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OPTIMAL DIVERSIFICATION OF BORROWING IN THE MULTINATIONAL FIRM FACING RISKY CURRENCY EXCHANGE RATES

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1 Introduction

The present theory of financial decisions of the firm is mainly concerned with the one-country environment. The growth of multinational firms has led to the question whether this theory is applicable in the multi-country setting as well, or whether significant modifications would be needed. A summary of recent research by Naumann-Etienne [2] points to the fact that many questions remain essentially unsolved, and that much additional research is needed in the various areas of financial decisions customarily distinguished in the one-country environment.

In addition to the questions analogous to the one-country environment, the multi-country environment presents problems which are unknown to the uninational firm. An area causing particular concern for multinational firms today is the Western world's move to floating currency exchange rates in 1972 and 1973. Changes in currency values create special problems for multinationals because any change in the values of other currencies of operation against the domestic currency of the company's headquarters has a "translation", or accounting, effect on the corporate earnings, and this effect is independent of the underlying profitability of the multinational operations. This results from the fact that, in addition to profits earned by foreign subsidiaries, the foreign balance sheet items are also subject to translation into the parent's currency in the year-end financial statements.

An example of the magnitude of these changes is given by the financial results reported by International Telephone and Telegraph for 1974 [1]. I.T.T. anticipated changes in the European currencies against the U.S.
dollar and entered into forward contracts in order to insulate its profits from currency fluctuations. Unfortunately, the company guessed wrong the direction of these changes and ended the year with a pre-tax loss of US $48 million for the hedging contracts.

In addition, its exposed assets were concentrated in the pound sterling, while its exposed liabilities were concentrated in the stronger currencies, particularly the Swiss franc, causing a big translation loss.

The I.T.T. case exemplifies one of the traditional ways of hedging against foreign exchange losses. This is to estimate the firm's exposure position, forecast the direction and magnitude of the change in the exchange rates, and enter into forward exchange contracts to cover the expected devaluation losses. As the I.T.T. example demonstrates, forecasting the direction of the change may be increasingly difficult task in the present-day world. Another possibility is to diversify the exposure among various currencies. A change in a specific exchange rate would then affect only a small part of exposed assets and liabilities.

The present paper is an attempt to demonstrate in the framework of a two-stage linear programming model how the optimal policy for the multinational firm shifts from the traditional hedging into diversifying of international borrowing sources when the forecasts of the future exchange rates, given in the form of discrete probability distributions, shift from an expected devaluation or revaluation of individual currencies into a floating pattern. Floating is approximated in our model by a distribution where the exchange rate can move in either direction.

We will not consider the possibility of entering into forward exchange contracts in the present version
of the model. Instead, we will include into the model the possibility of borrowing in the weak currency and transferring the funds between affiliates by making interaffiliate loans. It has already been demonstrated how to include the forward exchange contract as decision variables into the model of a multinational firm [3] and it is our intention to expand also the present model into this direction in the future.

2 Exposition of the Model and a Fictitious Numerical Example

2.1 Preliminaries

This chapter presents a two-stage linear programming formulation for joint optimal planning of external borrowing, interaffiliate loan granting, and production in the multinational firm, assuming risky currency exchange rates.

The model is kept as simple as possible in order to concentrate on the essential points. A great many details which should be taken into account in a real-life planning situation, but which are neither new nor theoretically essential, are omitted or simplified. Nevertheless, the model is constructed in a manner which allows the incorporation of the omitted details, when required.\footnote{For further discussion on this point see [3], p. 20.}

The presentation of the model has to be kept relatively brief, because of the limited space. Thus a host of the assumptions made has to be read directly from the mathematical presentation of the model.
A fictitious numerical example involving a multinational firm with affiliates in three countries is given simultaneously with the model. In the numerical example we shall refer to country 1 as the USA, country 2 as England, and country 3 as Finland for solely illustrative reasons. The USA is the home country of the multinational firm of the numerical example.

Figure 1 gives the model in a flow form. (See next page.)

The decision variables of the model will be identified by the letter X. The constants of the model will be identified by the letters A, B, and C. Furthermore, nc will denote the number of the affiliates involved (nc = 3 in the fictitious numerical example). i and j will be indexes indicating countries. i = 1 will indicate the home country of the multinational firm. (The USA in the example.)

2.2 SUBDIVIDING THE PLANNING HORIZON INTO TWO SUBPERIODS (Pre- and Post- the Potential Change in Exchange Rates)

The planning horizon is divided into two subperiods as delineated by Figure 2.

<table>
<thead>
<tr>
<th>change in the set of exchange rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1(j,i)</td>
</tr>
<tr>
<td>first subperiod</td>
</tr>
<tr>
<td>planning horizon</td>
</tr>
</tbody>
</table>

It is assumed that the set of the exchange rates can change only once during the planning horizon, lest the model becomes too complicated. The first subperiod
FIGURE 1  Flow Model

England (€)

affiliate 2

borrowing in €
X2(2,2)

variable cost
fixed cost
income tax

loan in €
X2(2,3)

interest in €
X3(2,3)

loan in Fmk
X2(3,2)

interest in Fmk
X3(3,2)

USA ($)

borrowing in $
X2(1,1)

production
X1(1)

interest in $

Finland (Fmk)

affiliate 3

borrowing in Fmk
X2(3,3)

production
X1(3)

variable cost
fixed cost
income tax

loan in Fmk
X2(3,1)

interest in Fmk
X3(3,1)

loan in $
corresponds to the period prior to the potential change, while the second subperiod corresponds to the period after it.

Denote:

\[ q \] = index for the state of the world (i.e. the set of exchange rates) that occurs.

\[ Q = 3 \] = number of possible states of the world that are accounted for in the model.

\[ p(q) \] = probability of occurrence of the q:th state of the world. (The central management's subjective estimate.)

The probabilities in the numerical examples will be given in the next chapter.

\[ A_1(j,i) \] = the exchange rate of currency j in country i during the first subperiod.

\[
\begin{align*}
A_1(1,1) &= 1.00\$/\$ \\
A_1(2,1) &= 2.30\$/\£ \\
A_1(3,1) &= 0.27\$/Fmk
\end{align*}
\]

\[
\begin{align*}
A_1(1,2) &= 0.44\£/\$ \\
A_1(2,2) &= 1.00\£/\£ \\
A_1(3,2) &= 0.12\£/Fmk
\end{align*}
\]

\[
\begin{align*}
A_1(3,1) &= 3.8Fmk/\$ \\
A_1(3,2) &= 8.6Fmk/\£ \\
A_1(3,3) &= 1.0Fmk/Fmk
\end{align*}
\]

\[ A_2(j,i,q) \] = the exchange rate of currency j in country i during the second subperiod if the q:th state of the world occurs.

The "second-subperiod exchange rates" in the numerical examples will be discussed in the next chapter.

---

2The fictitious numerical example is given stepwisely after each relevant point.
2.2 MATHEMATICAL MODEL

2.2.1 Sales/Capacity Constraints

Denote:

\[ X_1(i) = \text{the number of units of the product produced and sold in country } i. \]

\[ B_1(i) = \text{the minimum of the sales potential and the production capacity in country } i. \]

\[ B_1(1) = 50 \text{ units} \]
\[ B_1(2) = 50 \text{ units} \]
\[ B_1(3) = 20 \text{ units} \]

Neither the sales potential nor the production capacity must be exceeded in the model. Inventory levels are kept unchanged in our one-product model for simplicity. Thus we have:

\[ X_1(i) \leq B_1(i) \quad i=1,\ldots,nc. \]

2.2.2 Local Borrowing Constraints

Denote:

\[ X_2(i, i) = \text{the loans raised by affiliate } i \text{ from the local money market in country } i. \]

\[ B_2(i) = \text{the maximum amount of loans that can be raised from the local money market in country } i. \]

\[ B_2(1) = 20,000 \text{ ($)} \]
\[ B_2(2) = 3,000 \text{ (£)} \]
\[ B_2(3) = 5,000 \text{ (Fmk)} \]

We have:

\[ X_2(i, i) \leq B_2(i) \quad i=1,\ldots,nc. \]

In the numerical example the English and the Finnish affiliate are short of funds.
2.2.3 Decisions on the Rate of Interest on Interaffiliate Loans: Interaffiliate Interest Rate Constraints

It is shown below how the interest rate on each interaffiliate loan can be treated as a decision variable. Earlier planning models of the multinational firm have omitted these decision variables.

Denote:

\[ X_2(i,j) = \text{the loan granted by affiliate } i \text{ to affiliate } j \text{ in the currency of country } i. \text{ The loan is raised during the first subperiod, and repaid after the planning horizon.} \]

\[ X_3(i,j) = \text{interest paid on the interaffiliate loan } X_2(i,j). \text{ It is paid by affiliate } j \text{ to affiliate } i \text{ during the second subperiod in the currency of country } i. \]

\[ A_3(i,j) = \text{the lower bound on the absolute interest rate on the loan granted by affiliate } i \text{ to affiliate } j. \]

\[
\begin{align*}
A_3(1,2) &= 0.10 \\
A_3(2,1) &= 0.10 \\
A_3(1,3) &= 0.10 \\
A_3(2,3) &= 0.10 \\
A_3(3,1) &= 0.10 \\
A_3(3,2) &= 0.10
\end{align*}
\]

\[ A_4(i,j) = \text{the upper bound on the absolute interest rate on the loan granted by affiliate } i \text{ to affiliate } j. \]

\[
\begin{align*}
A_4(1,2) &= 0.10 \\
A_4(2,1) &= 0.10 \\
A_4(1,3) &= 0.10 \\
A_4(2,3) &= 0.10 \\
A_4(3,1) &= 0.10 \\
A_4(3,2) &= 0.10
\end{align*}
\]

The interest rates on interaffiliate loans can be manipulated within predetermined bounds only. If the interest rates are made too low or too high central banks and tax authorities are bound to step in. Thus we have:

Lower bounds

\[ A_3(i,j)X_2(i,j) - X_3(i,j) \leq 0 \quad i=1,\ldots,nc \\
\text{ } \text{ } \quad j=1,\ldots,nc \]

Upper bounds

\[ -A_4(i,j)X_2(i,j) + X_3(i,j) \leq 0 \]

\footnote{For a numerical example with differing lower and upper bounds on interaffiliate interest rates see [3], ch. 5.}
2.2.4 Cash Flow Constraints

Denote:

\[ X_4(i,q) = \text{a distress loan raised from the local money market during the second subperiod in the currency of country i, if the q:th state of the world occurs.} \]

\[ X_5(i,q) = \text{the profit before taxes shown in books by affiliate i for tax assessment in country i, if the q:th state of the world occurs.} \]

\[ A_5(i) = \text{the portion of the sales price in country i which is collected in cash.} \]
\[ A_5(1) = 0 \quad (\$) \]
\[ A_5(2) = 0 \quad (£) \]
\[ A_5(3) = 0 \quad (\text{Fmk}) \]

\[ A_6(i)^* = \text{the portion of the sales price in country i which is collected in accounts receivable.} \]
\[ A_6(1) = 300 \quad (\$) \]
\[ A_6(2) = 130 \quad (£) \]
\[ A_6(3) = 1000 \quad (\text{Fmk}) \]

\[ A_7(i) = \text{the cash portion of the variable cost of the product in country i.} \]
\[ A_7(1) = 210 \quad (\$) \]
\[ A_7(2) = 91 \quad (£) \]
\[ A_7(3) = 770 \quad (\text{Fmk}) \]

\[ A_8(i)^* = \text{the accounts-payable portion of the variable cost of the product in country i.} \]
\[ A_8(1) = 0 \quad (\$) \]
\[ A_8(2) = 0 \quad (£) \]
\[ A_8(3) = 0 \quad (\text{Fmk}) \]

\[ A_9(i) = \text{the absolute rate of interest paid on the loans from the local money market in country i. It is paid in cash.} \]
\[ A_9(1) = 0.11 \quad (11\%) \]
\[ A_9(2) = 0.11 \]
\[ A_9(3) = 0.11 \]

\[ A_{10}(i,j) = \text{the absolute stamp-duty rate on the inter-affiliate loan granted by affiliate i to affiliate j. It is paid by the granting affiliate in the currency of country i.} \]
\[ A_{10}(1,2) = 0.01 \quad A_{10}(1,3) = 0.01 \quad (1\%) \]
\[ A_{10}(2,1) = 0.01 \quad A_{10}(2,3) = 0.01 \]
\[ A_{10}(3,1) = 0.01 \quad A_{10}(3,2) = 0.01 \]

*These constants are not needed until the next set of constraints is presented.*
A11(i) = the absolute rate of interest paid on the
distress loan raised in country i.
A11(1) = 0.25  (25%)
A11(2) = 0.25
A11(3) = 0.25

A12(i) = the absolute tax rate (on profit) in country i. Taxes are paid in cash.
A12(1) = 0.45  (45%)
A12(2) = 0.45
A12(3) = 0.45

B3(i) = the initial cash in bank less the predetermined cash outflows plus the predetermined cash inflows less the required minimum closing cash in country i.
B3(1) = 12 500  ($)
B3(2) = 4 000   (£)
B3(3) = 10 000  (Fmk)

It must be required separately for each affiliate that
cash outflows less cash inflows ≤
initial cash less minimum closing cash.

Thus we have:

\[
\begin{align*}
\text{variable cost} & \quad \text{interaffiliate} \\
\text{cash sales} & \quad \text{in cash} & \quad \text{local loans} & \quad \text{local loans} \\
-\text{A5(i)X1(i)} & + \text{A7(i)X1(i)} & - \text{X2(i,i)} & + \text{A9(i)X2(i,i)} \\
\text{interaffiliate} & & & \\
\text{nc loans granted} & \quad \text{nc stamp-duty} & \quad \text{interest on} & \\
+ \sum_{j=1}^{j\neq i} \text{X2(i,j)} & + \sum_{j=1}^{j\neq i} \text{A10(i,j)X2(i,j)} & & \\
\text{interaffiliate} & & \text{interest} & \\
\text{nc loans received} & \quad \text{nc received} & & \\
- \sum_{j=1}^{j\neq i} \text{A1(j,i)X2(j,i)} & - \sum_{j=1}^{j\neq i} \text{X3(i,j)} & & \\
\text{interaffiliate} & \text{distress} & \text{interest on} & \\
\text{nc interest paid} & \text{loans} & \text{distress loan} & \\
+ \sum_{j=1}^{j\neq i} \text{A2(j,i,q)X3(j,i)} & - \text{X4(i,q)} & + \text{A11(i)X4(i,q)} & \\
\text{interaffiliate} & \text{initial cash less} & \text{minimum closing cash etc.} & \\
+ \text{A12(i)X5(i,q)} & \leq \text{B3(i)} & i=1,\ldots,\text{nc} & q=1,\ldots,Q.
\end{align*}
\]

\(^5\)In the model cash is held in the currency of the home country of the relevant affiliate. Any foreign currency is always converted into the pertinent currency.
2.2.5 Operational Treatment of Taxation: Equations for Book Profit and Loss

The book profit and loss are separately defined for each affiliate with the help of proper auxiliary equations. Conventionally this has not been done, but that manner of proceeding is incorrect, since it implies a) that book profits and losses are treated analogously in taxations, and b) that the relevant government, in the model, is supposed to subsidize in cash the pertinent affiliate in the case of a book loss.

Denote:

\[ X_6(i,q) = \text{the loss before taxes shown in books by affiliate } i \text{ for tax assessment in country } i, \text{ if the } q\text{th state of the world occurs.} \]

\[ A_{13}(i) = A_5(i) + A_6(i) - A_7(i) - A_8(i). \]

\[ B_4(i) = \text{the deductible fixed cost plus the depreciation shown in books by affiliate } i \text{ for tax assessment in country } i. \]

\[
\begin{align*}
B_4(1) &= 3\,500 \quad (\text{\$}) \\
B_4(2) &= 1\,300 \quad (\text{\£}) \\
B_4(3) &= 5\,000 \quad (\text{Fmk})
\end{align*}
\]

We have:

\[
\begin{align*}
\text{contribution} & \quad \text{interest on local loans} & \quad \text{nc stamp-duty} \\
A_{13}(i)X_1(i) & - A_9(i)X_2(i,i) & - \sum_{j=1}^{\text{nc}} A_{10}(i,j)X_2(i,j) \\
& & j \neq i
\end{align*}
\]

\[
\begin{align*}
&\quad \text{nc exchange gain on loans} & \quad \text{nc received exchange rate} \\
&+ \sum_{j=1}^{\text{nc}} \left[ A_1(j,i) - A_2(j,i,q) \right] X_2(j,i) & + \sum_{j=1}^{\text{nc}} X_3(i,j) \\
&j \neq i \text{ first subperiod exchange rate} & j \neq i
\end{align*}
\]

\[
\begin{align*}
&\quad \text{interest on interaffiliate} & \quad \text{interest on distress loan} \\
&- \sum_{j=1}^{\text{nc}} A_2(j,i,q)X_3(j,i) & - A_1(i)X_4(i,q) \\
&j \neq i
\end{align*}
\]

\[
\begin{align*}
&\quad \text{book profit} & \quad \text{book loss} = \text{deductible fixed cost + depreciation} \\
&- X_5(i,q) & + X_6(i,q) = B_4(i) \\
&i = 1, \ldots, \text{nc} & q = 1, \ldots, Q
\end{align*}
\]
2.2.6 Operational Treatment of the Objective Function: Equations for Internal Profit and Loss, and the Objective Function

The objective of the model is to maximize the global expected net internal income of the multinational firm over the planning horizon. The proper auxiliary equations define the internal profit and loss separately for each affiliate in the currency of the relevant host country. These profits and losses are converted in calculations into a common unit in the objective function. A "geocentric operating philosophy" is thus indicated. Technically the above manner of proceeding facilitates a flexible handling of the multiple-currency situation.

Denote:

\[ X_7(i,q) = \text{the profit after taxes in country } i \text{ in the currency of country } i \text{ from the firm's internal point of view, if the } q:th \text{ state of the world occurs.} \]

\[ X_8(i,q) = \text{the loss after taxes in country } i \text{ in the currency of country } i \text{ from the firm's internal point of view, if the } q:th \text{ state of the world occurs.} \]

\[ B_5(i) = \text{the fixed cost in country } i \text{ from the firm's internal point of view, in the currency of country } i. \]

\[
\begin{align*}
B_5(1) &= 3 \, 000 \quad (\$) \\
B_5(2) &= 1 \, 000 \quad (£) \\
B_5(3) &= 4 \, 000 \quad (Fmk)
\end{align*}
\]

\[ C_1(i,q) = \text{the translation coefficient on the net internal income realized in affiliate } i, \text{ set by the central management for the } q:th \text{ state of the world.} \]

The translation coefficients of the numerical examples will be given in the next chapter.

We have:

\[
\text{contribution margin} = A_3(i)X_1(i) - A_9(i)X_2(i,i) - \sum_{j=1, \, j \neq i}^{\text{nc}} A_{10}(i,j)X_2(i,j)
\]
The objective function is

\[
\text{maximize } E(P) = \sum_{q=1}^{Q} \sum_{i=1}^{\text{nc}} \text{p}(q) \text{C1}(i,q) [X_7(i,q) - X_8(i,q)].
\]

Finally it is required that all the decision variables of this two-stage linear programming\(^6\) model must be non-negative.

\(^6\)Known also as "linear programming under uncertainty".
3 Solution of Two Fictitious Numerical Examples

3.1 Hedging Against Devaluation

Consider, first, the fictitious numerical example with a devaluation of the pound to be expected. Assume that the central management of the multinational firm estimates that the pound will not be devalued during the planning horizon with probability \( p(1) = 0.4 \), that the pound will be devalued with 20% with probability \( p(2) = 0.4 \), and with 30% with probability \( p(3) = 0.2 \).

The second-subperiod exchange rates \( A_2(j,i,q) \) (cf. p. 6.) are then

\[
\begin{align*}
A_2(1,1,1) &= 1.00\$/\$ \\
A_2(2,1,1) &= 2.30\$/\£ \\
A_2(3,1,1) &= 0.27\$/\text{Fmk} \\
A_2(1,2,1) &= 1.00\$/\$ \\
A_2(2,2,1) &= 1.80\$/\£ \\
A_2(3,2,1) &= 0.27\$/\text{Fmk} \\
A_2(1,3,1) &= 3.8\$/\$ \\
A_2(2,3,1) &= 8.6\$/\£ \\
A_2(3,3,1) &= 1.0\$/\text{Fmk} \\
A_2(1,2,2) &= 0.56\$/\$ \\
A_2(2,2,2) &= 1.00\$/\£ \\
A_2(3,2,2) &= 0.15\$/\text{Fmk} \\
A_2(1,3,2) &= 3.8\$/\$ \\
A_2(2,3,2) &= 6.7\$/\£ \\
A_2(3,3,2) &= 1.0\$/\text{Fmk} \\
A_2(1,2,3) &= 0.63\$/\$ \\
A_2(2,2,3) &= 1.00\$/\£ \\
A_2(3,2,3) &= 0.17\$/\text{Fmk} \\
A_2(1,3,3) &= 3.8\$/\$ \\
A_2(2,3,3) &= 6.0\$/\£ \\
A_2(3,3,3) &= 1.0\$/\text{Fmk}.
\end{align*}
\]

The translation coefficients \( C_1(i,q) \) (cf. p. 12) are

\[
\begin{align*}
C_1(1,1) &= 1.00\$/\$ \\
C_1(2,1) &= 2.30\$/\£ \\
C_1(3,1) &= 0.27\$/\text{Fmk} \\
C_1(1,2) &= 1.00\$/\$ \\
C_1(2,2) &= 1.80\$/\£ \\
C_1(3,2) &= 0.27\$/\text{Fmk} \\
C_1(1,3) &= 1.00\$/\$ \\
C_1(2,3) &= 1.60\$/\£ \\
C_1(3,3) &= 0.27\$/\text{Fmk}.
\end{align*}
\]

The solution of this version of the fictitious numerical example is delineated by Figure 3. In the optimal solution the multinational firm concentrates its borrowing in one currency, i.e. the pound. The excess pounds are sent to the Finnish affiliate, which is short of funds, and to the affiliate in the U.S.A. These pounds are subsequently converted into Finnish marks and dollars. Thus the multinational firm is hedged against the potential devaluation of the pound.
3.2 DIVERSIFYING THE BORROWING SOURCES IN THE CASE OF FLOATING CURRENCIES

Let us consider what happens in the numerical example if, instead of the potential devaluation, the pound is made to float. Let us say that the pound may float in the (-9%, 9%) range relative to the dollar and the Finnish mark. Consider, for simplicity, the following three states of the world:

- q=1 The exchange rate of the pound remains unchanged, \( p(1) = 0.4 \) being the relevant subjective probability of the occurrence of this state of the world.
- q=2 The exchange rate of the pound floats downwards by 9%. \( p(2) = 0.3 \).
- q=3 The exchange rate of the pound floats upwards by 9%. \( p(3) = 0.3 \).

In addition to the probabilities of occurrence, i.e. the \( p(q) \)'s, only the second-subperiod exchange rates \( A_{2}(j,i,q) \) and the translation coefficients \( C_{1}(i,q) \) are altered in the fictitious numerical example as follows.

\[
\begin{align*}
A_{2}(1,1,1) &= 1.00 \$/\$ \\
A_{2}(2,1,1) &= 2.30 \$/£ \\
A_{2}(3,1,1) &= 0.27 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
A_{2}(1,1,2) &= 1.00 \$/\$ \\
A_{2}(2,1,2) &= 2.10 \$/£ \\
A_{2}(3,1,2) &= 0.27 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
A_{2}(1,1,3) &= 1.00 \$/\$ \\
A_{2}(2,1,3) &= 2.50 \$/£ \\
A_{2}(3,1,3) &= 0.27 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(1,1) &= 1.00 \$/\$ \\
C_{1}(1,2) &= 2.30 \$/£ \\
C_{1}(1,3) &= 0.27 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(2,1) &= 1.00 \$/\$ \\
C_{1}(2,2) &= 2.10 \$/£ \\
C_{1}(2,3) &= 0.27 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(3,1) &= 3.8 \$/Fmk \\
C_{1}(3,2) &= 6.7 \$/Fmk \\
C_{1}(3,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(4,1) &= 3.8 \$/Fmk \\
C_{1}(4,2) &= 6.7 \$/Fmk \\
C_{1}(4,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(5,1) &= 3.8 \$/Fmk \\
C_{1}(5,2) &= 6.7 \$/Fmk \\
C_{1}(5,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(6,1) &= 3.8 \$/Fmk \\
C_{1}(6,2) &= 6.7 \$/Fmk \\
C_{1}(6,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(7,1) &= 3.8 \$/Fmk \\
C_{1}(7,2) &= 6.7 \$/Fmk \\
C_{1}(7,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(8,1) &= 3.8 \$/Fmk \\
C_{1}(8,2) &= 6.7 \$/Fmk \\
C_{1}(8,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(9,1) &= 3.8 \$/Fmk \\
C_{1}(9,2) &= 6.7 \$/Fmk \\
C_{1}(9,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(10,1) &= 3.8 \$/Fmk \\
C_{1}(10,2) &= 6.7 \$/Fmk \\
C_{1}(10,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(11,1) &= 3.8 \$/Fmk \\
C_{1}(11,2) &= 6.7 \$/Fmk \\
C_{1}(11,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(12,1) &= 3.8 \$/Fmk \\
C_{1}(12,2) &= 6.7 \$/Fmk \\
C_{1}(12,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(13,1) &= 3.8 \$/Fmk \\
C_{1}(13,2) &= 6.7 \$/Fmk \\
C_{1}(13,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(14,1) &= 3.8 \$/Fmk \\
C_{1}(14,2) &= 6.7 \$/Fmk \\
C_{1}(14,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(15,1) &= 3.8 \$/Fmk \\
C_{1}(15,2) &= 6.7 \$/Fmk \\
C_{1}(15,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(16,1) &= 3.8 \$/Fmk \\
C_{1}(16,2) &= 6.7 \$/Fmk \\
C_{1}(16,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(17,1) &= 3.8 \$/Fmk \\
C_{1}(17,2) &= 6.7 \$/Fmk \\
C_{1}(17,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(18,1) &= 3.8 \$/Fmk \\
C_{1}(18,2) &= 6.7 \$/Fmk \\
C_{1}(18,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(19,1) &= 3.8 \$/Fmk \\
C_{1}(19,2) &= 6.7 \$/Fmk \\
C_{1}(19,3) &= 9.3 \$/Fmk
\end{align*}
\]

\[
\begin{align*}
C_{1}(20,1) &= 3.8 \$/Fmk \\
C_{1}(20,2) &= 6.7 \$/Fmk \\
C_{1}(20,3) &= 9.3 \$/Fmk
\end{align*}
\]

Figure 4 illustrates the new optimal solution. The borrowing is diversified. The multinational firm now borrows both pounds and Finnish marks in the numerical example.
FIGURE 4  Floating Exchange Rates

**England (£)**
- Production: 50 units
- Borrowing: 897 £
- Interest: 99 £

**Finland (Fmk)**
- Production: 20 units
- Borrowing: 281 Fmk
- Interest: 31 Fmk

**USA ($)**
- Production: 50 units
- Stamp-duty: 16 $
4 Conclusion

The uncertainty about the future exchange rates was reflected in the objective function in the present model. By modifying the distribution of the exchange rate for the pound sterling from the case of an expected devaluation in the first example into a case of expected movements in either direction in the second example, the optimal financing policy shifted from the use of the devaluation-prone currency into the use of local borrowing sources.

This tendency of diversifying the borrowing sources would be reinforced if the management wished to specify that the firm's exposure position with respect to each foreign currency must remain within certain limits. It has been shown elsewhere [3] how the exposure position can be calculated in the model. This position can then be simply limited by upper and/or lower bounds. Alternatively, we could specify that the various currency positions must be in a given relation to each other.

Naumann-Etienne suggested in his survey [2] that the multinational firm would diversify its international borrowing sources whenever it is impossible to forecast the direction of the changes in the exchange rates. However, it seems intuitively plausible to achieve the same diversification by borrowing from one source and by making simultaneous forward contracts in various currencies in order to achieve the desired distribution of the exposure. The demonstration of how this could be modelled in the framework of a two-stage linear programming remains the subject of another paper.
REFERENCES

