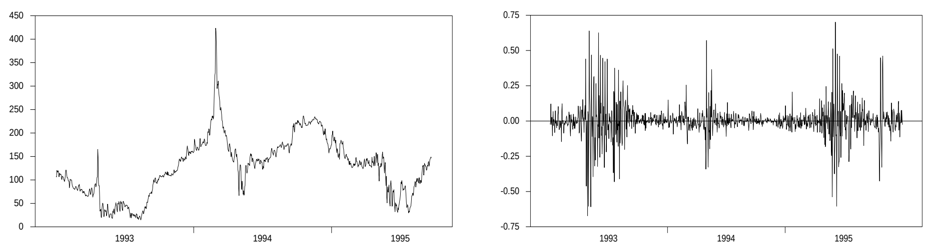

Figure 2: The price and logarithmic returns for the system price at Nord Pool, for the period 1 January 1993 to 31 December 1995, i.e., when only Norway participates in the power market.

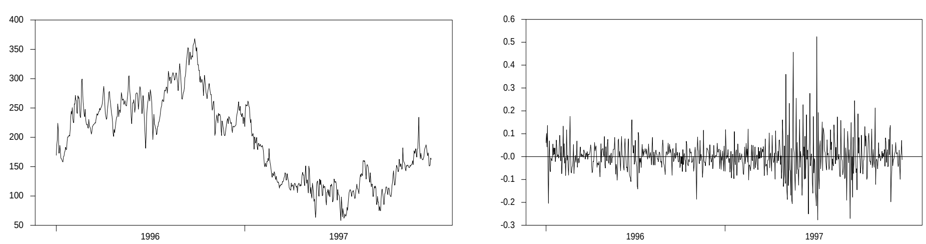

Figure 3: The price and logarithmic returns for the system price at Nord Pool, for the period 1 January 1996 to 28 December 1997, i.e., when only Norway and Sweden participate in the power market.

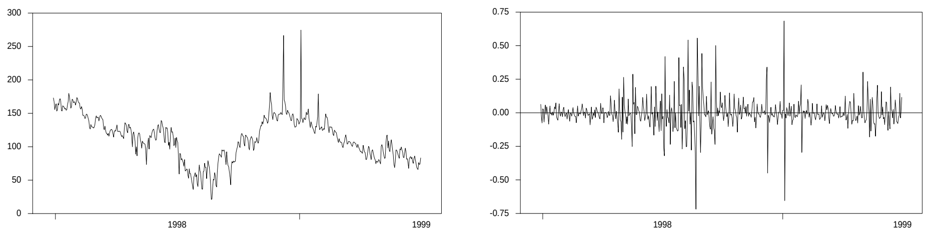

Figure 4: The price and logarithmic returns for the system price at Nord Pool, for the period 29 December 1997 to 30 June 1999, i.e., when only Norway, Sweden and Finland participate in the power market.

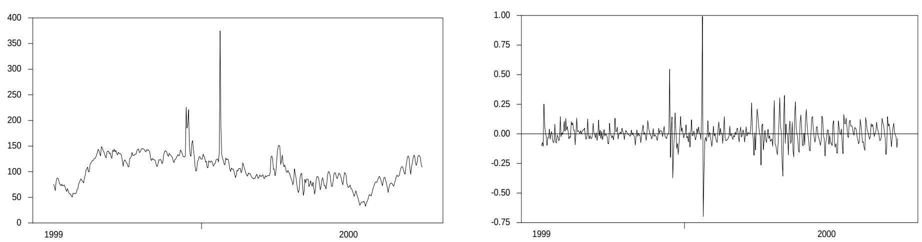

Figure 5: The price and logarithmic returns for the system price at Nord Pool, for the period 1 July 1999 to 30 September 2000, i.e., when only Norway, Sweden, Finland and Western Denmark participate in the power market.

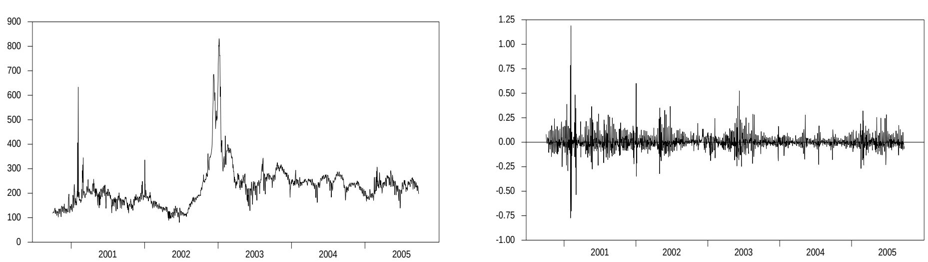

Figure 6: The price and logarithmic returns for the system price at Nord Pool, for the period 1 October 2000 to 25 September 2005, i.e., when Norway, Sweden, Finland and Denmark participate in the power market.

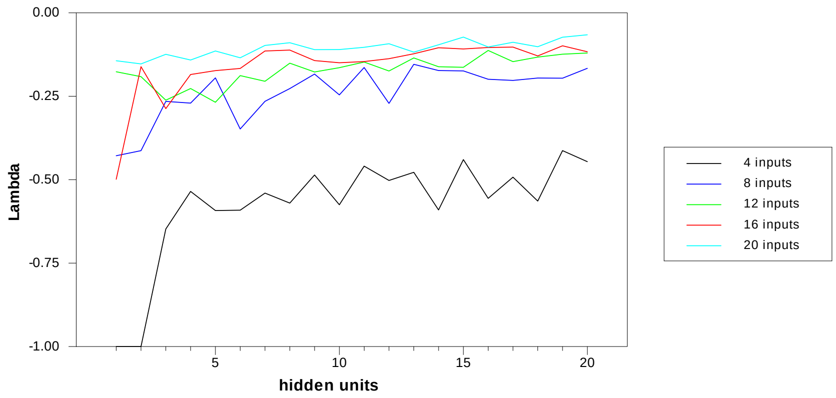

Figure 7: Graph over the stability of lambda over the number of hidden units for the different number of inputs in the neural network, for the period 1 January 1993 to 31 December 1995, i.e., when only Norway participates in the power market.

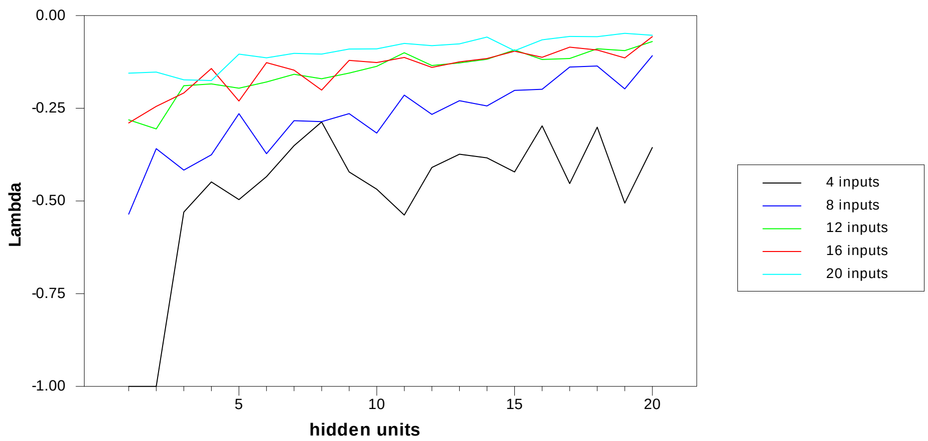

Figure 8: Graph over the stability of lambda over the number of hidden units for the different number of inputs in the neural network, for the period 1 January 1996 to 28 December 1997, i.e., when only Norway and Sweden participate in the power market.

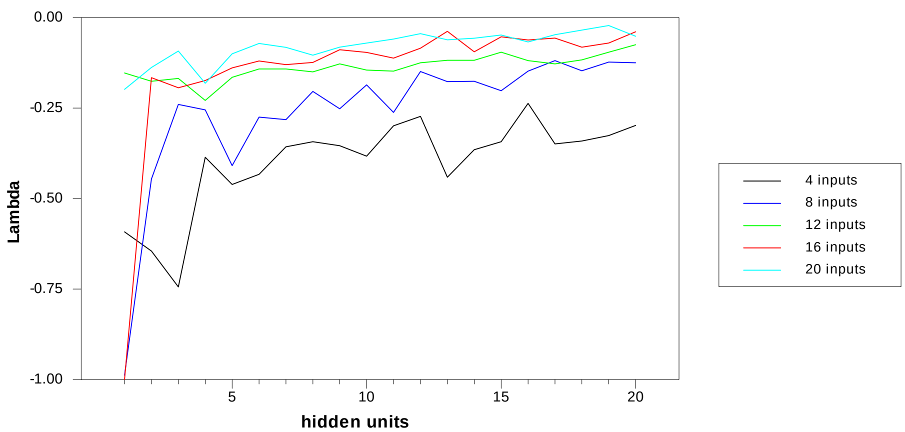

Figure 9: Graph over the stability of lambda over the number of hidden units for the different number of inputs in the neural network, for the period 29 December 1997 to 30 June 1999, i.e., when only Norway, Sweden and Finland participate in the power market.

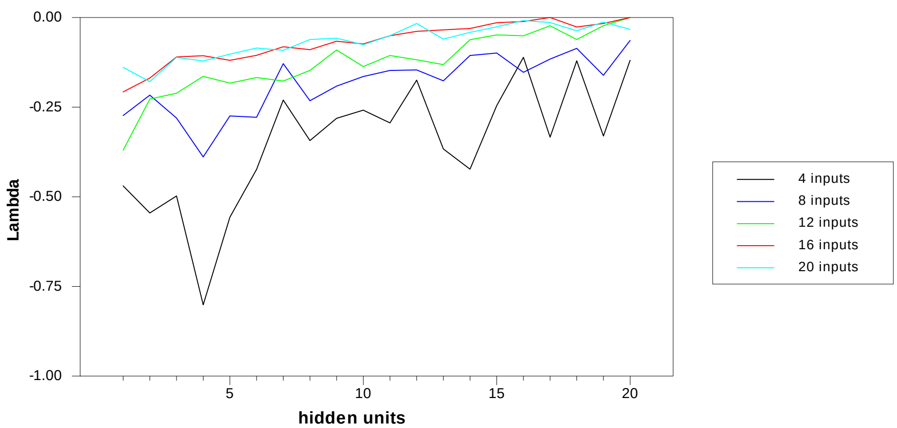

Figure 10: Graph over the stability of lambda over the number of hidden units for the different number of inputs in the neural network, for the period 1 July 1999 to 30 September 2000, i.e., when only Norway, Sweden, Finland and Western Denmark participate in the power market.

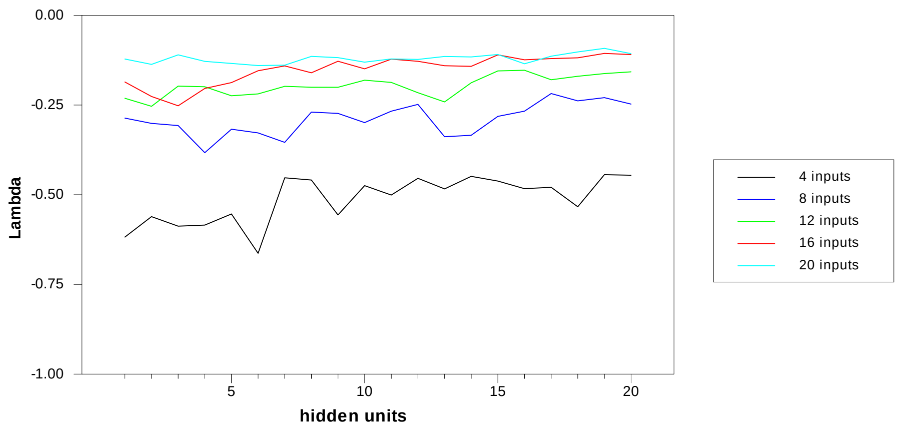

Figure 11: Graph over the stability of lambda over the number of hidden units for the different number of inputs in the neural network, for the period 1 October 2000 to 25 September 2005, i.e., when Norway, Sweden, Finland and Denmark participate in the power market.

Appendix

Countries	t **statistic**	**Significance**
Norway	-0,70	No
Norway and Sweden	-0,44	No
Norway, Sweden and Finland	-1,30	No
Norway, Sweden, Finland and Western Denmark	-0,36	No
Norway, Sweden, Finland and Denmark	-1,29	No

Table A.1: The Dickey-Fuller Unit Root Test for the daily mean system price at Nord Pool. The critical values are $1\% = -2,57$, $5\% = -1,94$ and $10\% = -1,62$.

Countries	t **statistic**	**Significance**
Norway	-10,67	1%
Norway and Sweden	-9,59	1%
Norway, Sweden and Finland	-8,67	1%
Norway, Sweden, Finland and Western Denmark	-6,13	1%
Norway, Sweden, Finland and Denmark	-15,33	1%

Table A.2: The Dickey-Fuller Unit Root Test for the logarithmic returns of the daily mean system price at Nord Pool. The critical values are $1\% = -2,57$, $5\% = -1,94$ and $10\% = -1,62$.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
4	1	-4,44	-3,54	-3,80
4	2	-4,43	-0,893	-1,39
4	3	-4,36	-0,458	-0,648
4	4	-4,40	-0,414	-0,535
4	5	-4,38	-0,380	-0,593
4	6	-4,44	-0,313	-0,591
4	7	-4,44	-0,315	-0,540
4	8	-4,40	-0,263	-0,570
4	**9**	**-4,51**	**-0,292**	**-0,486**
4	10	-4,22	-0,280	-0,575
4	11	-4,21	-0,286	-0,459
4	12	-4,35	-0,214	-0,502
4	13	-4,29	-0,250	-0,478
4	14	-4,27	-0,216	-0,591
4	15	-4,40	-0,167	-0,440
4	16	-4,40	-0,197	-0,556
4	17	-4,36	-0,252	-0,493
4	18	-4,13	-0,240	-0,564
4	19	-4,23	-0,187	-0,413
4	20	-4,24	-0,140	-0,446

Table A.3: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1993 to 31 December 1995, i.e., when only Norway participates in the power market. The total number of observations are 1096, which are reduced to 1092 observations when there are 4 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda}$
8	1	-4,50	-0,107	-0,428
8	2	-4,47	-0,101	-0,413
8	3	-4,51	-0,109	-0,265
8	4	-4,43	-0,0403	-0,270
8	5	-4,50	-0,0478	-0,195
8	6	-4,42	-0,0547	-0,348
8	7	-4,29	-0,0653	-0,265
8	8	-4,52	-0,0339	-0,227
8	9	-4,40	-0,0419	-0,184
8	**10**	**-4,57**	**-0,0605**	**-0,246**
8	11	-4,55	-0,00619	-0,164
8	12	-4,40	-0,0277	-0,271
8	13	-4,30	0,0346	-0,154
8	14	-4,46	0,00420	-0,173
8	15	-4,55	0,0123	-0,174
8	16	-4,38	0,0125	-0,199
8	17	-4,56	0,0237	-0,203
8	18	-4,55	0,00133	-0,195
8	19	-4,20	0,000917	-0,196
8	20	-4,44	0,0630	-0,166

Table A.4: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1993 to 31 December 1995, i.e., when only Norway participates in the power market. The total number of observations are 1096, which are reduced to 1088 observations when there are 8 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
12	1	-4,52	-0,0728	-0,177
12	2	-4,61	-0,0929	-0,191
12	3	-4,53	-0,0767	-0,262
12	4	-4,60	-0,0847	-0,227
12	**5**	**-4,68**	**-0,0606**	**-0,268**
12	6	-4,60	-0,0498	-0,188
12	7	-4,34	-0,0401	-0,205
12	8	-4,43	-0,0250	-0,151
12	9	-4,46	-0,00285	-0,177
12	10	-4,52	0,0295	-0,164
12	11	-4,40	0,0100	-0,147
12	12	-4,57	0,00822	-0,174
12	13	-4,49	0,0181	-0,135
12	14	-4,63	0,0360	-0,161
12	15	-4,55	0,0462	-0,163
12	16	-4,39	0,0570	-0,113
12	17	-4,53	0,0517	-0,146
12	18	-4,51	0,0463	-0,133
12	19	-4,39	0,0469	-0,124
12	20	-4,46	0,0861	-0,120

Table A.5: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1993 to 31 December 1995, i.e., when only Norway participates in the power market. The total number of observations are 1096, which are reduced to 1084 observations when there are 12 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
16	1	-4,44	-0,216	-0,499
16	2	-4,48	-0,0238	-0,161
16	**3**	**-4,66**	**-0,0515**	**-0,287**
16	4	-4,51	-0,0266	-0,185
16	5	-4,70	-0,0341	-0,173
16	6	-4,56	-0,0216	-0,167
16	7	-4,57	0,0115	-0,114
16	8	-4,55	0,0199	-0,112
16	9	-4,32	0,0215	-0,143
16	10	-4,55	0,0259	-0,150
16	11	-4,35	0,0215	-0,146
16	12	-4,38	0,0237	-0,137
16	13	-4,40	0,0258	-0,123
16	14	-4,26	0,0536	-0,105
16	15	-4,49	0,0855	-0,108
16	16	-4,28	0,0709	-0,104
16	17	-4,47	0,0645	-0,103
16	18	-4,22	0,0714	-0,129
16	19	-3,86	0,0549	-0,0988
16	20	-4,40	0,0961	-0,117

Table A.6: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1993 to 31 December 1995, i.e., when only Norway participates in the power market. The total number of observations are 1096, which are reduced to 1080 observations when there are 16 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
20	1	-4,49	-0,0414	-0,144
20	**2**	**-4,60**	**-0,0479**	**-0,153**
20	3	-4,58	-0,00734	-0,124
20	4	-4,43	-0,00970	-0,141
20	5	-4,40	0,0105	-0,114
20	6	-4,50	0,00104	-0,135
20	7	-4,32	0,0394	-0,0977
20	8	-4,26	0,0111	-0,0896
20	9	-4,26	0,0515	-0,110
20	10	-4,51	0,0419	-0,110
20	11	-4,43	0,0353	-0,103
20	12	-4,44	0,0540	-0,0926
20	13	-4,10	0,0425	-0,118
20	14	-4,17	0,0645	-0,0958
20	15	-3,84	0,0437	-0,0728
20	16	-3,94	0,0421	-0,102
20	17	-4,28	0,0998	-0,0883
20	18	-4,21	0,117	-0,102
20	19	-4,27	0,125	-0,0731
20	20	-4,27	0,144	-0,0659

Table A.7: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1993 to 31 December 1995, i.e., when only Norway participates in the power market. The total number of observations are 1096, which are reduced to 1076 observations when there are 20 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
4	1	-5,20	-0,820	-1,06
4	2	-5,15	-0,911	-1,44
4	3	-5,06	-0,364	-0,530
4	4	-5,09	-0,349	-0,448
4	**5**	**-5,10**	**-0,328**	**-0,496**
4	6	-5,05	-0,309	-0,434
4	7	-4,99	-0,146	-0,351
4	8	-4,97	-0,128	-0,287
4	9	-4,95	-0,196	-0,422
4	10	-4,83	-0,261	-0,468
4	11	-4,82	-0,261	-0,538
4	12	-4,91	-0,113	-0,410
4	13	-4,81	-0,153	-0,374
4	14	-4,72	-0,195	-0,384
4	15	-4,64	-0,0892	-0,422
4	16	-4,72	-0,0362	-0,297
4	17	-4,53	-0,201	-0,453
4	18	-4,57	-0,104	-0,301
4	19	-4,39	-0,274	-0,506
4	20	-4,55	-0,0800	-0,356

Table A.8: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1996 to 28 December 1997, i.e., when only Norway and Sweden participate in the power market. The total number of observations are 729, which are reduced to 725 observations when there are 4 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
8	1	-5,38	-0,103	-0,536
8	**2**	**-5,40**	**-0,0623**	**-0,359**
8	3	-5,35	-0,0677	-0,417
8	4	-5,23	-0,0728	-0,376
8	5	-5,32	-0,0240	-0,265
8	6	-5,23	-0,0568	-0,372
8	7	-5,14	-0,0140	-0,284
8	8	-5,16	-0,0377	-0,286
8	9	-5,21	-0,00205	-0,264
8	10	-5,16	0,0102	-0,317
8	11	-5,08	-0,00492	-0,215
8	12	-4,99	0,0189	-0,266
8	13	-4,93	0,0381	-0,230
8	14	-5,00	0,0361	-0,244
8	15	-4,89	0,0383	-0,202
8	16	-4,74	0,0151	-0,199
8	17	-4,86	0,0613	-0,139
8	18	-4,80	0,0739	-0,136
8	19	-4,30	0,0407	-0,198
8	20	-4,74	0,129	-0,108

Table A.9: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1996 to 28 December 1997, i.e., when only Norway and Sweden participate in the power market. The total number of observations are 729, which are reduced to 721 observations when there are 8 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	λ_1	λ
12	1	-5,37	-0,0645	-0,281
12	2	-5,30	-0,0958	-0,306
12	3	-5,15	-0,0562	-0,189
12	**4**	**-5,28**	**-0,0318**	**-0,184**
12	5	-5,24	-0,0256	-0,196
12	6	-5,16	-0,00594	-0,179
12	7	-5,04	0,0199	-0,158
12	8	-4,98	-0,00622	-0,170
12	9	-4,99	0,0362	-0,155
12	10	-4,92	0,0184	-0,137
12	11	-4,90	0,0428	-0,100
12	12	-4,84	0,0509	-0,135
12	13	-4,86	0,0575	-0,128
12	14	-4,72	0,0690	-0,118
12	15	-4,68	0,121	-0,0929
12	16	-4,49	0,102	-0,119
12	17	-4,59	0,116	-0,116
12	18	-4,53	0,121	-0,0896
12	19	-4,36	0,115	-0,0945
12	20	-4,37	0,140	-0,0701

Table A.10: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1996 to 28 December 1997, i.e., when only Norway and Sweden participate in the power market. The total number of observations are 729, which are reduced to 717 observations when there are 12 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
16	1	-5,37	-0,0402	-0,290
16	2	-5,24	-0,0327	-0,245
16	3	-5,23	-0,0173	-0,209
16	4	-5,23	-0,0176	-0,143
16	5	-5,17	-0,00453	-0,231
16	6	-5,05	0,0114	-0,127
16	**7**	**-5,07**	**0,0125**	**-0,147**
16	8	-4,88	0,0114	-0,201
16	9	-4,70	0,0250	-0,121
16	10	-4,82	0,0316	-0,127
16	11	-4,75	0,0564	-0,113
16	12	-4,54	0,0452	-0,140
16	13	-4,31	0,0431	-0,125
16	14	-4,43	0,0685	-0,116
16	15	-4,40	0,0962	-0,0962
16	16	-4,25	0,106	-0,113
16	17	-4,18	0,0866	-0,0852
16	18	-4,14	0,131	-0,0929
16	19	-3,96	0,117	-0,114
16	20	-4,02	0,154	-0,0568

Table A.11: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1996 to 28 December 1997, i.e., when only Norway and Sweden participate in the power market. The total number of observations are 729, which are reduced to 713 observations when there are 16 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda}$
20	**1**	**-5,33**	**-0,0362**	**-0,155**
20	2	-5,30	-0,0315	-0,152
20	3	-5,29	0,0173	-0,173
20	4	-5,10	0,00753	-0,175
20	5	-4,97	0,0135	-0,104
20	6	-4,95	0,0157	-0,114
20	7	-4,78	0,0325	-0,102
20	8	-4,86	0,0381	-0,104
20	9	-4,69	0,0385	-0,0903
20	10	-4,66	0,0498	-0,0898
20	11	-4,60	0,0794	-0,0752
20	12	-4,50	0,0952	-0,0813
20	13	-4,19	0,101	-0,0764
20	14	-3,90	0,105	-0,0580
20	15	-3,81	0,0790	-0,0948
20	16	-3,96	0,119	-0,0655
20	17	-4,23	0,205	-0,0563
20	18	-4,01	0,158	-0,0569
20	19	-3,93	0,200	-0,0479
20	20	-3,64	0,203	-0,0531

Table A.12: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 January 1996 to 28 December 1997, i.e., when only Norway and Sweden participate in the power market. The total number of observations are 729, which are reduced to 709 observations when there are 20 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
4	**1**	**-4,57**	**-0,424**	**-0,592**
4	2	-4,50	-0,461	-0,645
4	3	-4,49	-0,270	-0,744
4	4	-4,52	-0,208	-0,386
4	5	-4,44	-0,213	-0,461
4	6	-4,42	-0,191	-0,433
4	7	-4,27	-0,160	-0,357
4	8	-4,30	-0,139	-0,343
4	9	-4,34	-0,139	-0,354
4	10	-4,21	-0,0811	-0,383
4	11	-4,40	0,0433	-0,299
4	12	-4,30	0,0800	-0,273
4	13	-4,07	-0,103	-0,441
4	14	-4,07	-0,0579	-0,365
4	15	-4,03	-0,0229	-0,343
4	16	-4,10	0,101	-0,237
4	17	-3,80	-0,126	-0,349
4	18	-3,90	0,00954	-0,341
4	19	-3,88	-0,00588	-0,326
4	20	-4,03	0,147	-0,298

Table A.13: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 29 December 1997 to 30 June 1999, i.e., when only Norway, Sweden and Finland participate in the power market. The total number of observations are 550, which are reduced to 546 observations when there are 4 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
8	1	-4,78	-0,0697	-0,988
8	2	-4,78	-0,0907	-0,446
8	3	-4,83	-0,0293	-0,240
8	4	-4,63	-0,0615	-0,255
8	5	-4,79	-0,0365	-0,409
8	6	-4,82	-0,0174	-0,275
8	7	-4,74	-0,0205	-0,282
8	**8**	**-4,84**	**0,0196**	**-0,204**
8	9	-4,65	0,0601	-0,252
8	10	-4,77	0,0678	-0,186
8	11	-4,75	0,0989	-0,262
8	12	-4,46	0,0822	-0,149
8	13	-4,33	0,0573	-0,177
8	14	-4,58	0,0950	-0,176
8	15	-4,50	0,102	-0,202
8	16	-4,37	0,106	-0,148
8	17	-4,51	0,119	-0,119
8	18	-4,41	0,136	-0,147
8	19	-4,03	0,195	-0,123
8	20	-4,35	0,189	-0,125

Table A.14: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 29 December 1997 to 30 June 1999, i.e., when only Norway, Sweden and Finland participate in the power market. The total number of observations are 550, which are reduced to 542 observations when there are 8 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
12	1	-4,79	-0,0404	-0,153
12	2	-4,79	-0,0501	-0,176
12	**3**	**-4,89**	**-0,0421**	**-0,168**
12	4	-4,72	-0,0452	-0,229
12	5	-4,71	0,0499	-0,165
12	6	-4,56	0,00317	-0,142
12	7	-4,42	0,0197	-0,142
12	8	-4,75	0,0517	-0,150
12	9	-4,44	0,0135	-0,128
12	10	-4,60	0,0698	-0,145
12	11	-4,42	0,0937	-0,148
12	12	-4,16	0,0586	-0,125
12	13	-4,07	0,0551	-0,118
12	14	-4,38	0,125	-0,118
12	15	-4,17	0,133	-0,0957
12	16	-4,06	0,139	-0,119
12	17	-4,29	0,195	-0,128
12	18	-4,19	0,224	-0,117
12	19	-3,93	0,217	-0,0957
12	20	-3,95	0,181	-0,0750

Table A.15: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 29 December 1997 to 30 June 1999, i.e., when only Norway, Sweden and Finland participate in the power market. The total number of observations are 550, which are reduced to 538 observations when there are 12 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda}$
16	1	-4,44	-1,15	-2,06
16	2	-4,65	-0,0141	-0,166
16	3	-4,78	-0,0164	-0,194
16	**4**	**-4,79**	**0,00430**	**-0,174**
16	5	-4,73	0,0507	-0,139
16	6	-4,50	0,0319	-0,120
16	7	-4,65	0,0247	-0,130
16	8	-4,42	0,0522	-0,124
16	9	-3,45	0,121	-0,0889
16	10	-4,33	0,0932	-0,0962
16	11	-4,17	0,0831	-0,112
16	12	-4,00	0,0882	-0,0846
16	13	-4,18	0,103	-0,0380
16	14	-3,75	0,117	-0,0946
16	15	-3,92	0,213	-0,0532
16	16	-3,69	0,180	-0,0618
16	17	-4,06	0,213	-0,0567
16	18	-3,57	0,149	-0,0818
16	19	-3,60	0,200	-0,0704
16	20	-3,87	0,257	-0,0392

Table A.16: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 29 December 1997 to 30 June 1999, i.e., when only Norway, Sweden and Finland participate in the power market. The total number of observations are 550, which are reduced to 534 observations when there are 16 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
20	**1**	**-4,77**	**-0,0182**	**-0,198**
20	2	-4,62	-0,0189	-0,138
20	3	-4,59	0,00784	-0,0926
20	4	-4,37	0,00830	-0,182
20	5	-4,39	0,0159	-0,100
20	6	-4,52	0,0376	-0,0715
20	7	-4,29	0,0372	-0,0823
20	8	-4,31	0,0841	-0,104
20	9	-4,27	0,0854	-0,0817
20	10	-4,28	0,142	-0,0704
20	11	-4,18	0,118	-0,0593
20	12	-4,25	0,120	-0,0445
20	13	-3,78	0,126	-0,0614
20	14	-3,77	0,146	-0,0570
20	15	-3,90	0,168	-0,0482
20	16	-3,40	0,127	-0,0674
20	17	-4,11	0,183	-0,0473
20	18	-4,09	0,209	-0,0347
20	19	-4,05	0,296	-0,0220
20	20	-4,41	0,305	-0,0516

Table A.17: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 29 December 1997 to 30 June 1999, i.e., when only Norway, Sweden and Finland participate in the power market. The total number of observations are 550, which are reduced to 530 observations when there are 20 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
4	**1**	**-4,12**	**-0,402**	**-0,470**
4	2	-4,05	-0,464	-0,545
4	3	-4,09	-0,322	-0,498
4	4	-4,06	-0,292	-0,801
4	5	-3,96	-0,252	-0,557
4	6	-3,93	-0,183	-0,424
4	7	-3,92	-0,0493	-0,230
4	8	-3,87	-0,0291	-0,343
4	9	-3,81	-0,00740	-0,281
4	10	-3,67	-0,0295	-0,258
4	11	-3,78	0,0584	-0,294
4	12	-3,66	0,0773	-0,175
4	13	-3,54	-0,0221	-0,367
4	14	-3,45	0,0112	-0,423
4	15	-3,58	0,158	-0,246
4	16	-3,47	0,209	-0,111
4	17	-3,27	0,0601	-0,334
4	18	-3,43	0,231	-0,121
4	19	-3,02	-0,0618	-0,331
4	20	-3,46	0,429	-0,120

Table A.18: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 July 1999 to 30 September 2000, i.e., when only Norway, Sweden, Finland and Western Denmark participate in the power market. The total number of observations are 459, which are reduced to 455 observations when there are 4 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
8	**1**	**-4,23**	**-0,0664**	**-0,273**
8	2	-4,22	-0,0478	-0,217
8	3	-4,14	-0,0148	-0,280
8	4	-3,98	0,0160	-0,389
8	5	-4,06	0,0194	-0,275
8	6	-4,03	0,0151	-0,278
8	7	-4,00	0,0393	-0,129
8	8	-3,83	0,0505	-0,232
8	9	-3,68	0,0898	-0,191
8	10	-3,79	0,173	-0,164
8	11	-3,65	0,129	-0,148
8	12	-3,51	0,0979	-0,146
8	13	-3,41	0,189	-0,177
8	14	-3,33	0,118	-0,106
8	15	-3,90	0,340	-0,0991
8	16	-3,08	0,160	-0,153
8	17	-3,06	0,203	-0,116
8	18	-3,12	0,306	-0,0861
8	19	-2,61	0,196	-0,161
8	20	-3,12	0,523	-0,0640

Table A.19: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 July 1999 to 30 September 2000, i.e., when only Norway, Sweden, Finland and Western Denmark participate in the power market. The total number of observations are 459, which are reduced to 451 observations when there are 8 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
12	**1**	**-4,17**	**-0,0607**	**-0,370**
12	2	-4,05	-0,0283	-0,227
12	3	-4,04	-0,0178	-0,211
12	4	-3,94	0,0274	-0,164
12	5	-3,88	0,0131	-0,183
12	6	-3,74	0,0618	-0,167
12	7	-3,52	0,0464	-0,177
12	8	-3,37	0,0400	-0,148
12	9	-3,37	0,104	-0,0907
12	10	-3,28	0,180	-0,137
12	11	-3,26	0,168	-0,106
12	12	-3,12	0,141	-0,118
12	13	-2,84	0,140	-0,132
12	14	-3,03	0,251	-0,0618
12	15	-2,83	0,272	-0,0485
12	16	-2,79	0,348	-0,0510
12	17	-2,73	0,384	-0,0231
12	18	-2,31	0,250	-0,0613
12	19	-2,99	0,472	-0,0229
12	20	-2,69	0,641	0,00231

Table A.20: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 July 1999 to 30 September 2000, i.e., when only Norway, Sweden, Finland and Western Denmark participate in the power market. The total number of observations are 459, which are reduced to 447 observations when there are 12 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
16	**1**	**-4,12**	**-0,0453**	**-0,208**
16	2	-4,08	-0,00148	-0,169
16	3	-3,95	0,00432	-0,110
16	4	-3,78	0,0267	-0,107
16	5	-3,67	0,0484	-0,120
16	6	-3,49	0,0492	-0,105
16	7	-3,49	0,0937	-0,0816
16	8	-3,17	0,0791	-0,0897
16	9	-3,16	0,170	-0,0662
16	10	-3,01	0,146	-0,0737
16	11	-3,01	0,189	-0,0509
16	12	-2,83	0,223	-0,0387
16	13	-2,94	0,297	-0,0345
16	14	-2,90	0,275	-0,0307
16	15	-2,64	0,413	-0,0146
16	16	-2,65	0,372	-0,0111
16	17	-2,67	0,541	0,00773
16	18	-2,77	0,486	-0,0266
16	19	-2,32	0,518	-0,0167
16	20	-2,95	0,476	0,00571

Table A.21: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 July 1999 to 30 September 2000, i.e., when only Norway, Sweden, Finland and Western Denmark participate in the power market. The total number of observations are 459, which are reduced to 443 observations when there are 16 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
20	**1**	**-4,10**	**-0,0278**	**-0,139**
20	2	-4,05	0,0130	-0,179
20	3	-3,74	-0,00545	-0,111
20	4	-3,62	0,0923	-0,121
20	5	-3,55	0,0388	-0,102
20	6	-3,42	0,0768	-0,0852
20	7	-3,09	0,0637	-0,0915
20	8	-3,01	0,117	-0,0613
20	9	-2,74	0,150	-0,0577
20	10	-2,91	0,176	-0,0759
20	11	-2,55	0,232	-0,0507
20	12	-3,28	0,336	-0,0167
20	13	-1,87	0,188	-0,0602
20	14	-3,15	0,326	-0,0415
20	15	-2,46	0,439	-0,0261
20	16	-2,07	0,429	-0,00846
20	17	-3,89	0,452	-0,0141
20	18	-3,56	0,551	-0,0371
20	19	-2,79	0,413	-0,0128
20	20	3,93	0,142	-0,0332

Table A.22: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 July 1999 to 30 September 2000, i.e., when only Norway, Sweden, Finland and Western Denmark participate in the power market. The total number of observations are 459, which are reduced to 439 observations when there are 20 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda}$
4	1	-4,87	-0,558	-0,618
4	2	-4,83	-0,464	-0,561
4	3	-4,99	-0,403	-0,588
4	4	-4,93	-0,430	-0,585
4	5	-4,93	-0,311	-0,554
4	6	-4,90	-0,405	-0,663
4	7	-4,93	-0,282	-0,453
4	8	-4,90	-0,344	-0,459
4	**9**	**-5,02**	**-0,257**	**-0,556**
4	10	-4,89	-0,215	-0,475
4	11	-4,91	-0,281	-0,501
4	12	-4,93	-0,231	-0,454
4	13	-4,92	-0,188	-0,484
4	14	-4,94	-0,185	-0,449
4	15	-4,94	-0,189	-0,462
4	16	-4,90	-0,131	-0,483
4	17	-4,76	-0,302	-0,480
4	18	-4,77	-0,316	-0,534
4	19	-4,72	-0,314	-0,444
4	20	-4,81	-0,159	-0,446

Table A.23: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 October 2000 to 25 September 2005, i.e., when Norway, Sweden, Finland and Denmark participate in the power market. The total number of observations are 1823, which are reduced to 1829 observations when there are 4 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda}$
8	1	-5,03	-0,0706	-0,286
8	2	-5,06	-0,0775	-0,301
8	3	-5,21	-0,0486	-0,307
8	4	-5,04	-0,0852	-0,383
8	**5**	**-5,29**	**-0,0394**	**-0,317**
8	6	-5,15	-0,0500	-0,328
8	7	-5,10	-0,0464	-0,354
8	8	-5,20	-0,0138	-0,270
8	9	-5,13	-0,0213	-0,273
8	10	-5,21	-0,0186	-0,299
8	11	-5,12	-0,0246	-0,267
8	12	-5,07	-0,0106	-0,248
8	13	-5,07	-0,00681	-0,338
8	14	-5,05	-0,0175	-0,334
8	15	-5,09	-0,00383	-0,282
8	16	-4,97	-0,00833	-0,270
8	17	-5,05	0,00236	-0,218
8	18	-5,04	0,0202	-0,238
8	19	-5,00	0,0486	-0,229
8	20	-5,00	0,0276	-0,247

Table A.24: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 October 2000 to 25 September 2005, i.e., when Norway, Sweden, Finland and Denmark participate in the power market. The total number of observations are 1823, which are reduced to 1815 observations when there are 8 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	**Hidden units**	**SIC**	$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda}$
12	1	-5,02	-0,0538	-0,231
12	2	-5,10	-0,0581	-0,254
12	3	-5,03	-0,0184	-0,197
12	**4**	**-5,15**	**-0,0291**	**-0,199**
12	5	-5,10	-0,0271	-0,224
12	6	-5,07	-0,0354	-0,219
12	7	-5,10	-0,0213	-0,197
12	8	-5,09	-0,0195	-0,200
12	9	-5,07	-0,0191	-0,201
12	10	-5,06	0,00178	-0,181
12	11	-5,01	-0,00206	-0,187
12	12	-4,99	-0,00401	-0,216
12	13	-4,94	-0,00674	-0,241
12	14	-5,01	0,0201	-0,188
12	15	-4,98	0,0368	-0,155
12	16	-4,90	0,0327	-0,153
12	17	-4,95	0,0373	-0,180
12	18	-4,85	0,0322	-0,170
12	19	-4,79	0,0230	-0,162
12	20	-4,83	0,0409	-0,157

Table A.25: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 October 2000 to 25 September 2005, i.e., when Norway, Sweden, Finland and Denmark participate in the power market. The total number of observations are 1823, which are reduced to 1811 observations when there are 12 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}$
16	1	-5,03	-0,0307	-0,186
16	2	-5,06	-0,0104	-0,226
16	3	-5,09	-0,0349	-0,252
16	4	-5,16	-0,0210	-0,204
16	**5**	**-5,16**	**-0,0174**	**-0,187**
16	6	-5,11	-0,00801	-0,154
16	7	-5,14	-0,00247	-0,141
16	8	-4,92	-0,00576	-0,160
16	9	-4,89	0,00873	-0,128
16	10	-5,09	0,0191	-0,149
16	11	-5,01	0,00814	-0,122
16	12	-5,00	0,0246	-0,128
16	13	-5,03	0,0292	-0,140
16	14	-4,88	0,0263	-0,142
16	15	-5,05	0,0463	-0,110
16	16	-4,81	0,0308	-0,124
16	17	-4,99	0,0522	-0,121
16	18	-4,79	0,0488	-0,118
16	19	-4,76	0,0414	-0,106
16	20	-4,72	0,0451	-0,109

Table A.26: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 October 2000 to 25 September 2005, i.e., when Norway, Sweden, Finland and Denmark participate in the power market. The total number of observations are 1823, which are reduced to 1807 observations when there are 16 inputs in the neural network. The bold numbers are when SIC is minimized.

Inputs	Hidden units	SIC	λ_1	λ
20	1	-5,03	-0,0280	-0,122
20	2	-5,14	-0,0378	-0,137
20	3	-5,05	-0,0368	-0,110
20	4	-5,08	-0,0102	-0,128
20	**5**	**-5,17**	**-0,0143**	**-0,134**
20	6	-5,07	-0,00579	-0,140
20	7	-5,03	-0,00609	-0,139
20	8	-5,05	0,00345	-0,114
20	9	-5,01	0,0111	-0,118
20	10	-5,00	0,0114	-0,131
20	11	-5,02	0,0232	-0,122
20	12	-4,95	0,0286	-0,123
20	13	-4,81	0,0178	-0,115
20	14	-4,73	0,0327	-0,116
20	15	-4,86	0,0317	-0,109
20	16	-4,69	0,0415	-0,135
20	17	-4,82	0,0492	-0,114
20	18	-4,76	0,0621	-0,102
20	19	-4,75	0,0590	-0,0920
20	20	-4,47	0,0372	-0,106

Table A.27: Estimates of the largest Lyapunov exponent (λ_1) and the average of the exponents (λ) (the number of significant figures is 3), for the period 1 October 2000 to 25 September 2005, i.e., when Norway, Sweden, Finland and Denmark participate in the power market. The total number of observations are 1823, which are reduced to 1803 observations when there are 20 inputs in the neural network. The bold numbers are when SIC is minimized.