Asset Allocation and Non-Normality of Stock Returns

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Abstract

Stock index returns have higher kurtosis and deviate thus further from normality than individual stock returns. This study suggests to explain this well documented, though hardly noticed phenomenon with the asset alocation process in institutional portfolios. Asset allocators with limited access to value relevant information are forced to condition on past prices, thereby inducing excess kurtosis into stock index returns. For individual stocks this effect is mitigated though, as portfolio managers with better information incorporate values into prices through their trades at security selection level.

1 Introduction

It is well known that the returns of stocks and stock indices, like financial returns in general, are leptokurtic [1]. That is, their probability densities contain excessive mass in the very center and the tails compared to the normal distribution. It is less well known, though widely documented, that stock index returns are even more leptokurtic than those of their constituing stocks [2], thereby deviating even further from the benchmark of a normal distribution.

The difference in kurtosis may be large enough to eliminate the benefits from investing into the index at times when diversification is needed the most. For example Blair et al. (2002) investigate daily returns of the S&P100 index and its constituent stocks from 1983 to 1992. They find a decrease of the variance from the median stock to the index by a factor of three but an index kurtosis about four times as large as for the constituent stocks. While the index lost 23.7% in the famous crash of 19 October 1987, the median loss of the constituent stocks was only 21.5%. So it appears that medium returns get diversified away but extreme returns don't.

From a statistical point of view such "dependence in the tails"Mikosch (2003) is surprising. After all, indexes are just weighted averages of their constituents. So they should reflect more independent news than individual stocks. Consequently, the central limit theorem would lead one to expect index returns which are closer to the normal rather than further away.

To resolve this paradox I suggest to regard stock index levels as carrying less information than the stocks they consist of. Such unfamiliar view is suggested by closer inspection of the investment process in institutional portfolios[3]: Asset allocation between broad asset categories (e.g. stocks vs. bonds) and security selection within those categories are usually performed by different entities. The asset allocator is often an external client or sponsor with little market information who wishes to delegate the investment management to professionals, whereas security selection is usually done by professional portfolio managers in house. Even when both asset allocation and security selection decisions are made in the same investment house, the former are usually done by upper hierarchy levels. These have usually more duties than just making asset allocation decisions, which may prevent them to process valuation relevant information as efficiently as their portfolio managers at security selection level do.

[1] see e.g. Mandelbrot (1963), Fama (1965), Mantegna and Stanley (2000), Lux and Ausloos (2003), Mikosch (2003).

[2] see e.g. Lux (1996), Campbell et al. (1997), Blair et al. (2002). None of these authors, however, draw the readers attention to the apparent difference in kurtosis, nor do they offer an explanation for it.

[3] For an extensive review of the institutional investment process, see e.g. Davis and Steil (2001).

Now assume that inferior information or balance sheet considerations force asset allocators to condition on past prices. This will effectively turn them into positive feedback traders [4] and thereby induce trends and excess kurtosis into market returns, here taken to be the index. Better informed portfolio managers have no chance of mitigating this effect at index level, since the asset allocation decision is binding for them. At individual stock level they can incorporate more information into prices though, by their decision to overweight or underweight certain stocks relative to the index benchmark. This results in lower excess kurtosis for the stocks than for the index.

In order to make this point rigorous, I will simulate the investment process in institutional portfolios both with and without uninformed asset allocation beyond security selection between two risky stocks. Asset allocation is between the stock index and a riskless bond in infinite supply (cash). This will be compared to a balanced fund setup, where portfolio managers decide themselves how they allocate their funds between equity and bonds (cash). I hypothesise that in the asset allocation setup index returns will be more leptokurtic than those of the constituent stocks, similar to what is emprically observed. Furthermore, I expect the discrepancy between stock and index kurtosis to decrease in the balanced fund setup, where positive feedback trading at index level is absent.

The model will use the master equation approach from synergetics originally developed for physical systems by Haken (1983) and extended to the social sciences by Weidlich (1983), (2002). The science of synergetics studies collective phenomena in multi-component systems with interactions between the units of the system. Its best known application to financial markets is by Lux (1998) and the simulation studies by Lux and Marchesi (1999), (2000). This study uses similar methods as their work, with the following exceptions:

1. Lux and Marchesi consider only one risky asset in order to study the univariate time series properties of its returns. I add another risky stock in order to investigate the difference between the return patterns of individual stocks and the index. Furthermore, the introduction of a riskfree asset allows to assess the impact of different asset allocation policies.

2. Lux and Marchesi consider an order based strategy, where traders place orders proportional to the expected profit of their investment. That approach has been criticized by Farmer and Joshi (2002) as it may lead to both unbounded long and short positions. In order to avoid this, I choose a position based strategy with target portfolios rather than orders proportional to expected profits. This has the fur-

[4] See DeLong et al. (1990). For example, portfolio insurance strategies are positive feedback trading strategies.

ther advantage of incorporating both limited wealth and the short selling restrictions institutional investors face.

I follow Lux in using the heterogeneous agent framework, which attempts to explain financial market phenomena by mathematical modelling the interaction of market participants with diverse trading strategies. Early proponents of this approach were Zeeman (1974), Beja and Goldman (1980), Day and Huang (1990) and Chiarella (1992), among others. Since then, a rich literature upon the interaction of heterogenous agents has developed, see Farmer (1999), Delli Gatti et al. (2000), Kirman and Zimmermann (2001), Lux and Marchesi (2002), and Lux and Ausloos (2003) for reviews.

Recently, this literature has also started to tackle the impact of heterogeneous expectations upon more than one risky asset. Westerhoff (2004) suggests chartists trading activity as an explanation for the high correlation of stock returns. Böhm and Wenzelburger (2005) extend the classical capital asset pricing model (CAPM) to a dynamic context with heterogeneous beliefs. Chiarella et al. (2005) extend the concept of adaptive belief systems (predictors of future prices which are periodically updated based upon past forecasting performance) introduced by Brock and Hommes (1998) to multiple risky asset. Similar to my work, they consider the impact of chartists and fundamentalists demand upon the price dynamics of two risky assets. However, while they model traders demand based upon the CAPM, my model takes its starting point from the actual investment process in institutional portfolios. Furthermore, I use a different methodology by applying the master equation approach rather than adaptive belief systems.

2 Methodology

Rather than assuming a representative agent, the synergetics literature regards societies such as the investment community to be composed of a large number of members, who individually adopt different attitudes or "states"of behaviour. In our case the members will be portfolio managers or traders, and their individual states will be fully described by the asset they hold and the investment strategy they choose. The total number of portfolio managers N is assumed to be constant through time.

They hold individually only one of three asset types, either one of two risky stock issues or a bond issue in infinite supply (cash). The logarithmic trading prices of the stocks are denoted by p_1 and p_2, and the logarithm of their intrinsic values by p_{f1} and p_{f2}. Portfolio managers holding stocks may choose one of two investment strategies, chartist or fundamentalist. Chartists hold (do not hold) a stock because its price is appreciating (declining) and they expect the trend to continue. Fundamentalists hold (do not hold) a stock because its trading price is below (above) its fundamental price, to which they expect the trading price to converge in the long run. Shortselling is not allowed.

Denote the number of chartists invested in stock 1 or 2 with n_{c1} resp. n_{c2} and the number of fundamentalists invested in stock 1 or 2 with n_{f1} resp. n_{f2}. Each chartist wishes to invest t_c currency units into her favourate stock, whereas the desired investment exposure of fundamentalists is proportional to the mispricing of the stock they wish to hold. Denoting fundamentalists target exposure per unit mispricing with t_f, the aggregate target exposure in either stock is

$$E_i = n_{ci}t_c + n_{fi}t_f(p_{fi} - p_i), \qquad i = 1, 2. \tag{1}$$

I assume the target exposures t_c and t_f and the fundamental prices p_{f1} and p_{f2} to be constant over the time period considered. Trading demand for the stocks is generated by changes in aggregate exposure due to changes in mispricing or the composition of traders

$$ED_i = \frac{d}{dt}E_i = \dot{n_{ci}}t_c + \dot{n_{fi}}t_f(p_{fi} - p_i) - n_{fi}t_f\dot{p_i}. \tag{2}$$

This contrasts with the setup of Lux, who models excess demand directly by an equation corresponding to (1), thereby implicitely assuming that nonzero trader populations and mispricing alone are sufficient to generate trading. Farmer and Joshi (2002) note that such an approach may be problematic insofar as it may lead to unbounded positions as long as mispricing remains or chartists stay in the market. Infinite holdings contradict the assumption of finite wealth and appear unrealistic from a risk management point of view. They should certainly be avoided when investigating the interplay between institutional investment processes and the statistical properties of stock returns. Assuming overall *investment exposure* rather than *trading demand* to be proportional to trader populations and mispricing answers that critique. A further advantage is that it implicitely takes the shortselling restrictions into account which institutional investors face.

Market clearing ($ED_i = 0$) yields for the logarithmic trading prices of the stocks

$$\dot{p_i} = \frac{1}{n_{fi}}\left(\dot{n_{ci}}\frac{t_c}{t_f} + \dot{n_{fi}}(p_{fi} - p_i)\right), \qquad i = 1, 2. \tag{3}$$

The following points appear worth noting: Fast changes in the composition of traders and large mispricings speed up price changes, whereas large fundamentalist populations slow them down. On the chartist side, the speed of price adjustment depends on the target exposure of chartists relative to fundamentalists. Large chartists exposures speed up price changes whereas large fundamentalists exposures have the opposite effect. Overall, we recover the recurrent theme in the interacting agent literature, that fundamentalists have a stabilizing and trend followers have a destabilizing effect upon prices, from the mere existence of chartists and fundamentalists without having made any specific assumptions yet about how to model changes in the traders populations.

We turn to this issue now. The synergetics literature models interactions between members of the population in terms of Markov chains. That is, for each member of the population it postulates a transition probability to change its state of behaviour, or equivalently to move to another subpopulation, which depends only upon the systems current state. Suppose there are M subpopulations (trader types) $n_1, \ldots, n_i, \ldots, n_j, \ldots, n_M$ and denote the transition probability to move from subpopulation i to subpopulation j as p_{ij}. It is then possible to describe the evolution of expected population sizes through time in terms of first order differential equations, the so called *quasi-meanvalue equations* [5]

$$\dot{n}_i = \sum_{j \neq i}^{M} (n_j p_{ji} - n_i p_{ij}), \qquad i = 1, \ldots, M. \tag{4}$$

The quasi-meanvalue equations have a very intuitive interpretation. They simply state that the expected change in population size n_i consists of expected population inflows from all other states $\sum n_j p_{ji}$ minus all expected population outflows into other states $\sum n_i p_{ij}$. In the following, we shall apply this approach to three models of financial markets and institutional investment bahviour of increasing complexity: two risky stocks without riskfree asset, two risky stocks and a riskfree asset in balanced funds, and two risky stocks and a riskfree asset including seperate asset allocation.

2.1 Two Stocks and No Bond

Consider first a market consisting of no other assets than the two risky stocks. Since all traders must allocate their funds somewhere, we have

$$n_{c1} + n_{c2} + n_{f1} + n_{f2} = N. \tag{5}$$

Our task is now to specify the transition probabilities p_{ij} according to which traders change from one subgroup to the other. As in Lux, it is assumed that traders change their strategy according to the perceived profit of the other strategies compared to their own. Following Lux, the perceived profits or utilities of chartists $C_{1,2}$ and fundamentalists $F_{1,2}$ are modelled as

$$C_i = \dot{p}_i / v, \quad F_i = s|p_{fi} - p_i|, \quad i = 1, 2 \qquad , \tag{6}$$

[5] The quasi-meanvalue equations describe the dynamics of expected values only for unimodal probability distributions of the system. If the system bifurcates into a multimodial probability distribution, the evolution of individual systems is no longer meaningfully described by unconditional expected values, as they lie somewhere between the states of maximal probability. But the quasi-meanvalue equations (4) describe still the most probable dynamics of individual systems depending upon their current states, see Weidlich (2002), Chapter 12.

where $\dot{p}_i$ denotes the price trend in stock i and s is a discount factor, since reversals to the fundamental price are first expected in the future. This contrasts to price trends, which may be cashed in immediately. v denotes the frequency at which agents evaluate their utility per unit time. It must be included in the chartists utility in order to make it independent of arbitrary transformations in the unit time scale.

As in Weidlich and Lux, I will assume that the relative change in probability to switch from one strategy to another is proportional to the difference between the utilities of the respective strategies, i.e.

$$dp_{ij}/p_{ij} = \alpha\, d(U_j - U_i), \qquad i,j = c1, c2, f1, f2, \tag{7}$$

where α measures the strengh of attraction which apparently more profitable strategies exert upon the trader. Inserting the utililities (6) into (7) yields for the transition probabilities between the trader types

$$\begin{aligned} p_{cicj} &= ve^{\alpha(C_j - C_i)}, \quad p_{fifj} = ve^{\alpha(F_j - F_i)}, \qquad i,j = 1,2,\ i \neq j \\ p_{cifj} &= ve^{\alpha(F_j - C_i)}, \quad p_{ficj} = ve^{\alpha(C_j - F_i)}, \qquad i,j = 1,2 \quad , \end{aligned} \tag{8}$$

where p_{cicj} and p_{fifj} denote transitions from stock i to stock j within the chartist resp. the fundamentalist subgroup, and p_{cifj} and p_{ficj} denote transitions from chartists to fundamentalists and vice versa. [6] We may now insert these transition rates into the quasi-meanvalue equations (4) in order to obtain the following equations of motion for the expected values of the trader populations:

$$\begin{aligned} \dot{n_{c1}} = v \cdot [\quad & n_{c2}e^{\alpha(C_1 - C_2)} - n_{c1}e^{\alpha(C_2 - C_1)} \\ & + n_{f1}e^{\alpha(C_1 - F_1)} - n_{c1}e^{\alpha(F_1 - C_1)} \\ & + n_{f2}e^{\alpha(C_1 - F_2)} - n_{c1}e^{\alpha(F_2 - C_1)}] \end{aligned} \tag{9a}$$

$$\begin{aligned} \dot{n_{c2}} = v \cdot [\quad & n_{c1}e^{\alpha(C_2 - C_1)} - n_{c2}e^{\alpha(C_1 - C_2)} \\ & + n_{f1}e^{\alpha(C_2 - F_1)} - n_{c2}e^{\alpha(F_1 - C_2)} \\ & + n_{f2}e^{\alpha(C_2 - F_2)} - n_{c2}e^{\alpha(F_2 - C_2)}] \end{aligned} \tag{9b}$$

$$\begin{aligned} \dot{n_{f1}} = v \cdot [\quad & n_{c1}e^{\alpha(F_1 - C_1)} - n_{f1}e^{\alpha(C_1 - F_1)} \\ & + n_{c2}e^{\alpha(F_1 - C_2)} - n_{f1}e^{\alpha(C_2 - F_1)} \\ & + n_{f2}e^{\alpha(F_1 - F_2)} - n_{f1}e^{\alpha(F_2 - F_1)}] \end{aligned} \tag{9c}$$

[6] This frequently chosen formulation of transition rates as exponential functions does not contradict the requirement that the sum of all transition rates into other states may not exceed one, since this may always be achieved by choosing a suffiently small time unit.

$$
\begin{aligned}
\dot{n_{f2}} = v\cdot[\quad & n_{c1}e^{\alpha(F_2-C_1)} - n_{f2}e^{\alpha(C_1-F_2)} \\
& + n_{c2}e^{\alpha(F_2-C_2)} - n_{f2}e^{\alpha(C_2-F_2)} \\
& + n_{f1}e^{\alpha(F_2-F_1)} - n_{f2}e^{\alpha(F_1-F_2)}]
\end{aligned} \tag{9d}
$$

2.2 Balanced Fund Setup

Assume now that a riskfree bond is available and denote the number of traders invested in the bond with n_B, such that

$$
n_B + n_{c1} + n_{c2} + n_{f1} + n_{f2} = N. \tag{10}
$$

For the sake of mathematical tractability the bond is assumed to pay no interests (cash). Compared to the setup without a riskfree asset, there are eight additional transition probabilities to determine, four transitions from the bond to each of the four stock investors and four transitions back from the stock to the bond investors. Since the bond pays no interest, its utility is zero. Letting portfolio managers decide themselves whether or not to invest in bonds yields for the additional transition probabilities between bond and the four kind of stock investors

$$
\begin{aligned}
p_{Bci} = ve^{\alpha_B C_i}, \quad p_{ciB} = ve^{-\alpha_B C_i}, \\
p_{Bfi} = ve^{\alpha_B F_i}, \quad p_{fiB} = ve^{-\alpha_B F_i},
\end{aligned} \qquad i = 1, 2 \tag{11}
$$

where p_{Bci} (p_{ciB}) denote transition rates to (from) chartists, and p_{Bfi} (p_{fiB}) denote transitions to (from) fundamentalists. The transition rates within the stock investor groups (8) remain unchanged. The strength of attraction parameter between stocks and bonds α_B will in general differ from the strength of attraction parameter α within stocks. This is the case because the asset allocation between stocks and bonds, here performed by portfolio managers themselves as in balanced funds, involves a decision between assets of different risk, whereas security selection within stocks is between assets of comparable risk. Inserting into the quasi-meanvalue equations (4) yields for the population dynamics similar to (9):

$$
\begin{aligned}
\dot{n_{c1}} =& v \cdot [(n_B e^{\alpha_B C_1} - n_{c1}e^{-\alpha_B C_1}) + (n_{c2}e^{\alpha(C_1-C_2)} - n_{c1}e^{\alpha(C_2-C_1)}) \\
&+ (n_{f1}e^{\alpha(C_1-F_1)} - n_{c1}e^{\alpha(F_1-C_1)}) + (n_{f2}e^{\alpha(C_1-F_2)} - n_{c1}e^{\alpha(F_2-C_1)})]
\end{aligned} \tag{12a}
$$

$$
\begin{aligned}
\dot{n_{c2}} =& v \cdot [(n_B e^{\alpha_B C_2} - n_{c2}e^{-\alpha_B C_2}) + (n_{c1}e^{\alpha(C_2-C_1)} - n_{c2}e^{\alpha(C_1-C_2)}) \\
&+ (n_{f1}e^{\alpha(C_2-F_1)} - n_{c2}e^{\alpha(F_1-C_2)}) + (n_{f2}e^{\alpha(C_2-F_2)} - n_{c2}e^{\alpha(F_2-C_2)})]
\end{aligned} \tag{12b}
$$

$$
\begin{aligned}
\dot{n_{f1}} =& v \cdot [(n_B e^{\alpha_B F_1} - n_{f1}e^{-\alpha_B F_1}) + (n_{c1}e^{\alpha(F_1-C_1)} - n_{f1}e^{\alpha(C_1-F_1)}) \\
&+ (n_{c2}e^{\alpha(F_1-C_2)} - n_{f1}e^{\alpha(C_2-F_1)}) + (n_{f2}e^{\alpha(F_1-F_2)} - n_{f1}e^{\alpha(F_2-F_1)})]
\end{aligned} \tag{12c}
$$

$$\dot{n_{f2}} = v \cdot [(n_B e^{\alpha_B F_2} - n_{f2} e^{-\alpha_B F_2}) + (n_{c1} e^{\alpha(F_2 - C_1)} - n_{f2} e^{\alpha(C_1 - F_2)}) + (n_{c2} e^{\alpha(F_2 - C_2)} - n_{f2} e^{\alpha(C_2 - F_2)}) + (n_{f1} e^{\alpha(F_2 - F_1)} - n_{f2} e^{\alpha(F_1 - F_2)})] \tag{12d}$$

2.3 Asset Allocation Setup

Suppose next that asset allocation and securtity selection are performed by seperate entities, as depicted in figure 1.

[Insert Figure 1 about here]

The asset allocation decision splits portfolio managers into a bond investor and an equity investor group, denoted by n_E:

$$n_E = n_{c1} + n_{c2} + n_{f1} + n_{f2}, \qquad n_B + n_E = N. \tag{13}$$

Traders in the equity investor group choose their stock and investment strategy in the same way as before using the transition rates in (8). However, the asset allocation decision whether to invest in bonds or stocks is no longer in their hands as in the balanced fund setup, but has been transferred to a seperate entity, which we shall call the sponsor. Simultaneously with portfolio managers reconsidering their stock selection decision, sponsors decide how many traders to put on the equity and the bond investor side. This will again be modelled using transition rates denoted by p_{BE} from bond to equity investors and by p_{EB} from equity to bond investors.

As our goal is to model uninformed asset allocators, let us assume that the sponsor has no further information available than current and past stock index levels $(p_1 + p_2)/2$. It is then reasonable to model the sponsors utility I of an equity investment as the index return

$$I = \frac{\dot{p_1} + \dot{p_2}}{2v}, \tag{14}$$

where the speed of adjustment parameter v has again been included in order to make that utility independent of arbitrary changes in the time unit. The resulting transition rates between equities and bonds read then

$$p_{BE} = v e^{\alpha_B (\dot{p_1} + \dot{p_2})/v} \qquad \text{and} \qquad p_{EB} = v e^{-\alpha_B (\dot{p_1} + \dot{p_2})/v}, \tag{15}$$

where the factor 1/2 has been included in the strength of attraction parameter α_B. In the next step we need to specify, how the transitions between equity and bonds on asset allocation level translate into transition probabilities between the individual stock investors and the bondholders. I will assume here, that the asset allocation decision leaves the internal composition of stock investors unchanged. That is, the transition rates from each individual stock investor to bondholders equal just the transition rates between equity and bonds

$$p_{ciB} = p_{fiB} = p_{EB}, \qquad i = 1, 2 \quad , \tag{16}$$

whereas transitions from the bondholders to the equity investors must be weighted by the relative frequency of the relevant stock investor type

$$p_{Bci} = \frac{n_{ci}}{n_E} p_{BE}, \qquad p_{Bfi} = \frac{n_{fi}}{n_E} p_{BE}, \qquad i = 1, 2. \tag{17}$$

These may then again be inserted into the quasi-meanvalue equations (4) in order to obtain for the population dynamics in analogy to (9) and (12):

$$\begin{aligned} \dot{n_{c1}} =& v \cdot \Bigg[n_{c1} \left(\frac{n_B}{n_E} e^{\alpha_B (\dot{p_1} + \dot{p_2})/v} - e^{-\alpha_B (\dot{p_1} + \dot{p_2})/v} \right) \\ & + n_{c2} e^{\alpha(C_1 - C_2)} - n_{c1} e^{\alpha(C_2 - C_1)} \\ & + n_{f1} e^{\alpha(C_1 - F_1)} - n_{c1} e^{\alpha(F_1 - C_1)} \\ & + n_{f2} e^{\alpha(C_1 - F_2)} - n_{c1} e^{\alpha(F_2 - C_1)} \Bigg] \end{aligned} \tag{18a}$$

$$\begin{aligned} \dot{n_{c2}} =& v \cdot \Bigg[n_{c2} \left(\frac{n_B}{n_E} e^{\alpha_B (\dot{p_1} + \dot{p_2})/v} - e^{-\alpha_B (\dot{p_1} + \dot{p_2})/v} \right) \\ & + n_{c1} e^{\alpha(C_2 - C_1)} - n_{c2} e^{\alpha(C_1 - C_2)} \\ & + n_{f1} e^{\alpha(C_2 - F_1)} - n_{c2} e^{\alpha(F_1 - C_2)} \\ & + n_{f2} e^{\alpha(C_2 - F_2)} - n_{c2} e^{\alpha(F_2 - C_2)} \Bigg] \end{aligned} \tag{18b}$$

$$\begin{aligned} \dot{n_{f1}} =& v \cdot \Bigg[n_{f1} \left(\frac{n_B}{n_E} e^{\alpha_B (\dot{p_1} + \dot{p_2})/v} - e^{-\alpha_B (\dot{p_1} + \dot{p_2})/v} \right) \\ & + n_{c1} e^{\alpha(F_1 - C_1)} - n_{f1} e^{\alpha(C_1 - F_1)} \\ & + n_{c2} e^{\alpha(F_1 - C_2)} - n_{f1} e^{\alpha(C_2 - F_1)} \\ & + n_{f2} e^{\alpha(F_1 - F_2)} - n_{f1} e^{\alpha(F_2 - F_1)} \Bigg] \end{aligned} \tag{18c}$$

$$\begin{aligned} \dot{n_{f2}} =& v \cdot \Bigg[n_{f2} \left(\frac{n_B}{n_E} e^{\alpha_B (\dot{p_1} + \dot{p_2})/v} - e^{-\alpha_B (\dot{p_1} + \dot{p_2})/v} \right) \\ & + n_{c1} e^{\alpha(F_2 - C_1)} - n_{f2} e^{\alpha(C_1 - F_2)} \\ & + n_{c2} e^{\alpha(F_2 - C_2)} - n_{f2} e^{\alpha(C_2 - F_2)} \\ & + n_{f1} e^{\alpha(F_2 - F_1)} - n_{f2} e^{\alpha(F_1 - F_2)} \Bigg] \end{aligned} \tag{18d}$$

3 Theoretical Results

Combining the time development of the assets prices (3) with either one of the population dynamics (9) for a market without riskfree asset, (12) for the balanced fund setup, and (18) for the asset allocation setup, one obtains three closed systems of highly non-linear differential equations with state variables p_1, p_2, n_{c1}, n_{c2}, n_{f1} and n_{f2}.

It turns out that all three systems have "fundemantal" equilibria, in which the trading prices of both assets equal their respective fundamental values ($p_i = p_{fi}$, $i = 1, 2$) with further requirements upon the composition of traders detailed below.

Proposition 1. *Fundamental Equilibria.*

1. *The market without riskfree asset has a fundamental equilibrium at*

$$n_{c1} = n_{c2} = n_{f1} = n_{f2} = N/4.$$

2. *The market consisting of balanced funds has a fundamental equilibrium at*

$$n_B = n_{c1} = n_{c2} = n_{f1} = n_{f2} = N/5.$$

3. *The market with seperate asset allocation has a fundamental equilibrium at*

$$n_B = n_E = N/2, \qquad n_{c1} = n_{c2} = n_{f1} = n_{f2} = n_E/4.$$

Proof. See appendix. □

Intuitively, the equilibrium conditions follow quite naturally from the structure of the quasi-meanvalue equations (4) as follows. At fundamental equilibrium all fundamentalist and chartist utilities equal zero, because all trading prices equal their fundamental value and there is no price trend. All transition probabilities in the quasi-meanvalue equations equal therefore one, such that (4) simplifies to

$$\dot{n}_i = \sum_{j \neq i}^{M} (n_j - n_i), \qquad i = 1, \ldots, M. \tag{19}$$

It is then immediately clear from (19) that zero expected changes for all trader populations imply that there are equally many traders in every population. As regards the asset allocation setup one has to keep in mind that there are two subdynamics, one between bonds and stocks, and one within stocks, for each of which equations of the form (19) hold.

The stability of the fundamental equilibria depends only upon the strength of attraction parameters α and α_B, and on the target exposure of chartists relative to fundamentalits, t_c/t_f, as detailed below.

Proposition 2. *Stability of fundamental equilibria.*

1. *Stability of the fundamental equilibrium in the market without riskfree asset requires*
$$\alpha\frac{t_c}{t_f} < \frac{1}{8}.$$
2. *Stability of the fundamental equilibrium in the market consisting of balanced funds requires*
$$(\alpha_B + 4\alpha)\frac{t_c}{t_f} < \frac{1}{2}.$$
3. *Stability of the fundamental equilibrium in the market with seperate asset allocation requires*
$$(\alpha_B + \alpha)\frac{t_c}{t_f} < \frac{1}{4}.$$

Proof. See appendix. □

The conditions for stability of the fundamental equilibria above conform with intuition. Large strength of attraction parameters imply that small deviations from equilibrium trigger fast changes in the trader populations, leading to fast price changes as well. Large target exposures of chartists relative to fundamentalists speed up price changes as was already mentioned in the discussion of (3).

It is also reasonable that stability depends only upon the ratio of chartists to fundamentalist target exposures t_c/t_f, since otherwise the dynamics would depend upon arbitrary choices of the currency unit. Similarly the speed of adjustment parameter v does not appear in the stability conditions, because it would introduce a dependence upon the arbitrary time unit. An inspection of the proof for the stability conditions reveals that the discount factor s is only absent because economically it is confined to values less than one. If that was not the case, large enough discount factors could lead to instability due to "exaggerated"price corrections by fundamentalists.

4 Simulation Results

Having established the existence and stability conditions for fundamental equilibria, the goal is to compare the statistical properties—in particular the kurtosis— of the individual stock returns with those of the index in the three market setups. That will be done using computer simulations similar to the work by Lux and Marchesi. In order to illustrate the appraoch, this section will contain simulation results applied to their work (Lux (1998), Lux and Marchesi (1999), (2000)).

[Insert Figure 2 about here]

Lux and Marchesi consider only one asset as depicted in figure 2. Their model consists of n_c chartists and n_f fundamentalists, where the chartists may be further subdivided into n_+ bullish and n_- bearish speculators. As in our model, the number of the three kind of traders evolves through time according to the perceived investment returns (utilities) of the respective groups, following the quasi-meanvalue equations (4).

Lux and Marchesi use nominal rather than logarithmic prices. The utility of their fundamentalists is therefore $s|(p_f - p)/p|$, where s is the discount factor, p_f is the fundamental, and p the trading price of the asset. Bullish chartists, who invest into the risky security, receive its nominal dividend r and the price change $\dot{p}$, but forego the average rate of return of the economy R, such that their utility is $(r + \dot{p})/p - R$. Bearish chartists, who short the risky asset in order to invest into the overall economy receive $R - (r + \dot{p})/p$. The transition rates from fundamentalists to the two kind of chartists $p_{f+/-}$ and vice versa $p_{+/-f}$ are governed by the differences in utility for the respective groups:[7]

$$\begin{aligned} p_{f+} &\propto \exp\left[\left(\tfrac{r+\dot{p}}{p} - R\right) - s\left|\tfrac{p_f - p}{p}\right|\right], \quad p_{+f} \propto \exp\left[s\left|\tfrac{p_f - p}{p}\right| - \left(\tfrac{r+\dot{p}}{p} - R\right)\right], \\ p_{f-} &\propto \exp\left[\left(R - \tfrac{r+\dot{p}}{p}\right) - s\left|\tfrac{p_f - p}{p}\right|\right], \quad p_{-f} \propto \exp\left[s\left|\tfrac{p_f - p}{p}\right| - \left(R - \tfrac{r+\dot{p}}{p}\right)\right]. \end{aligned} \tag{20}$$

Strategy shifts within the chartist subgroup are assumed to be a function of both the prevailing opinion as measured by the difference between the two kind of chartists $(n_+ - n_-)/n_c$ (herding) and the price change $\dot{p}$ (chartism). The transition rates read then

$$p_{-+} \propto \exp\left(\frac{n_+ - n_-}{n_c} + \dot{p}\right), \qquad p_{+-} \propto \exp\left(\frac{n_- - n_+}{n_c} - \dot{p}\right). \tag{21}$$

Lux and Marchesi assume excess demand rather than target portfolios proportional to trader populations and mispricing

$$ED = n_c t_c + n_f t_f (p_f - p), \tag{22}$$

where t_c and t_f denote funds to *reallocate* in each evaluation round of the utility differences, rather than target positions to *hold*. This may lead to the beforementioned buildup of infinitely large positions in Lux' model, which is avoided in my setup. Furthermore, Lux and Marchesi assume price adjustment by a market maker ($\dot{p} \propto ED$), whereas I assume instant market clearing ($ED_i = 0$).

[Insert Figure 3 about here]

[7]Strength of attraction and scaling parameters will in the following be omitted for readability. For a detailed description of the model, see Lux (1998), Lux and Marchesi (1999), (2000).

Figure 3 shows a simulation of both fundamental and trading price over 120000 time steps. The fundamental value of the stock follows geometric Brownian motion with the same parameters as in Lux and Marchesi (1999). The trading price tracks fundamental value so closely that it is hard to spot any difference. The picture changes however when returns rather than prices are considered.

[Insert Figure 4 about here]

Figure 4 shows the logreturns generated from the price series in figure 3. The lognormally distributed increments in fundamental value are *not* reflected by similarly distributed trading returns. Instead the white noise underlying the changes in fundamental value has been transformed into a highly heteroscedastic return series with clusters of volatility similar to what is empirically observed in financial markets.

A closer inspection of the model reveals that the bursts of volatility are related to the number of chartists. With many chartists in the market, small increments in fundamental value get quickly reinforced to solid trends until a wide discrepency between fundamental and trading prices causes fundamentalists to trigger a sharp reversal. This again creates a new trend into the opposite direction which may be further reinforced if too many traders have converted to the chartist strategy.

[Insert Figure 5 about here]

Figure 5 illustrates how extreme returns may be traced back to large chartist populations. The stability analysis of Lux' model shows that there is a critical size of the chartist population above which the fundamental equilibrium becomes unstable. Indeed, when the fraction of chartists within the traders population exceeded the critical line indicated in figure 5, its value had to be reset just below the critical line in order to avoid the breakdown of simulations.

By an application of Jensens inequality, the transformation of the lognormally distributed price increments into heteroscedastic trading returns results in excess kurtosis of the transformed relative to the original process [8]. That is also the case here, as depicted in Figure 6.

[Insert Figure 6 about here]

It shows the tail probabilities of standardized trading returns at different levels of time aggregation. The positive and negative tail have been merged by using absolute returns. For comparison, the solid line shows the half-normal distribution on which the absolute scaled increments of the fundamental price would collapse at all levels of time aggregation. It is seen

[8] see Bollerslev et al. (1994).

that trading returns have clearly more mass in the tails than fundamental returns, as expected. For increasing levels of time aggregation, however, the tail probabilities of the trading returns approach those of the normal distribution, as it is also empirically observed in financial markets. The reason is that by decreasing the sampling frequency intermediate bubbles and crashes may already have burst, implying that the corresponding large returns do not appear in the sample but average out with other returns in the unobserved period.

For the highest observation frequencies, the tail probabilities obey approximate power-law scaling. Performing a log-log regression on the 30% largest observations the estimated slope is -2.57 ± 0.101 at unit time steps. This is similar to the value found by Lux (-2.64 ± 0.077) and to those of various financial returns at daily frequencies. [9]

Figures 7 and 8 show the estimation of the self similarity parameter H for raw (fig. 7) and absolute returns (fig. 8) derived from the fundamental and trading prices, using detrended fluctuation analysis as introduced by Peng et al. (1994). The average fluctuation of self-similar processes around their local trend during the interval t, $<F(t)>$ (the standard deviation of the logarithmic price, in our case) is expected to scale as a power law: $<F(t)> \propto t^H$ from which the Hurst exponent H can be extracted by performing a regression in log-coordinates.

[Insert Figure 7 about here]
[Insert Figure 8 about here]

As is shown in figure 7, the Hurst exponents of the raw returns for both fundamental $H = 0.47 \pm 0.09$ and trading prices $H = 0.45 \pm 0.08$ are statistically indistinguishable from the theoretical value for white noise processes $H = 0.5$. There is thus no persistence in either of the series.

The picture changes when considering absolute returns as estimators of volatility in figure 8. While fundamental volatility with a self similarity parameter of $H = 0.53 \pm 0.10$ shows no sign of long memory, the volatility of trading returns shows strong persistence with $H = 0.83 \pm 0.24$. These findings are in harmony with both the results by Lux and empirical observations in financial markets [10].

Lux and Marchesi (2000) document then how the main characteristics of the models trading returns: volatility clustering, fat tails, and long range dependence, remain conserved even when the fundamental price is assumed constant. This time long range dependence is investigated by considering the fractional differencing parameter d which is related to the self-similarity parameter H through the relation $d = H - 1/2$. Therefore, $d > 0$ indicates long range dependence. Lux and Marchesi simulate 20000 trading

[9] see e.g. Jansen and de Vries (1991), Loretan and Phillips (1994), Longin (1996).
[10] see e.g. Crato and de Lima (1994), Lobato and Savin (1996)

returns for each of four parameter sets within the stable regime of the fundamental equilibrium. These are then split into 10 samples of 2000 observtions each, in which the fractional differencing parameter d is obtained from log-periodogram regressions following the procedure by Geweke and Porter-Hudak (1983).

[Insert Table 1 about here]

Table 1 shows the estimates of d for squared and absolute returns as volatility estimators of two such simulations together with the results by Lux and Marchesi. The estimates of d are always positive, often significantly so, indicating long range dependence in return volatility. Absence of long memory is also more often rejected for absolute than for squared returns, which is in harmony with empirical findings in financial markets [11].

[Insert Figure 9 about here]

Long memory is also evident from the plot of the autocorrelation functions for raw, squared and absolute returns, here for simulation 2 of parameter set IV (the other plots look similar). Slow declines in the autocorrelation of volatility measures as shown here for squared and in particular absolute returns are fingerprints of long memory processes and have similarly been observed in financial markets [12]. The trading returns themselves, on the other hand, do not display significant autocorrelations over extended time periods, which is also in harmony with empirical observations [13]. The observed pattern is consistent with multiscaling, a concept to describe long memory of varying degree for different powers of random processes introduced by Mandelbrot et al. (1997).

[Insert Table 2 about here]

Table 2 contains kurtosis and tail index estimates for the trading returns series of the four parameter sets obtained from the estimator proposed by Hill (1975). All time series are heavily leptokurtic. The tail index which measures the asymptotic decay in the probability of large returns, hoovers in a range between 2 and 4, implying the existence of the variance, but not the kurtosis of a stationary process with the same tail index as the series investigated here. Sample estimates of the variance do therefore converge to a finite value, but sample estimates of kurtosis don't, which is shown as an example for one simulation of parameter set IV in figure 10. Temporary increases in the variance following return shocks level off after a while, which stands in contrast to the permanent increases in the level of kurtosis

[11] see e.g. Ding et al. (1993), Mills (1997).
[12] see e.g. Beran (1994), Ding et al. (1993).
[13] see e.g. Fama (1970).

following extreme returns. This is also the explanation for the occasional discrepancies in the empirical kurtosis estimates for the different simulation runs in table 2. Kurtosis estimates may differ considerably depending upon the frequency and intensity of extreme returns which happened to be in the sample, as is expected for tail indexes smaller than four.

[Insert Figure 10 about here]

5 Conclusion

I have suggested to explain the discrepancy in kurtosis between indexes and stocks with the investment process in institutional portfolios. This is certainly not the only mechanism at work. For example, large capitalization stocks may be expected to show the fastest reaction to major economic shocks due to their higher liquidity and representation in institutional benchmark indexes. As a result, kurtosis in capitalization weighted indexes may increase compared to the median constituent stock. As another example, an efficient pricing story (Fama (1970)) would claim that major shocks tend to affect the majority of stocks simultaneously, which would also increase index kurtosis relative to those of individual stocks. However, none of these stories suffice as an explanation, since higher index kurtosis is present in equally weighted indexes as well (Campbell et al. (1997)), and the largest index movements occur with little or no news (Cutler et al. (1989)).

Futures on major stock indexes may play a role, as they allow the fastest movements in and out of rapidly moving markets. On the other hand, such contracts are regularly hedged in the spot market, and it is hard to imagine effects of uncovered positions on return periods of a full trading day and above.

Whatever the reason may be, the topic deserves further investigation. Understanding the interaction between the institutional investment process and the likelihood of bubbles and crashes in the markets is essential both for financial market regulation purposes and from a risk management point of view.

Appendix

A Proof of Proposition 1

A fundamental equilibrium requires

$$\dot{n_{c1}} = \dot{n_{c2}} = \dot{n_{f1}} = \dot{n_{f2}} = 0 \quad \text{at} \quad p_1 \equiv p_{f1} \text{ and } p_2 \equiv p_{f2}. \tag{A.1}$$

Consider first the market without riskfree asset. Using the identity

$$\begin{aligned} & n_j e^{\alpha(U_i-U_j)} - n_i e^{\alpha(U_j-U_i)} \\ =& (n_i+n_j)\cdot\left[\tanh(\alpha(U_i-U_j)) - \frac{n_i-n_j}{n_i+n_j}\right]\cosh(\alpha(U_i-U_j)) \end{aligned} \tag{A.2}$$

the equations of motion for the trader populations in the market without riskfree asset (9) may be rewritten in terms of hyperbolic funcions as

$$\begin{aligned} \dot{n_{c1}} = v\cdot\Bigg\{ & (n_{c1}+n_{c2})\left[\tanh(\alpha(C_1-C_2)) - \frac{n_{c1}-n_{c2}}{n_{c1}+n_{c2}}\right]\cosh(\alpha(C_1-C_2)) \\ & + (n_{c1}+n_{f1})\left[\tanh(\alpha(C_1-F_1)) - \frac{n_{c1}-n_{f1}}{n_{c1}+n_{f1}}\right]\cosh(\alpha(C_1-F_1)) \\ & + (n_{c1}+n_{f2})\left[\tanh(\alpha(C_1-F_2)) - \frac{n_{c1}-n_{f2}}{n_{c1}+n_{f2}}\right]\cosh(\alpha(C_1-F_2))\Bigg\} \\ \dot{n_{c2}} = v\cdot\Bigg\{ & (n_{c2}+n_{c1})\left[\tanh(\alpha(C_2-C_1)) - \frac{n_{c2}-n_{c1}}{n_{c2}+n_{c1}}\right]\cosh(\alpha(C_2-C_1)) \\ & + (n_{c2}+n_{f1})\left[\tanh(\alpha(C_2-F_1)) - \frac{n_{c2}-n_{f1}}{n_{c2}+n_{f1}}\right]\cosh(\alpha(C_2-F_1)) \\ & + (n_{c2}+n_{f2})\left[\tanh(\alpha(C_2-F_2)) - \frac{n_{c2}-n_{f2}}{n_{c2}+n_{f2}}\right]\cosh(\alpha(C_2-F_2))\Bigg\} \\ \dot{n_{f1}} = v\cdot\Bigg\{ & (n_{f1}+n_{c1})\left[\tanh(\alpha(F_1-C_1)) - \frac{n_{f1}-n_{c1}}{n_{f1}+n_{c1}}\right]\cosh(\alpha(F_1-C_1)) \\ & + (n_{f1}+n_{c2})\left[\tanh(\alpha(F_1-C_2)) - \frac{n_{f1}-n_{c2}}{n_{f1}+n_{c2}}\right]\cosh(\alpha(F_1-C_2)) \\ & + (n_{f1}+n_{f2})\left[\tanh(\alpha(F_1-F_2)) - \frac{n_{f1}-n_{f2}}{n_{f1}+n_{f2}}\right]\cosh(\alpha(F_1-F_2))\Bigg\} \\ \dot{n_{f2}} = v\cdot\Bigg\{ & (n_{f2}+n_{c1})\left[\tanh(\alpha(F_2-C_1)) - \frac{n_{f2}-n_{c1}}{n_{f2}+n_{c1}}\right]\cosh(\alpha(F_2-C_1)) \\ & + (n_{f2}+n_{c2})\left[\tanh(\alpha(F_2-C_2)) - \frac{n_{f2}-n_{c2}}{n_{f2}+n_{c2}}\right]\cosh(\alpha(F_2-C_2)) \\ & + (n_{f2}+n_{f1})\left[\tanh(\alpha(F_2-F_1)) - \frac{n_{f2}-n_{f1}}{n_{f2}+n_{f1}}\right]\cosh(\alpha(F_2-F_1))\Bigg\} \end{aligned} \tag{A.3}$$

In order to fulfil the condition (A.1) it suffices that all squared brackets above equal zero. That is the case for $n_{c1} = n_{c2} = n_{f1} = n_{f2} = N/4$, as claimed.

Similarly the equations of motion in the balanced fund setup (12) may be written as

$$
\begin{aligned}
\dot{n_{c1}} = v\cdot \Bigg\{ &(n_{c1}+n_B)\left[\tanh(\alpha_B C_1) - \frac{n_{c1}-n_B}{n_{c1}+n_B}\right]\cosh(\alpha_B C_1) \\
&+ (n_{c1}+n_{c2})\left[\tanh(\alpha(C_1-C_2)) - \frac{n_{c1}-n_{c2}}{n_{c1}+n_{c2}}\right]\cosh(\alpha(C_1-C_2)) \\
&+ (n_{c1}+n_{f1})\left[\tanh(\alpha(C_1-F_1)) - \frac{n_{c1}-n_{f1}}{n_{c1}+n_{f1}}\right]\cosh(\alpha(C_1-F_1)) \\
&+(n_{c1}+n_{f2})\left[\tanh(\alpha(C_1-F_2)) - \frac{n_{c1}-n_{f2}}{n_{c1}+n_{f2}}\right]\cosh(\alpha(C_1-F_2)) \Bigg\} \\
\dot{n_{c2}} = v\cdot \Bigg\{ &(n_{c2}+n_B)\left[\tanh(\alpha_B C_2) - \frac{n_{c2}-n_B}{n_{c2}+n_B}\right]\cosh(\alpha_B C_2) \\
&+ (n_{c2}+n_{c1})\left[\tanh(\alpha(C_2-C_1)) - \frac{n_{c2}-n_{c1}}{n_{c2}+n_{c1}}\right]\cosh(\alpha(C_2-C_1)) \\
&+ (n_{c2}+n_{f1})\left[\tanh(\alpha(C_2-F_1)) - \frac{n_{c2}-n_{f1}}{n_{c2}+n_{f1}}\right]\cosh(\alpha(C_2-F_1)) \\
&+(n_{c2}+n_{f2})\left[\tanh(\alpha(C_2-F_2)) - \frac{n_{c2}-n_{f2}}{n_{c2}+n_{f2}}\right]\cosh(\alpha(C_2-F_2)) \Bigg\} \\
\dot{n_{f1}} = v\cdot \Bigg\{ &(n_{f1}+n_B)\left[\tanh(\alpha_B F_1) - \frac{n_{f1}-n_B}{n_{f1}+n_B}\right]\cosh(\alpha_B F_1) \\
&+ (n_{f1}+n_{c1})\left[\tanh(\alpha(F_1-C_1)) - \frac{n_{f1}-n_{c1}}{n_{f1}+n_{c1}}\right]\cosh(\alpha(F_1-C_1)) \\
&+ (n_{f1}+n_{c2})\left[\tanh(\alpha(F_1-C_2)) - \frac{n_{f1}-n_{c2}}{n_{f1}+n_{c2}}\right]\cosh(\alpha(F_1-C_2)) \\
&+(n_{f1}+n_{f2})\left[\tanh(\alpha(F_1-F_2)) - \frac{n_{f1}-n_{f2}}{n_{f1}+n_{f2}}\right]\cosh(\alpha(F_1-F_2)) \Bigg\} \\
\dot{n_{f2}} = v\cdot \Bigg\{ &(n_{f2}+n_B)\left[\tanh(\alpha_B F_2) - \frac{n_{f2}-n_B}{n_{f2}+n_B}\right]\cosh(\alpha_B F_2) \\
&+ (n_{f2}+n_{c1})\left[\tanh(\alpha(F_2-C_1)) - \frac{n_{f2}-n_{c1}}{n_{f2}+n_{c1}}\right]\cosh(\alpha(F_2-C_1)) \\
&+ (n_{f2}+n_{c2})\left[\tanh(\alpha(F_2-C_2)) - \frac{n_{f2}-n_{c2}}{n_{f2}+n_{c2}}\right]\cosh(\alpha(F_2-C_2)) \\
&+(n_{f2}+n_{f1})\left[\tanh(\alpha(F_2-F_1)) - \frac{n_{f2}-n_{f1}}{n_{f2}+n_{f1}}\right]\cosh(\alpha(F_2-F_1)) \Bigg\}
\end{aligned}
\tag{A.4}
$$

and the equations of motion in the asset allocation setup (18) as

$$\begin{aligned}
\dot{n_{c1}} = v \cdot \Bigg\{ & \frac{n_{c1}}{n_E} \left[\tanh(\alpha_B(\dot{\pi}_1 - \dot{\pi}_2)) - (2n_E - 1)\right] \cosh(\alpha_B(\dot{\pi}_1 - \dot{\pi}_2)) \\
& + (n_{c1} + n_{c2}) \left[\tanh(\alpha(C_1 - C_2)) - \frac{n_{c1} - n_{c2}}{n_{c1} + n_{c2}}\right] \cosh(\alpha(C_1 - C_2)) \\
& + (n_{c1} + n_{f1}) \left[\tanh(\alpha(C_1 - F_1)) - \frac{n_{c1} - n_{f1}}{n_{c1} + n_{f1}}\right] \cosh(\alpha(C_1 - F_1)) \\
& + (n_{c1} + n_{f2}) \left[\tanh(\alpha(C_1 - F_2)) - \frac{n_{c1} - n_{f2}}{n_{c1} + n_{f2}}\right] \cosh(\alpha(C_1 - F_2)) \Bigg\} \\
\dot{n_{c2}} = v \cdot \Bigg\{ & \frac{n_{c2}}{n_E} \left[\tanh(\alpha_B(\dot{\pi}_1 - \dot{\pi}_2)) - (2n_E - 1)\right] \cosh(\alpha_B(\dot{\pi}_1 - \dot{\pi}_2)) \\
& + (n_{c2} + n_{c1}) \left[\tanh(\alpha(C_2 - C_1)) - \frac{n_{c2} - n_{c1}}{n_{c2} + n_{c1}}\right] \cosh(\alpha(C_2 - C_1)) \\
& + (n_{c2} + n_{f1}) \left[\tanh(\alpha(C_2 - F_1)) - \frac{n_{c2} - n_{f1}}{n_{c2} + n_{f1}}\right] \cosh(\alpha(C_2 - F_1)) \\
& + (n_{c2} + n_{f2}) \left[\tanh(\alpha(C_2 - F_2)) - \frac{n_{c2} - n_{f2}}{n_{c2} + n_{f2}}\right] \cosh(\alpha(C_2 - F_2)) \Bigg\} \\
\dot{n_{f1}} = v \cdot \Bigg\{ & \frac{n_{f1}}{n_E} \left[\tanh(\alpha_B(\dot{\pi}_1 - \dot{\pi}_2)) - (2n_E - 1)\right] \cosh(\alpha_B(\dot{\pi}_1 - \dot{\pi}_2)) \\
& + (n_{f1} + n_{c1}) \left[\tanh(\alpha(F_1 - C_1)) - \frac{n_{f1} - n_{c1}}{n_{f1} + n_{c1}}\right] \cosh(\alpha(F_1 - C_1)) \\
& + (n_{f1} + n_{c2}) \left[\tanh(\alpha(F_1 - C_2)) - \frac{n_{f1} - n_{c2}}{n_{f1} + n_{c2}}\right] \cosh(\alpha(F_1 - C_2)) \\
& + (n_{f1} + n_{f2}) \left[\tanh(\alpha(F_1 - F_2)) - \frac{n_{f1} - n_{f2}}{n_{f1} + n_{f2}}\right] \cosh(\alpha(F_1 - F_2)) \Bigg\} \\
\dot{n_{f2}} = v \cdot \Bigg\{ & \frac{n_{f2}}{n_E} \left[\tanh(\alpha_B(\dot{\pi}_1 - \dot{\pi}_2)) - (2n_E - 1)\right] \cosh(\alpha_B(\dot{\pi}_1 - \dot{\pi}_2)) \\
& + (n_{f2} + n_{c1}) \left[\tanh(\alpha(F_2 - C_1)) - \frac{n_{f2} - n_{c1}}{n_{f2} + n_{c1}}\right] \cosh(\alpha(F_2 - C_1)) \\
& + (n_{f2} + n_{c2}) \left[\tanh(\alpha(F_2 - C_2)) - \frac{n_{f2} - n_{c2}}{n_{f2} + n_{c2}}\right] \cosh(\alpha(F_2 - C_2)) \\
& + (n_{f2} + n_{f1}) \left[\tanh(\alpha(F_2 - F_1)) - \frac{n_{f2} - n_{f1}}{n_{f2} + n_{f1}}\right] \cosh(\alpha(F_2 - F_1)) \Bigg\}
\end{aligned} \tag{A.5}$$

where $\dot{\pi}_1$ and $\dot{\pi}_2$ denote the scale free time changes

$$\dot{\pi}_1 = \dot{p_1}/v, \qquad \dot{\pi}_2 = \dot{p_2}/v. \tag{A.6}$$

The equilibrium points for the balanced and asset allocation setups follow by requiring all squared brackets in their respective equations of motion (A.4) and (A.5) to equal zero.

B Proof of Proposition 2

Consider again first the market without riskfree asset. Stability of the fundamental equilibrium will be considered by inspecting the Jacobian of the system of differential equations for the trader populations (9) and prices (3)[14]. Since the Jacobian contains only derivatives of first order, it is sufficient to consider the following first order approximation of (A.3) around the fundamental equilibrium with utilities $C_1 = C_2 = F_1 = F_2 = 0$:

$$\begin{aligned} \dot{n_{c1}} = v\cdot[\quad & (n_{c1} + n_{c2})\alpha(\dot{\pi}_1 - \dot{\pi}_2) + n_{c2} - n_{c1} \\ & + (n_{c1} + n_{f1})\alpha(\dot{\pi}_1 - F_1) + n_{f1} - n_{c1} \\ & + (n_{c1} + n_{f2})\alpha(\dot{\pi}_1 - F_2) + n_{f2} - n_{c1}] \end{aligned} \tag{B.1a}$$

$$\begin{aligned} \dot{n_{c2}} = v\cdot[\quad & (n_{c2} + n_{c1})\alpha(\dot{\pi}_2 - \dot{\pi}_1) + n_{c1} - n_{c2} \\ & + (n_{c2} + n_{f1})\alpha(\dot{\pi}_2 - F_1) + n_{f1} - n_{c2} \\ & + (n_{c2} + n_{f2})\alpha(\dot{\pi}_2 - F_2) + n_{f2} - n_{c2}] \end{aligned} \tag{B.1b}$$

$$\begin{aligned} \dot{n_{f1}} = v\cdot[\quad & (n_{f1} + n_{c1})\alpha(F_1 - \dot{\pi}_1) + n_{c1} - n_{f1} \\ & + (n_{f1} + n_{c2})\alpha(F_1 - \dot{\pi}_2) + n_{c2} - n_{f1} \\ & + (n_{f1} + n_{f2})\alpha(F_1 - F_2) + n_{f2} - n_{f1}] \end{aligned} \tag{B.1c}$$

$$\begin{aligned} \dot{n_{f2}} = v\cdot[\quad & (n_{f2} + n_{c1})\alpha(F_2 - \dot{\pi}_1) + n_{c1} - n_{f2} \\ & + (n_{f2} + n_{c2})\alpha(F_2 - \dot{\pi}_2) + n_{c2} - n_{f2} \\ & + (n_{f2} + n_{f1})\alpha(F_2 - F_1) + n_{f1} - n_{f2}] \end{aligned} \tag{B.1d}$$

Introducing the new variables $c_1 = n_{c1}/N$, $c_2 = n_{c2}/N$, $f_1 = n_{f1}/N$, $f_2 = n_{f2}/N$, and $a = v\alpha$ and making use of (5) this may be rewritten as:

$$\begin{aligned} \dot{c1} &= v(1 - 4c_1) + a[(1 + 2c_1)\dot{\pi}_1 - c_1(\dot{\pi}_2 + F_1 + F_2) - c_2\dot{\pi}_2 - f_1F_1 - f_2F_2] \\ \dot{c2} &= v(1 - 4c_2) + a[(1 + 2c_2)\dot{\pi}_2 - c_2(\dot{\pi}_1 + F_1 + F_2) - c_1\dot{\pi}_1 - f_1F_1 - f_2F_2] \\ \dot{f1} &= v(1 - 4f_1) + a[(1 + 2f_1)F_1 - f_1(\dot{\pi}_1 + \dot{\pi}_2 + F_2) - c_1\dot{\pi}_1 - c_2\dot{\pi}_2 - f_2F_2] \\ \dot{f2} &= v(1 - 4f_2) + a[(1 + 2f_2)F_2 - f_2(\dot{\pi}_1 + \dot{\pi}_2 + F_1) - c_1\dot{\pi}_1 - c_2\dot{\pi}_2 - f_1F_1] \end{aligned} \tag{B.2}$$

Similarly, the equations for the trading prices (3) may be rewritten as

$$\dot{\pi}_i = \frac{1}{f_i}\left(t\dot{c}_i + (\pi_{fi} - \pi_i)\dot{f}_i\right), \qquad i = 1, 2 \quad , \tag{B.3}$$

[14]see e.g. Gandolfo (1996).

where we have introduced the leverage parameter $t = t_c/(vt_f)$ and rescaled prices

$$\pi_i = \frac{p_i}{v}, \quad \pi_{f1} = \frac{p_{fi}}{v}, \qquad i = 1, 2. \tag{B.4}$$

Both the differential equations for the trader populations (B.2) and for the trading prices (B.3) have time derivatives on both sides. Solving for the time derivatives $(\dot{c_1}, \dot{c_2}, \dot{f_1}, \dot{f_2}, \dot{\pi}_1, \dot{\pi}_2)$ yields for the price equations

$$\begin{aligned}\dot{\pi}_1 &= \frac{A_1(D_1 + E_1) + B_1(D_2 + G_1)}{B_1 B_2 - A_1 A_2}, \\ \dot{\pi}_2 &= \frac{A_2(D_2 + G_2) + B_2(D_1 + E_2)}{B_1 B_2 - A_1 A_2},\end{aligned} \tag{B.5}$$

where

$$\begin{aligned}A_{1/2} &= a\left((c_{2/1} + f_{1/2})(\pi_{f1/2} - \pi_{1/2}) + (c_1 + c_2)t\right), \\ B_{1/2} &= a\left((2c_{2/1} + 1)t + (c_{2/1} + f_{2/1})(\pi_{f2/1} - \pi_{2/1})\right) - f_{2/1}, \\ D_{1/2} &= a\left((f_1 + f_2)F_{1/2} - (2f_{2/1} + 1)F_{2/1} + (4f_{2/1} - 1)v\right)(\pi_{f2/1} - \pi_{2/1}), \\ E_{1/2} &= \left(a\left((c_2 + f_{2/1})F_{2/1} + (c_2 + f_{1/2})F_{1/2}\right) + (4c_2 - 1)v\right)t, \\ G_{1/2} &= \left(a\left((c_1 + f_{2/1})F_{2/1} + (c_1 + f_{1/2})F_{1/2}\right) + (4c_1 - 1)v\right)t.\end{aligned}$$

Inserting these into (B.2) eliminates all time derivatives also on the right hand side of the trader populations' equations of motion.

The partial derivatives of the price equations (B.5) evaluated at the fundamental equilibrium read

$$\frac{\partial\,\dot{\pi}_1}{\partial c_1} = \frac{\partial\,\dot{\pi}_2}{\partial c_2} = \frac{16tv(6at - 1)}{(4at - 1)(8at - 1)}, \tag{B.6a}$$

$$\frac{\partial\,\dot{\pi}_1}{\partial c_2} = \frac{\partial\,\dot{\pi}_2}{\partial c_1} = \frac{32at^2v}{(4at - 1)(8at - 1)}, \tag{B.6b}$$

$$\frac{\partial\,\dot{\pi}_1}{\partial f_1} = \frac{\partial\,\dot{\pi}_2}{\partial f_2} = \frac{\partial\,\dot{\pi}_1}{\partial f_2} = \frac{\partial\,\dot{\pi}_2}{\partial f_1} = 0, \tag{B.6c}$$

$$\frac{\partial\,\dot{\pi}_1}{\partial \pi_1} = \frac{\partial\,\dot{\pi}_2}{\partial \pi_1} = \frac{2atF_1{}'(\pi_{f1})}{4at - 1}, \tag{B.6d}$$

$$\frac{\partial\,\dot{\pi}_1}{\partial \pi_2} = \frac{\partial\,\dot{\pi}_2}{\partial \pi_2} = \frac{2atF_2{}'(\pi_{f2})}{4at - 1}. \tag{B.6e}$$

Application of the chain rule to (B.2) yields for the chartists populations at fundamental equilibrium

$$\frac{\partial \dot{c}_{1/2}}{\partial c_{1/2}} = \frac{a}{2}\left(3\frac{\partial\,\dot{\pi}_{1/2}}{\partial c_{1/2}} - \frac{\partial\,\dot{\pi}_{2/1}}{\partial c_{1/2}}\right) - 4v, \tag{B.7a}$$

$$\frac{\partial \dot{c}_{1/2}}{\partial c_{2/1}} = \frac{a}{2}\left(3\frac{\partial\,\dot{\pi}_{1/2}}{\partial c_{2/1}} - \frac{\partial\,\dot{\pi}_{2/1}}{\partial c_{2/1}}\right), \tag{B.7b}$$

$$\frac{\partial \dot{c}_1}{\partial f_1} = \frac{\partial \dot{c}_1}{\partial f_2} = \frac{\partial \dot{c}_2}{\partial f_1} = \frac{\partial \dot{c}_2}{\partial f_2} = 0, \tag{B.7c}$$

$$\frac{\partial \dot{c}_{1/2}}{\partial \pi_1} = \frac{a}{2}\left(3\frac{\partial\,\dot{\pi}_{1/2}}{\partial \pi_1} - \frac{\partial\,\dot{\pi}_{2/1}}{\partial \pi_1} - {F_1}'(\pi_{f1})\right), \tag{B.7d}$$

$$\frac{\partial \dot{c}_{1/2}}{\partial \pi_2} = \frac{a}{2}\left(3\frac{\partial\,\dot{\pi}_{1/2}}{\partial \pi_2} - \frac{\partial\,\dot{\pi}_{2/1}}{\partial \pi_2} - {F_2}'(\pi_{f2})\right), \tag{B.7e}$$

and for the fundamentalist populations

$$\frac{\partial \dot{f}_1}{\partial c_1} = \frac{\partial \dot{f}_2}{\partial c_1} = -\frac{a}{2}\left(\frac{\partial\,\dot{\pi}_1}{\partial c_1} + \frac{\partial\,\dot{\pi}_2}{\partial c_1}\right), \tag{B.8a}$$

$$\frac{\partial \dot{f}_1}{\partial c_2} = \frac{\partial \dot{f}_2}{\partial c_2} = -\frac{a}{2}\left(\frac{\partial\,\dot{\pi}_1}{\partial c_2} + \frac{\partial\,\dot{\pi}_2}{\partial c_2}\right), \tag{B.8b}$$

$$\frac{\partial \dot{f}_1}{\partial f_1} = \frac{\partial \dot{f}_2}{\partial f_2} = -4v, \tag{B.8c}$$

$$\frac{\partial \dot{f}_1}{\partial f_2} = \frac{\partial \dot{f}_2}{\partial f_1} = 0, \tag{B.8d}$$

$$\frac{\partial \dot{f}_{1/2}}{\partial \pi_{1/2}} = \frac{a}{2}\left(3{F_{1/2}}'(\pi_{f1/2}) - \frac{\partial\,\dot{\pi}_1}{\partial \pi_{1/2}} - \frac{\partial\,\dot{\pi}_2}{\partial \pi_{1/2}}\right), \tag{B.8e}$$

$$\frac{\partial \dot{f}_{1/2}}{\partial \pi_{2/1}} = -\frac{a}{2}\left({F_{2/1}}'(\pi_{f2/1}) + \frac{\partial\,\dot{\pi}_1}{\partial \pi_{2/1}} + \frac{\partial\,\dot{\pi}_2}{\partial \pi_{2/1}}\right), \tag{B.8f}$$

where (B.6c) has already been taken into account. Inserting these into the Jakobian matrix of (B.2) and (B.5)

$$J = \begin{pmatrix} \frac{\partial \dot{c}_1}{\partial c_1} & \frac{\partial \dot{c}_1}{\partial c_2} & \frac{\partial \dot{c}_1}{\partial f_1} & \frac{\partial \dot{c}_1}{\partial f_2} & \frac{\partial \dot{c}_1}{\partial \pi_1} & \frac{\partial \dot{c}_1}{\partial \pi_2} \\ \frac{\partial \dot{c}_2}{\partial c_1} & \frac{\partial \dot{c}_2}{\partial c_2} & \frac{\partial \dot{c}_2}{\partial f_1} & \frac{\partial \dot{c}_2}{\partial f_2} & \frac{\partial \dot{c}_2}{\partial \pi_1} & \frac{\partial \dot{c}_2}{\partial \pi_2} \\ \frac{\partial \dot{f}_1}{\partial c_1} & \frac{\partial \dot{f}_1}{\partial c_2} & \frac{\partial \dot{f}_1}{\partial f_1} & \frac{\partial \dot{f}_1}{\partial f_2} & \frac{\partial \dot{f}_1}{\partial \pi_1} & \frac{\partial \dot{f}_1}{\partial \pi_2} \\ \frac{\partial \dot{f}_2}{\partial c_1} & \frac{\partial \dot{f}_2}{\partial c_2} & \frac{\partial \dot{f}_2}{\partial f_1} & \frac{\partial \dot{f}_2}{\partial f_2} & \frac{\partial \dot{f}_2}{\partial \pi_1} & \frac{\partial \dot{f}_2}{\partial \pi_2} \\ \frac{\partial \dot{\pi}_1}{\partial c_1} & \frac{\partial \dot{\pi}_1}{\partial c_2} & \frac{\partial \dot{\pi}_1}{\partial f_1} & \frac{\partial \dot{\pi}_1}{\partial f_2} & \frac{\partial \dot{\pi}_1}{\partial \pi_1} & \frac{\partial \dot{\pi}_1}{\partial \pi_2} \\ \frac{\partial \dot{\pi}_2}{\partial c_1} & \frac{\partial \dot{\pi}_2}{\partial c_2} & \frac{\partial \dot{\pi}_2}{\partial f_1} & \frac{\partial \dot{\pi}_2}{\partial f_2} & \frac{\partial \dot{\pi}_2}{\partial \pi_1} & \frac{\partial \dot{\pi}_2}{\partial \pi_2} \end{pmatrix} \tag{B.9}$$

yields the following set of eigenvalues:

$$\lambda_1 = \lambda_2 = -4v, \tag{B.10a}$$
$$\lambda_3 = \lambda_4 = 0, \tag{B.10b}$$
$$\lambda_5 = \frac{4v}{8at-1}, \tag{B.10c}$$
$$\lambda_6 = \frac{4v + 2at\left({F_1}'(\pi_{f1}) + {F_2}'(\pi_{f2})\right)}{4at-1}. \tag{B.10d}$$

Stability of the fundamental equilibrium requires all eigenvalues to be non-positive. The first four eigenvalues are always less or equal to zero independent of the choice of parameters. λ_5 is less or equal to zero for

$$at < \frac{1}{8} \quad \Leftrightarrow \quad \alpha\frac{t_c}{t_f} < \frac{1}{8}. \tag{B.11}$$

As regards the last eigenvalue λ_6 a complication arises from the fact that ${F_1}'(\pi_{f1})$ and ${F_2}'(\pi_{f2})$ are not defined because

$$F_{1/2}(\pi_{1/2}) = vs|\pi_{f1/2} - \pi_{1/2}| \tag{B.12}$$

implies a jump of the derivative $F'_{1/2}$ at the respective fundamental price

$${F_{1/2}}'(\pi_{1/2}) = \pm vs \qquad \text{for} \qquad \pi_{f1/2} \gtrless \pi_{1/2}. \tag{B.13}$$

It is therefore necessary to investigate each of the two regimes for each of the two potentials seperately:

$$\lambda_6 = \begin{cases} \frac{4v(1+ats)}{4at-1} & \text{for } \pi_{f1} > \pi_1 \text{ and } \pi_{f2} > \pi_2; \\ \frac{4v}{4at-1} & \text{for } \pi_{f1} < \pi_1,\ \pi_{f2} > \pi_2 \text{ or } \pi_{f1} > \pi_1,\ \pi_{f2} < \pi_2; \\ \frac{4v(1-ats)}{4at-1} & \text{for } \pi_{f1} < \pi_1 \text{ and } \pi_{f2} < \pi_2. \end{cases} \tag{B.14}$$

The dominator will always be negative, when condition (B.11) is fulfilled. $\lambda_6 \leq 0$ requires thus the numerator in all three cases to be non-negative, that is

$$ats \leq 1. \tag{B.15}$$

This is however always true when condition (B.11) is fulfilled, because s as a discount factor satisfies $0 < s < 1$. Condition (B.11) is therefore the only necessary condition for stability of the fundamental equilibrium, as was to be shown.

The proves for the stability conditions in the setups including a riskfree asset follow the same logic and will be presented elsewhere.

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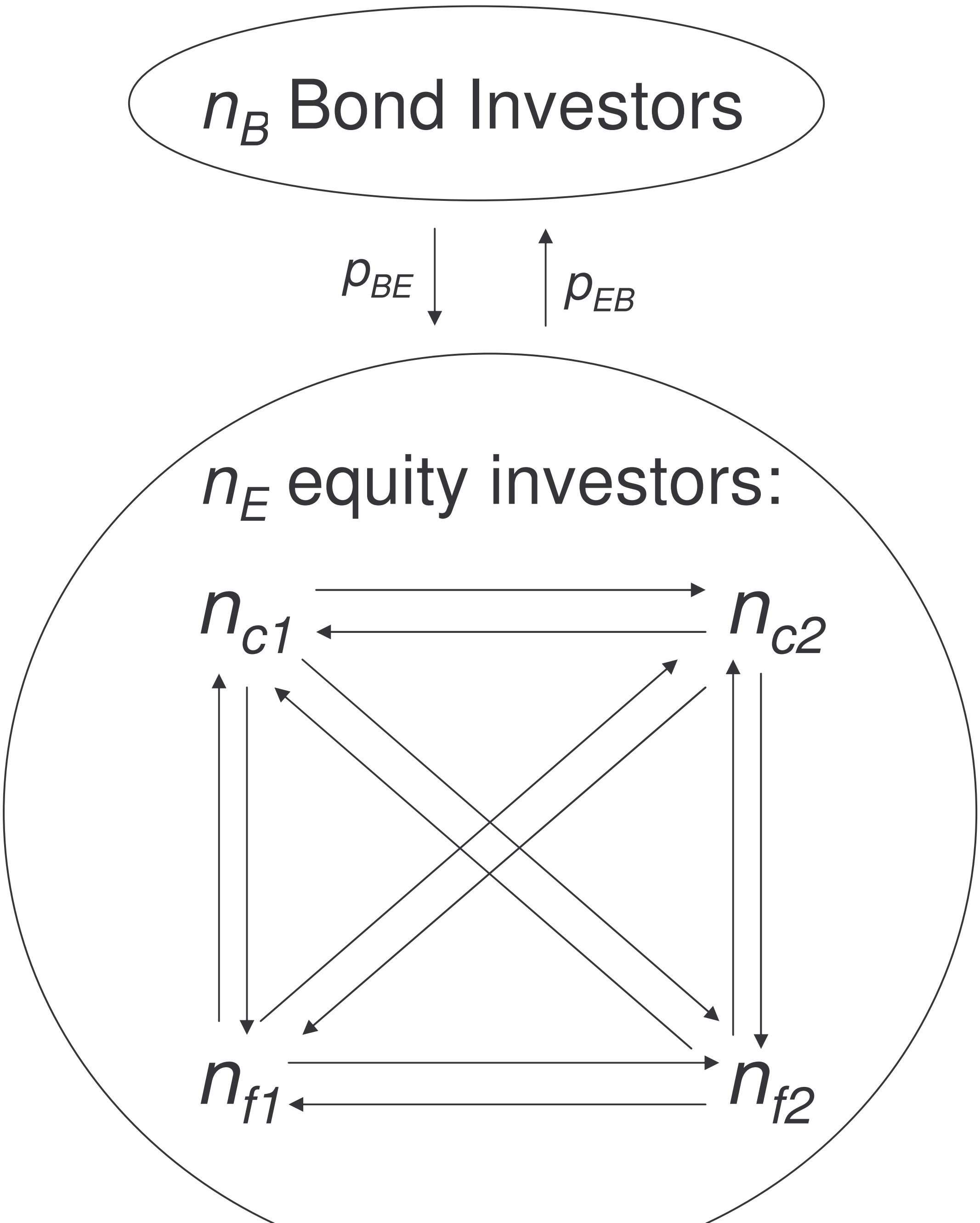
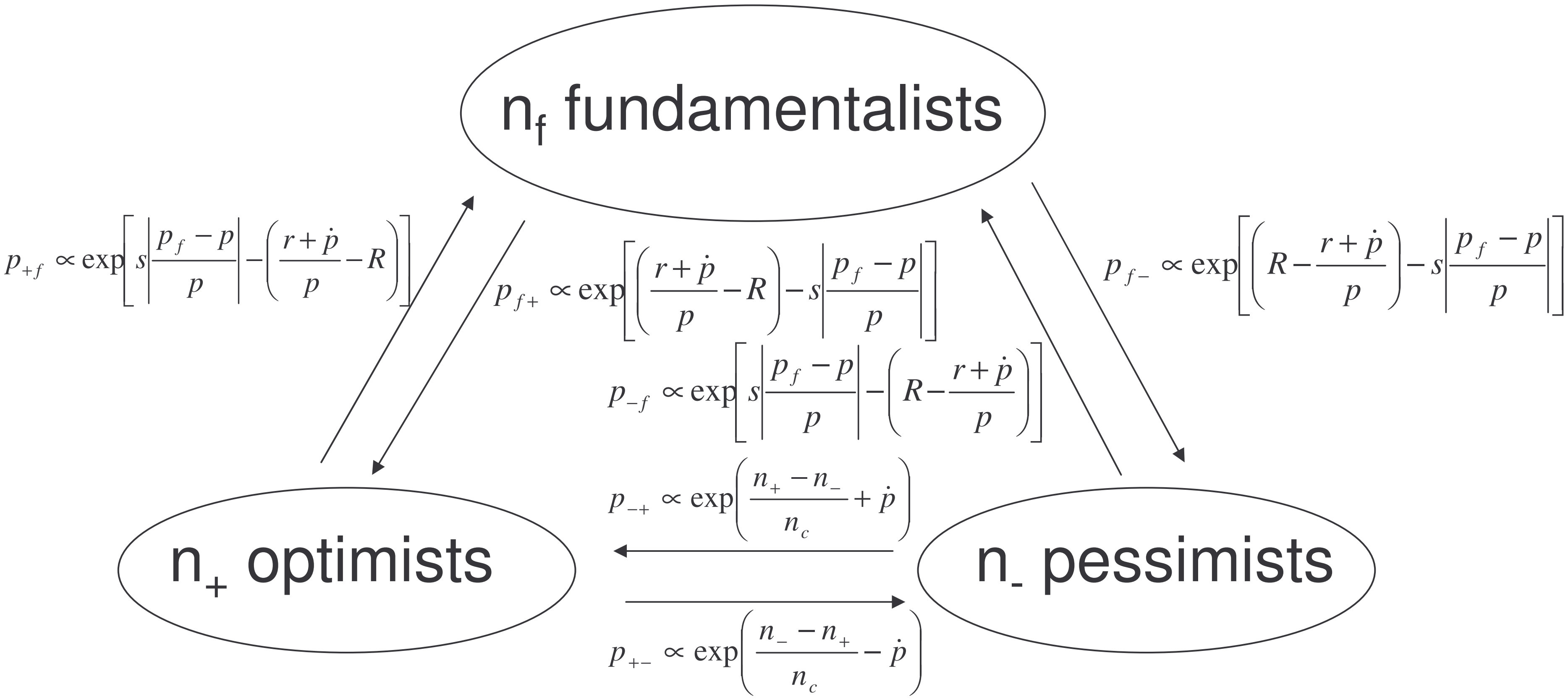
Figure 1



n_B Bond Investors
p_{BE}
p_{EB}
n_E equity investors:
n_{c1}
n_{c2}
n_{f1}
n_{f2}

Figure 2

Lux 1998, Lux/Marchesi 1999/2000:



$$\dot{p} \propto (ED_c + ED_f) \qquad ED_c \propto (n_+ - n_-) \qquad ED_f \propto n_f (p_f - p)$$

Figure 2

Figure 3

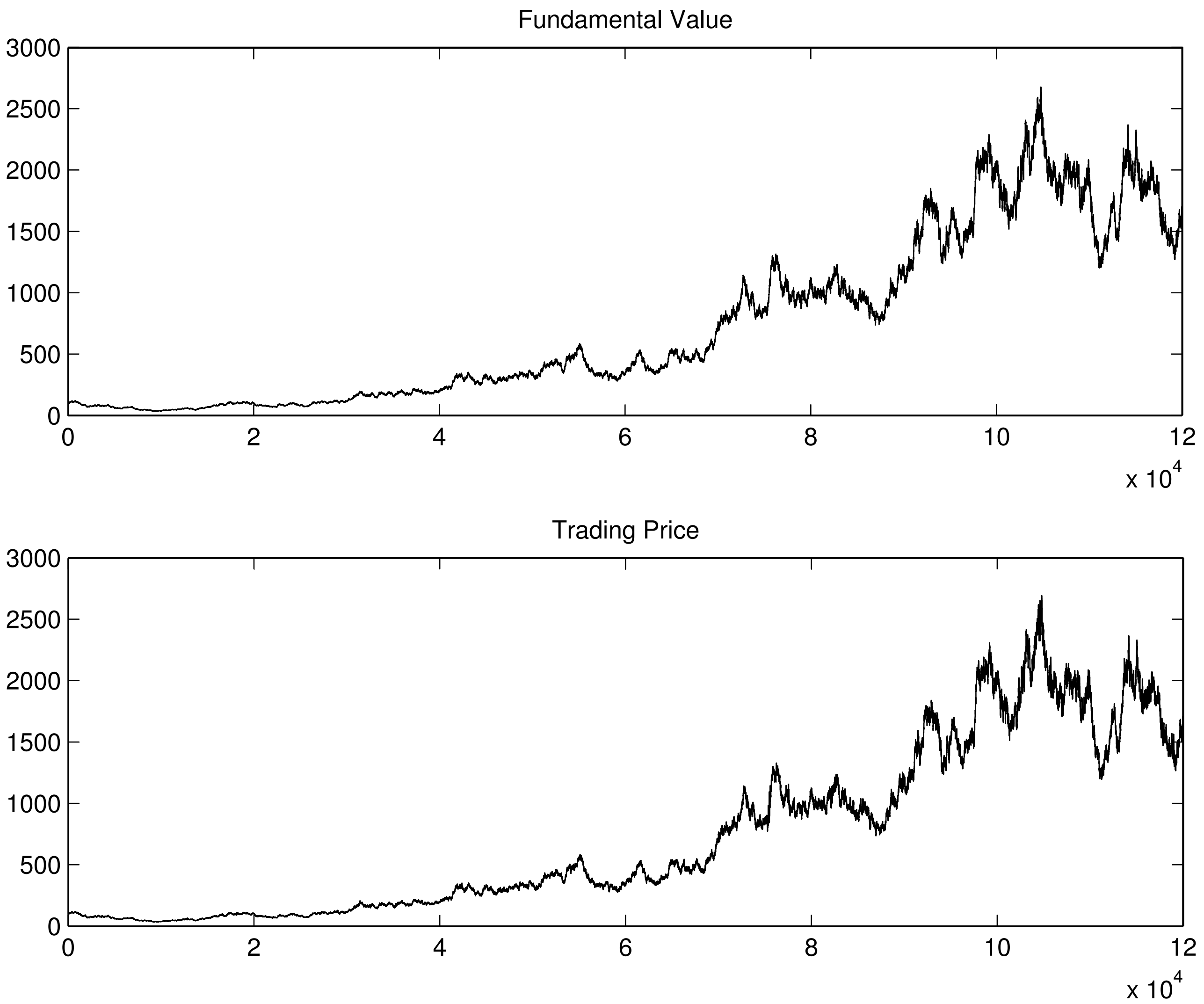
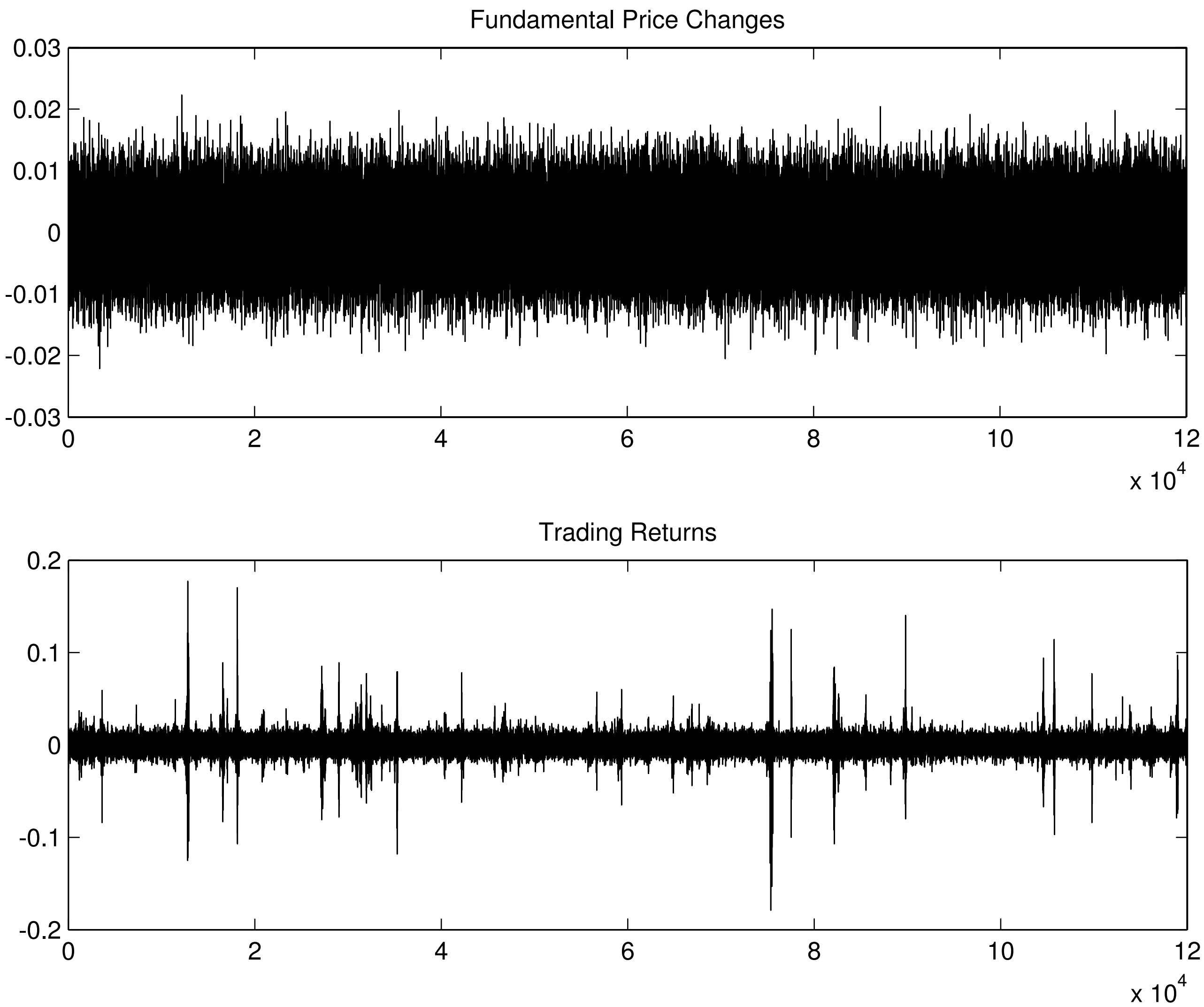
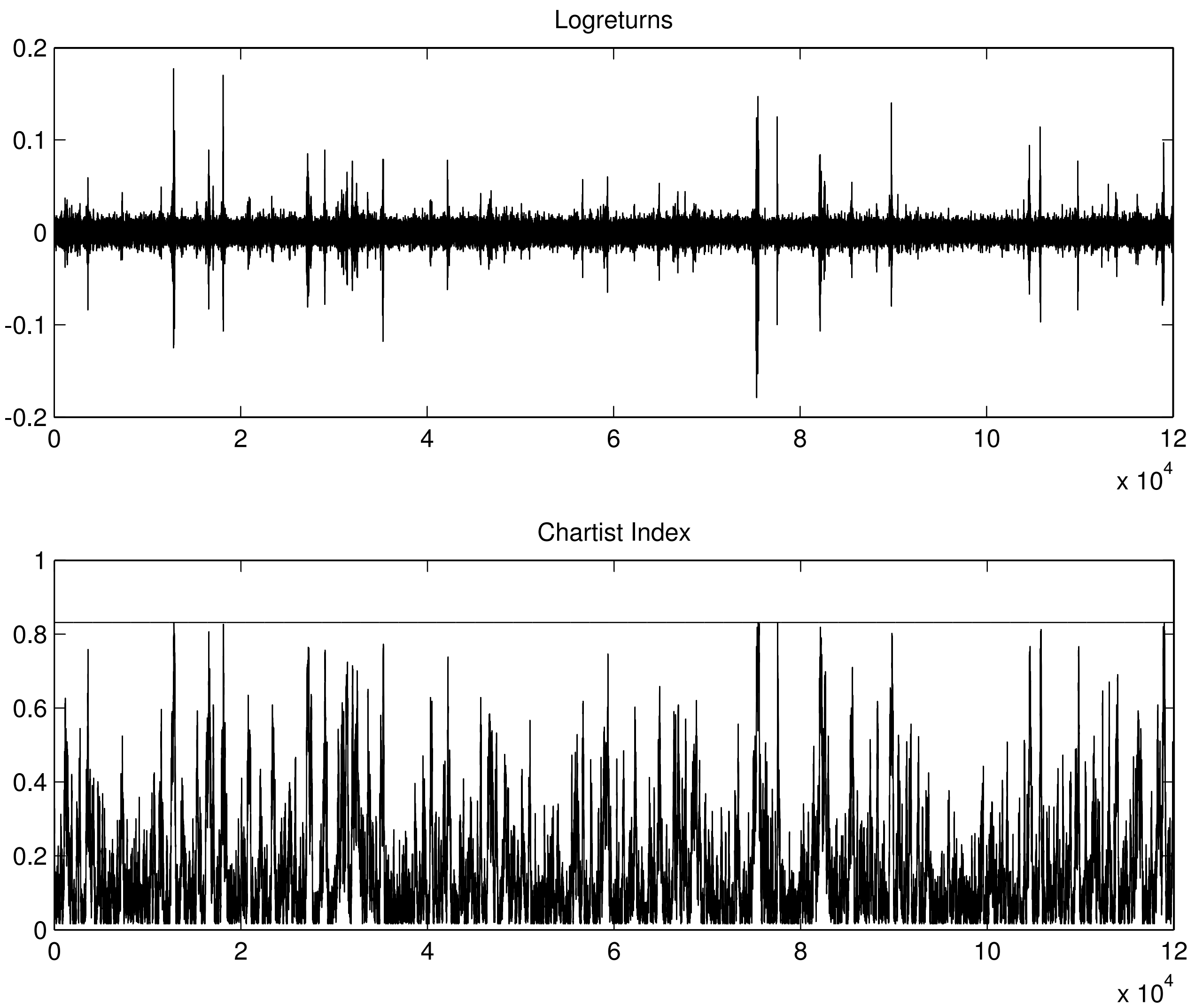

Fundamental Value
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
12
x 10^4
Trading Price
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
12
x 10^4

Figure 4



Fundamental Price Changes
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
0
2
4
6
8
10
12
x 10^4
Trading Returns
0.2
0.1
0
-0.1
-0.2
0
2
4
6
8
10
12
x 10^4

Figure 5



Logreturns
0.2
0.1
0
-0.1
-0.2
0
2
4
6
8
10
12
x 10^4
Chartist Index
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
x 10^4

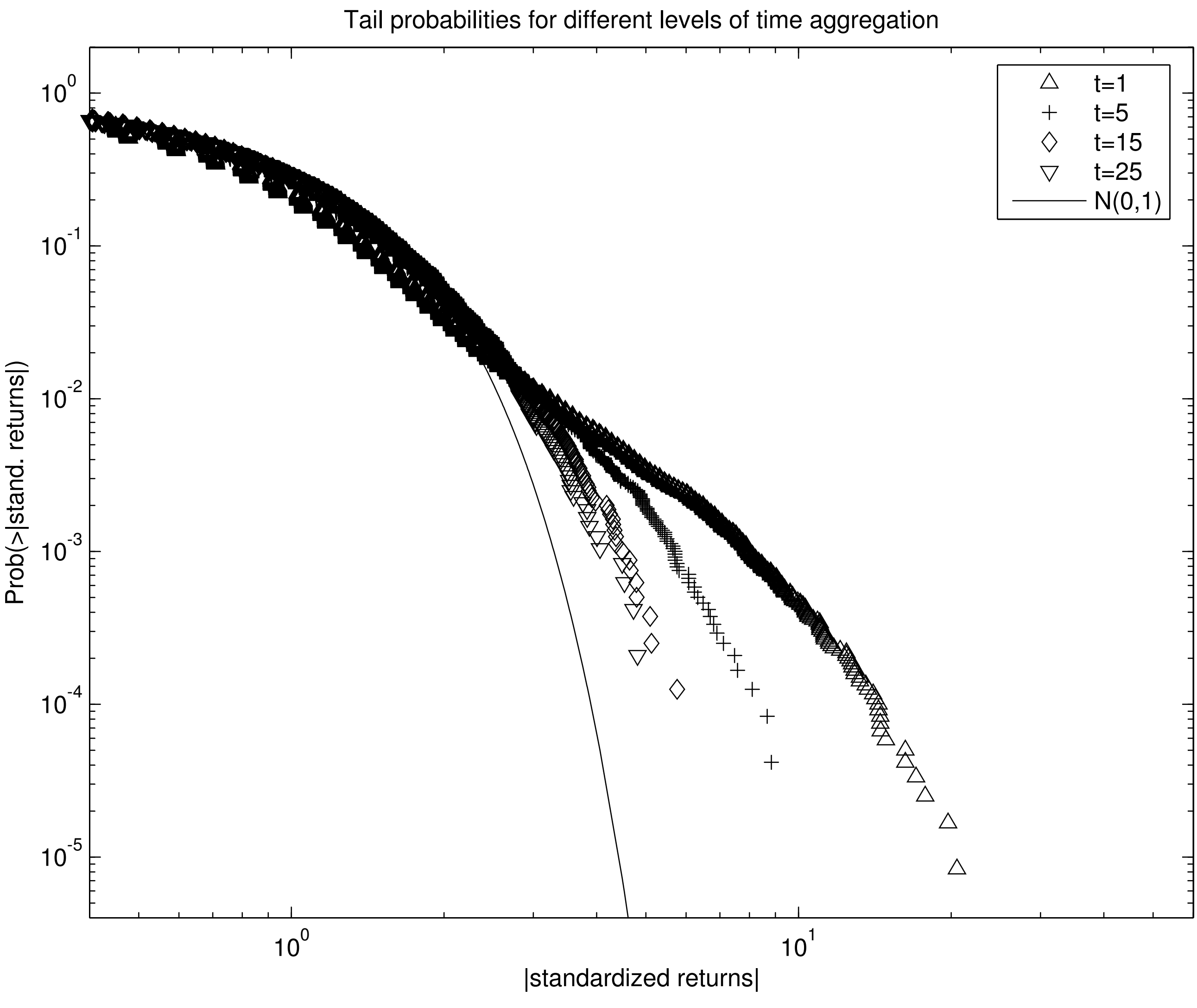

Tail probabilities for different levels of time aggregation
t=1
t=5
t=15
t=25
N(0,1)
Prob(>|stand. returns|)
|standardized returns|
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^0
10^1

Figure 7

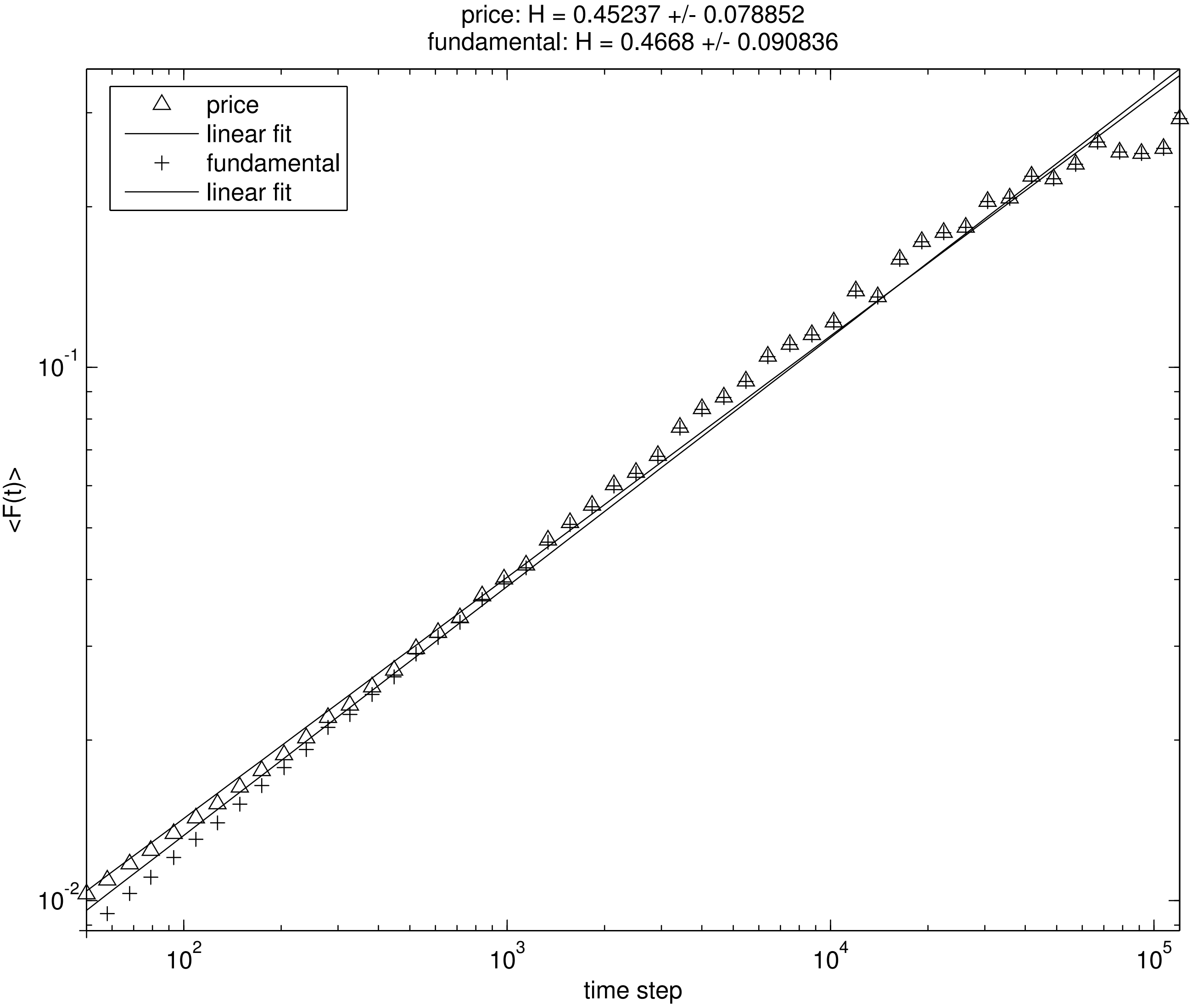
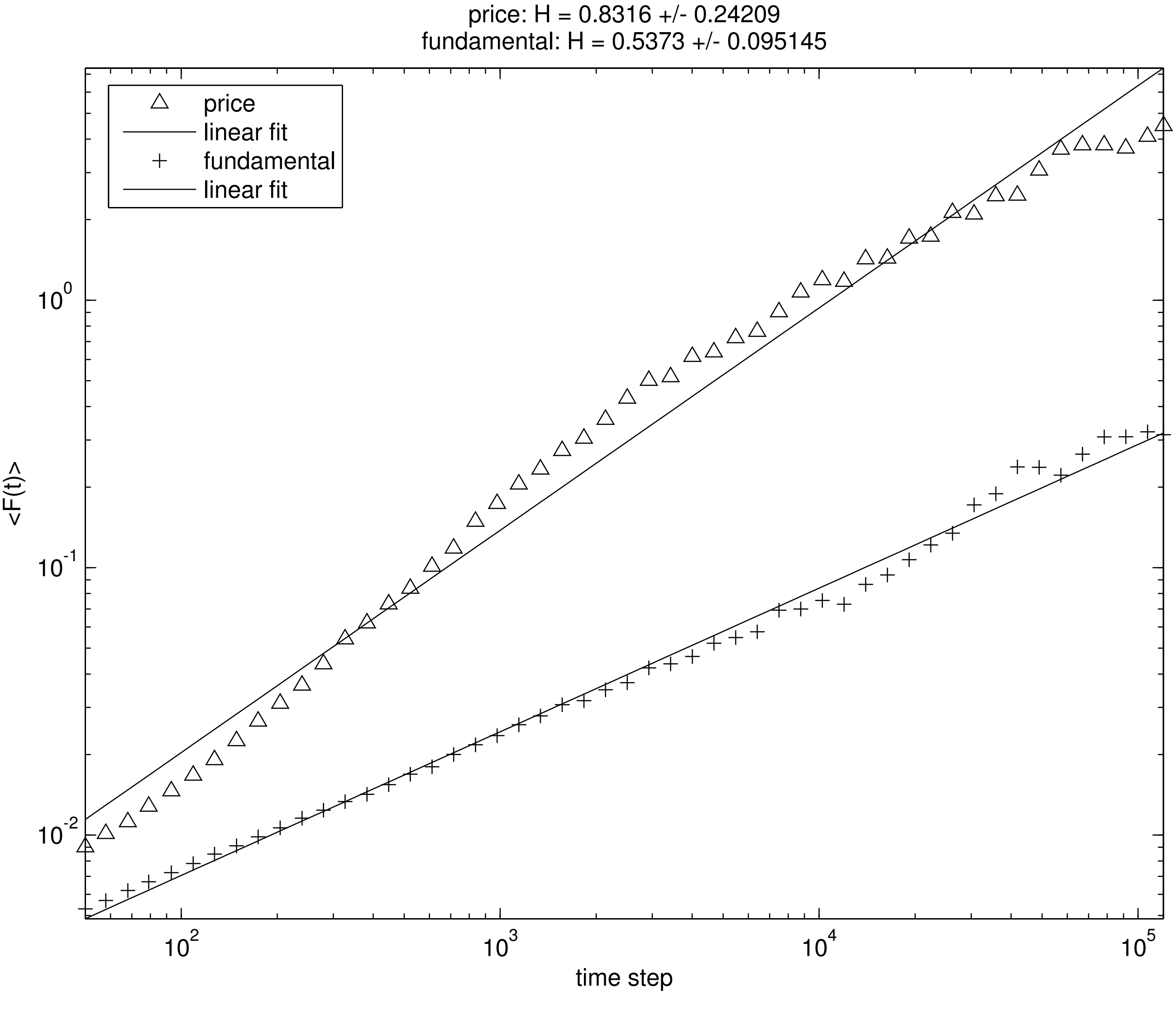

price: H = 0.45237 +/- 0.078852
fundamental: H = 0.4668 +/- 0.090836
price
linear fit
fundamental
linear fit
10^{-1}
10^{-2}
<F(t)>
10^{2}
10^{3}
10^{4}
10^{5}
time step

Figure 8



price: H = 0.8316 +/- 0.24209
fundamental: H = 0.5373 +/- 0.095145
price
linear fit
fundamental
linear fit
10^{0}
10^{-1}
10^{-2}
<F(t)>
10^{2}
10^{3}
10^{4}
10^{5}
time step

Figure 9

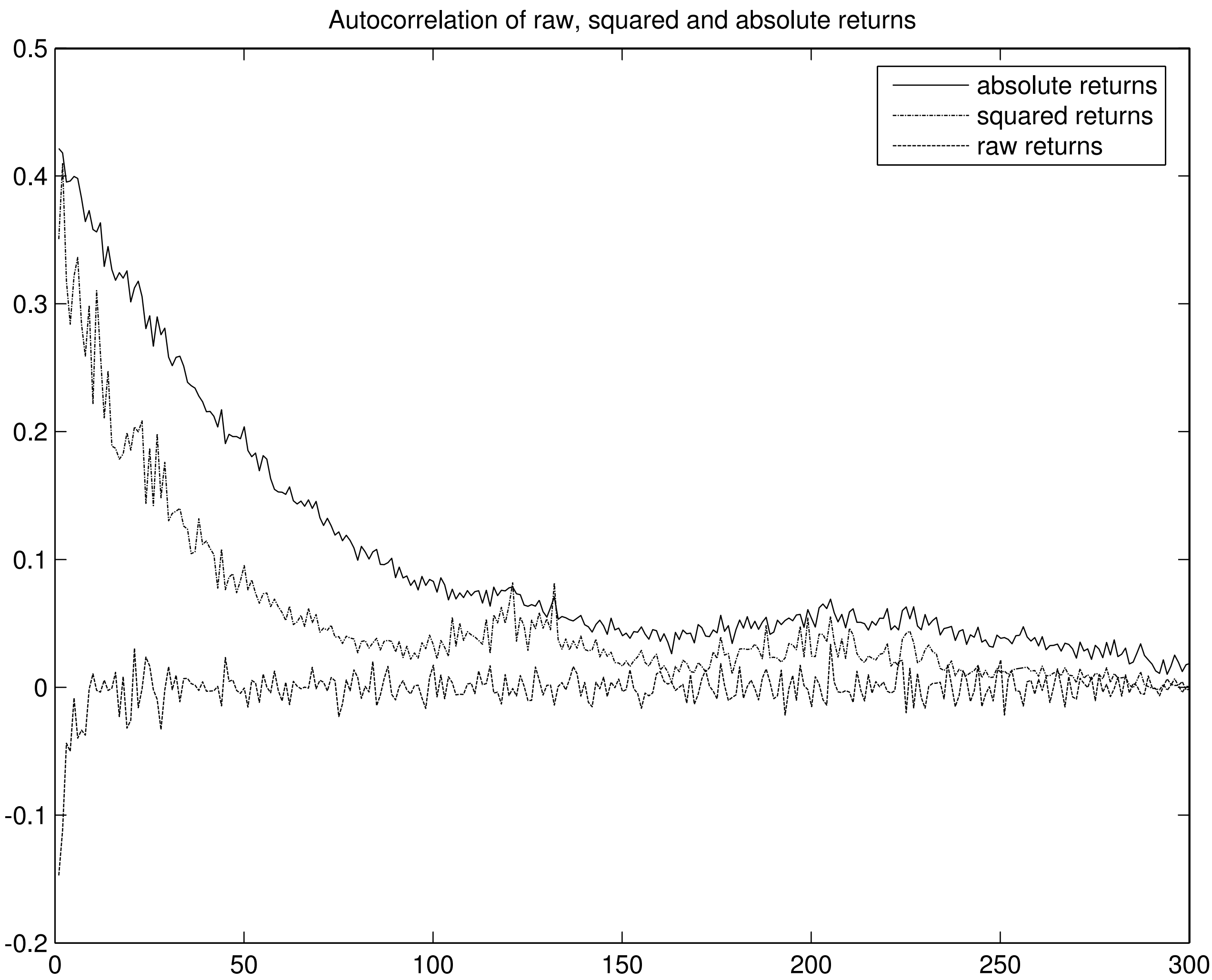
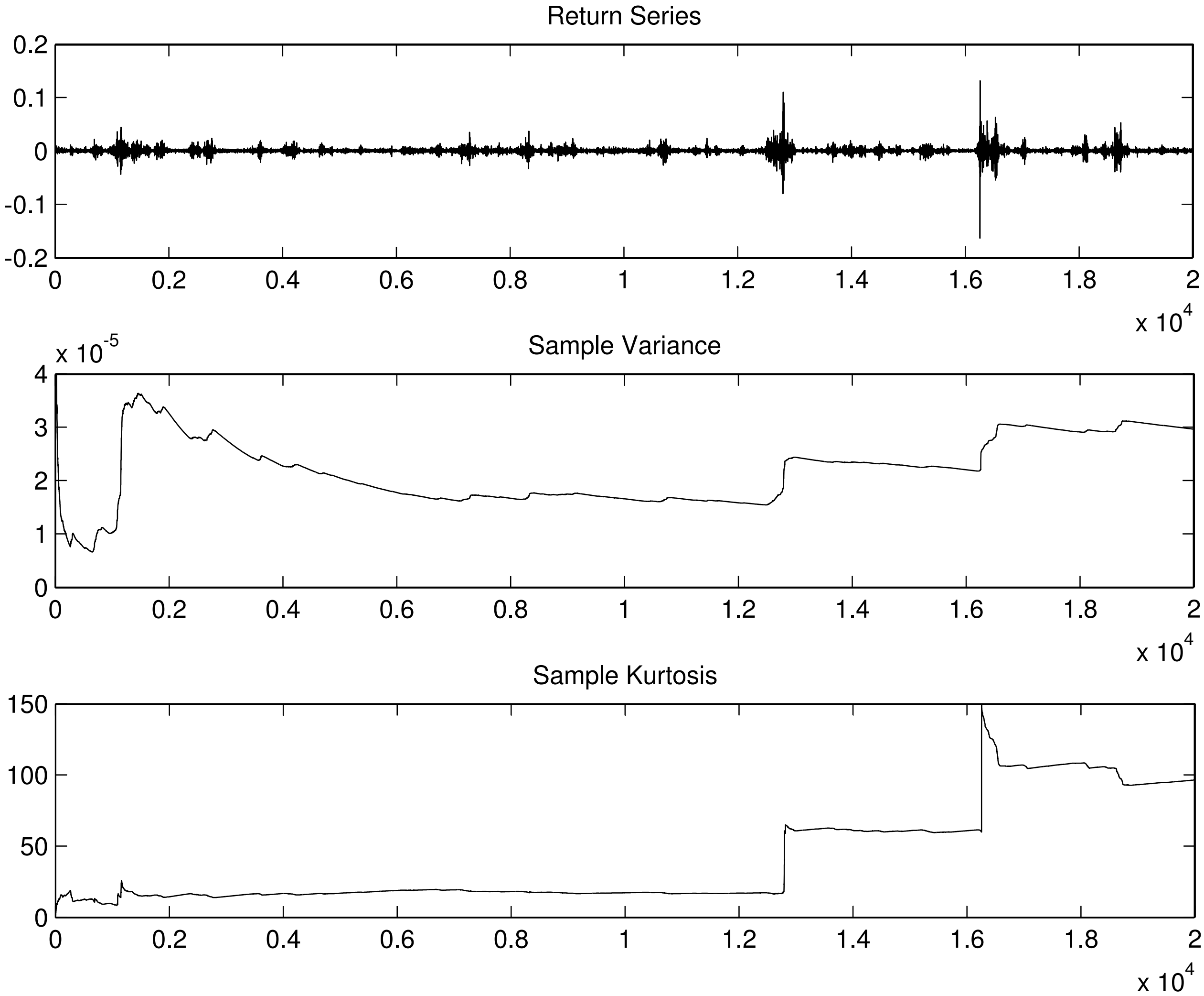

Autocorrelation of raw, squared and absolute returns
absolute returns
squared returns
raw returns
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
50
100
150
200
250
300

Figure 10



Return Series
0.2
0.1
0
-0.1
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x 10^4
Sample Variance
x 10^-5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x 10^4
Sample Kurtosis
150
100
50
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x 10^4

Table 1: Long-term dependence in squared and absolute returns

		squared returns		absolute returns	
		median d in 10 samples of 2000 obs.	% rejections of d=0 (95% conf.)	median d in 10 samples of 2000 obs.	% rejections of d=0 (95% conf.)
Set I:	Simulation 1	0.31	20 %	0.39	30 %
	Simulation 2	0.24	20 %	0.39	40 %
	Lux/Marchesi	0.17	40 %	0.38	80 %
Set II:	Simulation 1	0.32	10 %	0.55	60 %
	Simulation 2	0.39	30 %	0.52	70 %
	Lux/Marchesi	0.54	100 %	0.63	100 %
Set III:	Simulation 1	0.47	60 %	0.56	80 %
	Simulation 2	0.53	70 %	0.62	90 %
	Lux/Marchesi	0.50	100 %	0.64	100 %
Set IV:	Simulation 1	0.47	60 %	0.61	80 %
	Simulation 2	0.40	40 %	0.58	100 %
	Lux/Marchesi	0.52	90 %	0.64	90 %

Table 2: Fat tail property of the data: kurtosis and tail index estimates

		kurtosis	median tail index estimates (10 samples / 2000 obs. each) 2.5% tail	5% tail	10% tail
Set I:	Simulation 1	147.47	3.34	2.57	2.27
	Simulation 2	27.59	3.23	2.51	2.17
	Lux/Marchesi	135.73	2.04	2.11	1.93
Set II:	Simulation 1	43.16	2.61	2.32	2.19
	Simulation 2	23.46	3.43	2.96	2.35
	Lux/Marchesi	16.10	2.82	2.52	2.18
Set III:	Simulation 1	147.95	3.34	2.93	2.28
	Simulation 2	34.65	3.21	2.81	2.67
	Lux/Marchesi	27.11	4.63	3.48	2.86
Set IV:	Simulation 1	22.51	3.70	2.90	2.83
	Simulation 2	96.44	3.25	2.98	2.33
	Lux/Marchesi	37.74	3.08	2.46	1.97