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John R. Darling
Attitudes Toward the Marketing Concept in Developing Countries

Timo Salmi
Multiperiod Production and Financial Planning with Two-Stage Linear Programming

Antti Korhonen
Finnish Beta Coefficients: Evidence of Stationarity and Stability

Antti Korhonen
Finnish Stock Prices: Evidence on Leads and Lags

Reginald Jägerhorn
Sandiland-raportti ja inflaation huomioon ottaminen laskentatoimessa (Summary p. 513)

Eero Pitkänen
Yrityksen yhteiskunnallinen laskentatoimi (Summary p. 535)

Kalevi Kyläkoski
Yrityksen strategiasuunnittelusysteemi (Summary p. 559)

ISSN 0024-3469

The Finnish Journal of Business Economics
Address: Runeberginkatu 22—24, 00100 Helsinki 10, Finland.
Multiperiod Production and Financial Planning with Two-Stage Linear Programming
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Abstract

This paper suggests a two-stage linear programming approach to multiperiod joint production and financial decisions of the firm under a probabilistic future. It is demonstrated that it is not necessary to increase the size of the model exponentially in the multiperiod case. The two-stage nature of the model is retained even with the introduction of multiple time periods. The application of a multi-stage structure is avoided by defining only a single revelation of the future state of the world instead of the several successive revelations conventionally defined in similar multi-period problems. Also the preparation of predictions is made easier with the application of the present approach. Hence the pragmatic applicability of the model for joint production and financial planning under risk is enhanced with the smaller model size and the relaxation of the requirements on the forecasts needed.

1. Introduction

It is the purpose of this paper to present a multiperiod model for planning production and short-term financing in a firm facing probabilistic demand for its products as well as probabilistic costs and revenues by applying the two-stage linear programming approach to a multiperiod planning situation under risk. The model solves the production, inventory, subcontracting and

*I am indebted to Associate Professors Markku Kallio and Jouko Manninen for their helpful comments. I also wish to acknowledge the contribution made by Kaija Korolainen, M.Sc. (Econ.) in the preparation of the elucidating numerical example utilized in this paper. Any errors and inadequacies are, however, my sole responsibility.
sales policy of the firm together with the selection between the on-going operations from current earnings and loan raising from different sources. The solution can be used for preparing flexible budgets for the firm. Thus an extension of the application of the linear programming approach to budgeting is made in this paper. A numerical example illustrates the approach to be presented.

The conventional linear programming approaches to managerial decision-making problems presuppose that the parameters of the model must be estimated as if they were known under conditions of certainty. There are, for example, many production and financial planning situations where ignoring the stochastic nature of the parameters may lead to poor solutions. Two-stage linear programming has been introduced as one remedy to provide for the probabilistic nature (risk) in the technology matrix, the right-hand side, and the objective, when discrete probability distributions are assumed for the parameters. The pioneering application to managerial allocation problems was presented by Ferguson and Dantzig [10] in 1956, and to production planning by Elmaghraby [9] in 1959. The relevant mathematical foundations have been laid especially by Dantzig [6], Dantzig and Madansky [7], and later by El Agizy [8], and Walkup and Wets [29].

The fundamental idea of two-stage linear programming is to divide the decision variables into two distinct categories, viz. the first-stage decision variables to indicate the choice of current action, and the second-stage decision variables for each considered potential state of the world to indicate the relevant future action or to reflect the consequences which result from the revelation of the true state of the world.

Only a single time period, which is subdivided into two stages, is usually covered in the two-stage linear programming applications. Examples of such one-period applications are given e.g. by the bank asset and liability management models, developed to account for the uncertainties of the money market, presented by Thore [25], [26] in 1968 and 1971, Cohen and Thore [5] in 1970, and Aghili, Cramer and Thompson [2] in 1975. Examples of one-period production planning models are given by the models presented

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1. For a comprehensive discussion on the linear programming approach to budgeting see [14] or [15].

2. In this paper I use the term "two-stage linear programming" rather than "linear programming under uncertainty", since the former more accurately specifies the method under discussion. See [21] and [23] for a discussion on different kinds of linear programming under uncertainty (risk).

3. To be precise: the first-stage variables indicate action which is by definition unchanged for the different states of the world. Usually, but not necessarily, this will mean the current action in managerial applications. The first-stage variables can actually also relate to actions after the revelation, as demonstrated in [24, Ch. 5].

In order to rectify the simplification caused by the application of the one-period approach to decision-making problems under risk, multi-stage linear programming has been suggested. Wagner [27] presents a multiperiod production scheduling example in his well-known textbook. Booth [3] presented in 1972 a two-period model for programming bank portfolios under risk. His approach is based on the formulation by Walkup and Wets [29] and it can actually be regarded as a reduced three-stage linear programming model. Kallio and Zambruno [17] presented in 1976 a probabilistic investment and financing model for a T-period problem. After assuming a penalty cost on short-term cash shortage and surplus, they utilize a reformulation to scale problem down to a manageable size.

The multi-stage linear programming approach entails several successive revelations of the state of the world instead of the single revelation of the two-stage approach. The model size grows prohibitively with the increased number of revelations. On the other hand, if only a single revelation from an assumed set of potential multiperiod futures were postulated, the two-stage linear programming approach would be fully applicable and legitimate for multiperiod problems also. To my knowledge this fact was first utilized by Lane [19] in his two-stage linear programming model for short-term money management for bank portfolios under risk. This paper elaborates the approach and present a new application of the principle discussed.

2. Review of Relevant Two-Stage Linear Programming Features

In this chapter the statement of the two-stage linear programming problem is reviewed. The correspondence between the stages of two- and multi-stage linear programming and the periods of the planning horizon is discussed and illustrated.

21. Statement of the Two-Stage Linear Programming Problem

As an introduction to two-stage linear programming consider a decision-making situation delineated by the following two-stage linear programming
model.\textsuperscript{4} This formulation retains the linearity of a linear programming model in the face of a risky planning situation providing discrete probability distributions for the risky parameters.

\[
\max E(w) = \sum_{j=1}^{k} E(c_j)x_j + \sum_{q=j}^{Q} p_q \sum_{j=k+1}^{n} d_{qj}y_{qj}
\]

subject to

\[
\sum_{j=1}^{k} a_{ij}x_j = b_i \quad i = 1, \ldots, g \quad \text{(first-stage constraints)}
\]

\[
\sum_{j=1}^{k} a_{qj}x_j + \sum_{j=k+1}^{n} h_{qj}y_{qj} = b_{qj} \quad i = g+1, \ldots, m \quad q = 1, \ldots, Q
\]

all \(x_j \geq 0\) and all \(y_{qj} \geq 0\).

\textsuperscript{5} In matrix-notation this can be written as

\[
\max E(w) = E(c'x + d'y)
\]

subject to

\[
A x = b
\]

\[
A x + Hy = b
\]

\[
x \geq 0
\]

\[
y \geq 0.
\]

At the beginning of the planning horizon the values of the first-stage variables \(\tilde{x}\) must be decided. This decision is made in the light of subsequent knowledge. The constants \(a_{ij}\) (\(i = 1, \ldots, g; j = 1, \ldots, k\)) and \(b_i\) (\(i = 1, \ldots, g\)) are known with certainty. It is assumed that there are \(Q\) possible outcomes \((d_{qj}, a_{qj}, b_{qj})\) for which the probability of occurrence is \(P\{d_{qj}, a_{qj}, b_{qj}\} = p_{q}\). The existence of these \(Q\) potential states of the world is the source of risk in the model. After the first-stage variables \(\tilde{x}\) have been fixed, the true state of the

\textsuperscript{4} The formulation is adapted from [28].

\textsuperscript{5} If continuous probability distributions were assumed, a quadratic programming problem would result.
world is revealed (revelation). After this the values of the proper elements of the second-stage variable vector $\tilde{y}$ reflect the further action. (These elements are $y_{qj}$ ($j=k+1, \ldots, n$), if the $q$:th state of the world occurs.) Since the true state of the world is not known until the revelation, the economic consequences of all potential $\tilde{y}$ values have to be reflected in the objective function. (The values of the second-stage variables for the different potential states of the world are based on the first-stage decisions.) The economic consequences of the recourse actions (contingency plans) $y_{qj}$ ($j = k+1, \ldots, n$) are given by

\[
(1) \sum_{j=k+1}^{n} d_{qj} y_{qj}
\]

in the case of the occurrence of the $q$:th state of the world. This expression has to be weighed by the concomitant probability of occurrence, i.e. by $p_{qj}$ because the expected value of the objective functional is to be maximized. — Thus the economic consequences of the current decisions are appropriately reflected in the model.

Expression (1) reflects the economic consequences of the risk related to recourse actions in the model. It was assumed that only $Q$ different states of the world are accounted for in the planning situation. The second-stage variables can behave in accordance only. The question naturally arises what happens if none of the predicted states of the world occur. This is not essential, however, because in two-stage linear programming the decision maker is actually interested in the current decision, i.e. in assessing the $\tilde{x}$ values. The $\tilde{y}$ values are not strictly designated as decisions on true future actions. Their basic function is to reflect the economic impact of management’s current decision on the objective functional in accordance with the decision makers’ subjective estimates of the future.

The two-stage linear programming formulation which was presented above is called the slack formulation of two-stage linear programming. In applying the slack formulation the model is conventionally formulated in a way which technically preserves feasibility for all considered states of the world.\(^6\) Another formulation, called the fat formulation of two-stage linear programming, can be used.\(^7\) In the fat formulation the possibility of infeasibility in the problem is not deliberately avoided.

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\(^6\) The application to be presented will illustrate this fact.
\(^7\) See [22, pp. 104–106] for a discussion on the two formulations.
This section illustrates the correspondence between the length of the planning horizon and the structure of two- and multiperiod linear programming approaches.

Figure 2—1 delineates the conventional correspondence between the stages of the two-stage linear programming approach and a one-period planning horizon. As indicated earlier in this paper, this usage has been customary in applying the two-stage linear programming technique to managerial decision-making problems.

Figure 2—1

<table>
<thead>
<tr>
<th>revelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>first stage</td>
</tr>
<tr>
<td>the one-period planning horizon</td>
</tr>
<tr>
<td>different states of the world</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>y</td>
</tr>
<tr>
<td>first-stage decisions</td>
</tr>
<tr>
<td>y</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>y</td>
</tr>
<tr>
<td>second-stage decisions</td>
</tr>
</tbody>
</table>

Figure 2—2 delineates the correspondence between the stages of the multiperiod linear programming approach and a multiperiod planning horizon. It is obvious from the figure that the number of variables will grow so rapidly with the number of periods that the approach is not applicable as such in practice.

Figure 2—3 delineates an alternative correspondence between the stages of the two-stage linear programming approach and a multiperiod planning horizon. Only a single revelation of the assumed set of potential multi-period futures is postulated. A decision-making model constructed along this line of thought clearly does not exhibit an exponential growth with an increased number of periods of the planning horizon.
Figure 2-2

<table>
<thead>
<tr>
<th>first stage</th>
<th>second stage</th>
<th>third stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>first revelation</td>
<td>second revelation</td>
<td>third revelation</td>
</tr>
</tbody>
</table>

the multiperiod planning horizon

first set of different states of the world

x

first-stage decisions

y

second-stage decisions

z

z

third-stage decisions

z

z

z

Figure 2-3

revelation

<table>
<thead>
<tr>
<th>first stage</th>
<th>second stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>first revelation</td>
<td>second revelation</td>
</tr>
</tbody>
</table>

the multiperiod planning horizon

different states of the world

x

first-stage decisions

y

second-stage decisions

y

y

third-stage decisions

y

y

fourth period
It can also be argued in favor of the approach depicted by Figure 2—3 that it is compatible with the common practice of predicting a few alternative futures rather than predicting the combinatorial structure of futures indicated by Figure 2—2. The economic benefits from constructing and utilizing in actual practice an accurate combinatorial prognosis are often more than lost by the increase in information processing costs as compared with using the "single-revelation" approach. Furthermore, the limited ability to predict the future may also render the combinatorial approach impractical. Consequently, for example the common procedure of preparing a pessimistic, a neutral, and an optimistic estimate of future conditions seems one relevant possibility for the single-revelation approach. Naturally, other similar schemes can be relevant, too.

The production and short-term financing model to be presented for a risky planning situation utilizes the single-revelation approach depicted by Figure 2—3.

Mathematically the combinatorial approach represented by Figure 2—2 and the single-revelation approach represented by Figure 2—3 can be considered interchangeable, if the necessary number of states of the world is defined for the single revelation approach.³ For real-life decision-making applications the lines of thought represented by the two figures differ in a fundamental way, nevertheless.

3. Exposition of the Planning Model

31. General Description of the Hypothetical Planning Situation

Consider a firm producing and storing I different products for sales to its customers. It is the aim of the model to maximize the undiscounted expected net income to the firm over a planning horizon of T periods. Taxation is omitted for convenience.⁹

The demand for the firm's products as well as the costs and revenues are assumed to be probabilistic (i.e. a planning situation under risk is assumed). Q different combinations of these risky parameters are considered by the decision maker. It is assumed that the decision maker can assess a subjective probability of occurrence, denoted by \( p(q) \), for each outcome, i.e. for each state of the world considered. Because of the probabilistic nature of the demand for the products, either shortages or overproduction may occur. In

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³ See e.g. [30] and [17].
⁹ For the inclusion of taxation into two-stage linear programming models for decision making see [24, Ch. 4].
the case of shortages, the required units have to be purchased from the
competitors at a high price to ensure customer satisfaction, which is
considered to be of overriding importance in the firm's line of business.\textsuperscript{10} In
the case of overproduction either sales at a reduced price (dumping) or
inventory-carrying can be resorted to.

The production, purchase, sales, and inventory policy causes the firm both
cash inflows and outflows. Sufficient cash inflows must be generated by the
current operations and the financial activities of the firm to cover the cash
outflows. In the planning situation under observation it is assumed that cash
inflows are generated from the operations and from loan making; and that
cash outflows are caused by the operations, repayment of loans and interest
payments.\textsuperscript{11} Both the physical and the financial operations are to be planned
simultaneously in order to attain the goal of (expected) net income maximiza-
tion over the planning horizon.

The probabilistic nature of the parameters of the planning situation also
affects the potential cash flows. It is even possible that for some alternative
futures no feasible solution generating adequate cash inflows over cash
outflows can be found. If even one such state of the world exists for the
planning situation, no feasible solution is found for the legitimate states of the
world, either, when the two-stage linear programming approach is applied
without defining proper recourse actions to provide for the infeasibilities. In
the planning situation under observation, this contingency can be circumven-
ted by defining a distress loan, with a high penalty cost, which can be raised in
the case of an emergency.

This fact indicates that the slack formulation of two-stage linear program-
ming is used in constructing the model.

The probabilistic nature of the parameters of the planning situation is
accounted for by assessing separate purchase, inventory, sales and distress
borrowing strategies for each considered state of the world in order to be able
to maximize the expected value of the net income. The decisions which are
made conditional on the occurring state of the world will be defined as
second-stage variables in the decision-making model. The second-stage
decisions are interrelated through the first-stage decisions, which must be
fixed before the revelation of the true state of the world.\textsuperscript{12} In the planning

\textsuperscript{10} Although a particular planning situation is thus specified for expository purposes,
the formulation can be easily modified to accommodate also different multi-period
planning situations under risk.

\textsuperscript{11} The cash flows related to other factors are treated as predetermined.

\textsuperscript{12} Note that the application of a separate model for each state of the world would not
ensure a global optimum because of the interaction.
situation under observation the first-stage decisions cover the production and the normal borrowing policy over the planning horizon.\textsuperscript{13}

The rest of this chapter presents the two-stage linear programming model for solving the optimal production, purchase, sales, inventory, and borrowing policy for the depicted firm. The detailed discussion of the model has to be confined to selected items only, because of space limitations. Thus a host of the assumptions made have to be deduced directly from the mathematical presentation of the model.

32. Decision Variables of the Two-Stage Linear Programming Model

The decision variables indicated by the description of the planning situation in the previous section are defined below. The input data constituting the constants of the model are listed in the appendix at the end of the paper. All the decision variables are required to be non-negative.

The first-stage decision variables are:

\begin{align*}
\text{PROD}_{it} & = \text{the number of units of product } i \text{ produced in period } t. \\
\text{LOAN}_{it} & = \text{the normal borrowing in period } t. \text{ This loan is to be paid back in period } t+1.
\end{align*}

The second-stage decision variables are:

\begin{align*}
\text{INVE}_{itq} & = \text{the inventory of product } i \text{ at the end of period } t \text{ if the } q:\text{th state of the world occurs.} \\
\text{PURC}_{itq} & = \text{the number of units of product } i \text{ purchased from competitors in period } t \text{ in order to fulfil the customers' demands if the } q:\text{th state of the world occurs.} \\
\text{SALE}_{itq} & = \text{the number of units of product } i \text{ sold in period } t \text{ if the } q:\text{th state of the world occurs.} \\
\text{DUMP}_{itq} & = \text{the number of units of product } i \text{ sold at a reduced price (dumping) if the } q:\text{th state of the world occurs.} \\
\text{DIST}_{itq} & = \text{the distress loan raised in period } t \text{ if the } q:\text{th state of the world occurs. It is to be paid back in period } t+1. \\
\text{CLCA}_{itq} & = \text{the closing cash in period } t \text{ if the } q:\text{th state of the world occurs.}
\end{align*}

To demonstrate the growth of the model if the combinatorial approach (cf. Figure 2—2) were applied, consider for example the closing inventory. To simplify, assume that exactly $q$, different states of the world were possible for each period. In that case the number of variables needed for the closing inventories would be $I \cdot \prod_{t=1}^{T} q_t$ in the combinatorial approach compared

\textsuperscript{13}Nothing would be changed, in principle, if only the current production and/or borrowing decision were to be determined prior to the revelation.
with $I \cdot T \cdot Q$ variables in the single-revelation approach. For example, if $I = 1$, $T = 4$, and $q = q_t = 3$ ($t = 1, \ldots, 4$), we would have $1 \cdot 3^4 = 81$ relevant decision variables for the combinatorial approach and $1 \cdot 4 \cdot 3 = 12$ for the single-revelation approach. A similar pattern could be observed in the number of constraints.

33. Objective Function

The objective of the model is to maximize the firm's undiscounted expected pre-tax net income over the multiperiod planning horizon. To obtain the expected value of the chosen criterion, the economic consequences of the alternative second-stage actions are weighed by the subjective probabilities of occurrence, i.e. by $p(q)$, of each considered state of the world, since the true state of the world is not known when the first-stage decisions have to be fixed.

Weighing by the relevant probabilities of occurrence to obtain an expected value of the net income, but ignoring the variance of the potential outcomes, means the implicit assumption that the decision maker is neutral to risk.\(^{14}\)

Net income is adopted as the criterion. This is naturally not the only possible choice.\(^{15}\) The net income is used as the criterion is not discounted in our model for convenience.\(^{16}\)

\[
\begin{align*}
\text{Maximize} & \quad \sum_{q=1}^{Q} p(q) \sum_{t=1}^{T} \left\{ A2_{ttq} \text{SALE}_{itq} + \sum_{i=1}^{I} A4_{ttq} \text{DUMP}_{itq} \right\} \\
& - \sum_{i=1}^{I} A7_{ttq} \text{PURC}_{itq} - \sum_{i=1}^{I} A9_{ttq} \text{INVE}_{itq} - \text{All-DIST}_{ttq} \\
& - \sum_{i=1}^{I} A6_{ttq} \text{PROD}_{it} - A10_q \text{LOAN}_t \\
\end{align*}
\]

\(^{14}\) This assumption is often revised by defining additional restrictions on the solution to preclude the possibility of highly undesirable outcomes.

\(^{15}\) For a discussion see e.g. [20, pp. 9–12] and [11, pp. 678–679].

\(^{16}\) It has even been demonstrated that not discounting the income stream is well-founded on certain grounds [4]. See also [1] for a critical discussion about the validity of discounting.
34. Operating Constraints

This section gives the constraints relating to the physical activities of the firm.

Capacity Constraints

\[
\sum_{i=1}^{I} A_{ik} \text{PROD}_{it} \leq B_{kt} \quad (k=1, \ldots, K; \ t=1, \ldots, T)
\]

It is assumed that K different dimensions of capacity are needed in producing the products. The capacities available must not be exceeded. The capacities are treated as predetermined in our model.\(^{17}\) Since the capacity constraints are unaffected by the state of the world, they are all first-stage constraints in our model. All the other constraints will be second-stage constraints. As was mentioned earlier, only one production schedule is defined in our model. Thus PROD\(_{it}\) (i = 1, \ldots, I; t = 2, \ldots, T) are also first-stage variables although they relate to actions after the revelation. This definition is realistic when a rigid production structure is assumed. The contrary assumption would entail no difficulties.

Inventory Balance Equations

\[
\begin{align*}
\text{previous inventory} & - \text{INVE}_{i(t-1)q} & - \text{PROD}_{i} & - \text{PURC}_{i\cdot q} \\
\text{normal sales} & + \text{SALE}_{i\cdot q} & + \text{DUMP}_{i\cdot q} & + \text{INVE}_{i\cdot q} = 0 \\
\end{align*}
\]

(i = 1, \ldots, I; t = 1, \ldots, T; q = 1, \ldots, Q)

The inventory balance equations define the closing inventory for each product, period, and state of the world. For t = 1 INVE\(_{i(t-1)q}\) is a constant indicating the initial inventory at the beginning of the planning horizon.

\(^{17}\)The treatment of decisions on investments in additional production capacity should be straightforward, in principle, along the lines established by Jääskeläinen [12].
Sales Equations

\[ \text{SALE}_{itq} = B2_{itq} \quad (i = 1, \ldots, I; t = 1, \ldots, T; q = 1, \ldots, Q) \]

The sales equations define the normal demand for each product in each period for each state of the world.\(^{18}\) Because of the required policy of always meeting the demand, the normal sales are equated with the demand. As is recalled, purchases from competitors are resorted to in our assumed planning situation if inventories and production are not sufficient to meet the demand. It could be argued here that it is quite common to allow unsatisfied demand, which is either backlogged or lost. A similar formulation would be relevant in that case, too. A penalty cost would have to be assessed for the unsatisfied demand.

35. Financial Constraints

This section gives the constraints relating to the financial flows in the firm.

Cash Balance Equations

The cash balance equations define the closing cash for each period and state of the world.

\[ \begin{array}{cc}
\text{cash from} & \text{collected cash from} \\
\text{normal sales} & \text{normal sales} \\
\hline
\sum_{i=1}^{I} A3_{itq} \text{SALE}_{itq} & + \sum_{i=1}^{I} (A2_{it(t-1)q} - A3_{it(t-1)q}) \text{SALE}_{it(t-1)q} \\
\text{cash from} & \text{collected cash} \\
\text{dumping} & \text{from dumping} \\
\sum_{i=1}^{I} A5_{itq} \text{DUMP}_{itq} & + \sum_{i=1}^{I} (A4_{it(t-1)q} - A5_{it(t-1)q}) \text{DUMP}_{it(t-1)q} \\
\text{normal} & \text{distress} \\
\text{borrowing} & \text{previous} \\
\text{distress borrowing} & \text{closing cash} \\
\text{previous} & \\
\text{closing cash} & \\
+ \text{LOAN}_t + \text{DIST}_{tq} + \text{CLCA}_{(t-1)q} \\
\end{array} \]

\(^{18}\) These equations could naturally be substituted into the other constraints. This is not done for expository purposes.
variable cost of production  

\[ I - \sum_{i=1}^{l} A6_{i\tau} \text{PROD}_{i\tau} - \sum_{i=1}^{l} A8_{i\tau} \text{PURC}_{i\tau} \]

present distress purchases

\[ I - \sum_{i=1}^{l} A8_{(t-1)q} \text{PURC}_{(t-1)q} - \sum_{i=1}^{l} A9_{i\tau} \text{INVE}_{i\tau} \]

earlier distress purchases

inventory carrying expenses

\[ I - \sum_{i=1}^{l} (A8_{(t-1)q} - A7_{(t-1)q}) \text{PURC}_{(t-1)q} - \sum_{i=1}^{l} A9_{i\tau} \text{INVE}_{i\tau} \]

interest on repayment

repayment

closing

cash

normal borrowing

predetermined

and fixed

outflows

less

inflows

\[ - A10_{q} \text{LOAN}_{t} - \text{LOAN}_{t-1} - \text{DIST}_{t-1} - \text{CLCA}_{t\tau} = B3_{t\tau} \]

\((t=1, \ldots, T; \ q=1, \ldots, Q)\)

For \( t = 1 \) CLCA\(_{(t-1)q}\) is a constant indicating the initial cash for the planning horizon. The loans are always repaid during the next period. Borrowing for longer than the one period is, however, indicated by a proper renewing of the loans. Consequently no actual simplification is involved. The slack-formulation of two-stage linear programming is used. Hence the distress borrowing DIST\(_{t\tau}\) with a high penalty cost in the objective function is defined.

**Maximum Borrowing Capability Constraints**

\[ \text{normal} \quad \text{maximum amount of}\]

\[ \text{borrowing} \quad \text{outstanding loans}\]

\[ \text{LOAN}_{t} \leq B4_{t} \ (t = 1, \ldots, T) \]

The borrowing capability must not be exceeded in any of the periods. Because the loans raised are technically treated as one-period loans, the right-hand side of the constraint is the maximum amount of outstanding loans in the relevant period. Only a single source of borrowing with a linear interest is defined in the above for expository convenience. Adding the number of borrowing sources and defining a piecewise increasing interest cost is easily achieved by proper redefinitions.
Minimum Cash Balance Constraints

\[ \text{CLCA}_{tq} \geq B_5, (t = 1, \ldots, T; q = 1, \ldots, Q) \]

A minimum liquidity\(^{19}\) defined by management policies must be maintained to reflect factors not covered by the model. (Such constraints can be classified as management-policy constraints.)

4. Numerical Illustration\(^{20}\)

This chapter gives a hypothetical numerical example elucidating the model.

41. Description of Input Data

The description of the input data for a hypothetical numerical example is given. Before the model can be solved, the data must be converted to correspond the input parameters of the model, which are listed in the appendix at the end of the paper. Since the conversion involves some trivial calculations only, it is not described in this paper.

The firm under observation produces two products, radios and tape recorders for industrial use. The firm has two separate job centers, called the machining line and the assembly line. The capacity of both centers is needed in production. Two different states of the world, a favorable and an unfavorable future, are considered.

<table>
<thead>
<tr>
<th></th>
<th>( q = 1 )</th>
<th>( q = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective probability of occurrence</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>Demand for radios in period 1</td>
<td>3000</td>
<td>2000</td>
</tr>
<tr>
<td>in period 2</td>
<td>3200</td>
<td>2300</td>
</tr>
<tr>
<td>Demand for tape recorders in period 1</td>
<td>2500</td>
<td>1500</td>
</tr>
<tr>
<td>in period 2</td>
<td>2700</td>
<td>1800</td>
</tr>
<tr>
<td>Sales price for normal sales of radios</td>
<td>$80</td>
<td>$70</td>
</tr>
<tr>
<td>in period 1</td>
<td>$80</td>
<td>$70</td>
</tr>
<tr>
<td>in period 2</td>
<td>$80</td>
<td>$70</td>
</tr>
</tbody>
</table>

\(^{19}\) \text{CLCA}_{tq} \geq 0 \text{ naturally always holds by definition.}

\(^{20}\) The numerical example illustrating the model to be presented was prepared and solved by Korolainen in her master's thesis [18].
<table>
<thead>
<tr>
<th>Description</th>
<th>q = 1 favorable future</th>
<th>q = 2 unfavorable future</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales price for normal sales of tape recorders in period 1</td>
<td>$75</td>
<td>$65</td>
</tr>
<tr>
<td>in period 2</td>
<td>$75</td>
<td>$65</td>
</tr>
<tr>
<td>Sales price for dumping sales of radios in period 1</td>
<td>$40</td>
<td>$20</td>
</tr>
<tr>
<td>in period 2</td>
<td>$40</td>
<td>$20</td>
</tr>
<tr>
<td>Sales price for dumping sales of tape recorders in period 1</td>
<td>$30</td>
<td>$10</td>
</tr>
<tr>
<td>in period 2</td>
<td>$30</td>
<td>$10</td>
</tr>
<tr>
<td>Variable cost of a radio (when produced by the firm itself) in period 1</td>
<td>$50</td>
<td>$50</td>
</tr>
<tr>
<td>in period 2</td>
<td>$50</td>
<td>$50</td>
</tr>
<tr>
<td>Variable cost of a tape recorder (when produced by the firm itself) in period 1</td>
<td>$55</td>
<td>$55</td>
</tr>
<tr>
<td>in period 2</td>
<td>$55</td>
<td>$55</td>
</tr>
<tr>
<td>Purchase price of a radio (when bought from competitors) in period 1</td>
<td>$90</td>
<td>$90</td>
</tr>
<tr>
<td>in period 2</td>
<td>$90</td>
<td>$90</td>
</tr>
<tr>
<td>Purchase price of a tape recorder (when bought from competitors) in period 1</td>
<td>$80</td>
<td>$80</td>
</tr>
<tr>
<td>in period 2</td>
<td>$80</td>
<td>$80</td>
</tr>
<tr>
<td>Inventory carrying cost calculated on the ending balance for a radio</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>for a tape recorder</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacity requirements for producing a radio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>on machining line</td>
<td>2h/unit</td>
<td></td>
</tr>
<tr>
<td>on assembly line</td>
<td>2h/unit</td>
<td></td>
</tr>
<tr>
<td>Capacity requirements for producing a tape recorder</td>
<td></td>
<td></td>
</tr>
<tr>
<td>on machining line</td>
<td>3h/unit</td>
<td></td>
</tr>
<tr>
<td>on assembly line</td>
<td>1h/unit</td>
<td></td>
</tr>
<tr>
<td>Capacities available</td>
<td></td>
<td></td>
</tr>
<tr>
<td>on machining line in period 1</td>
<td>9000 h</td>
<td></td>
</tr>
<tr>
<td>in period 2</td>
<td>9000 h</td>
<td></td>
</tr>
<tr>
<td>on assembly line in period 1</td>
<td>7000 h</td>
<td></td>
</tr>
<tr>
<td>in period 2</td>
<td>7000 h</td>
<td></td>
</tr>
<tr>
<td>Initial inventories</td>
<td></td>
<td></td>
</tr>
<tr>
<td>radios</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>tape recorders</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Required minimum closing inventory
radios
  0
tape recorders
  0

Initial cash
$ 10000

Accounts receivable (will be received in cash during period 1)
$130000

Short-term borrowing and accounts payable (must be paid out in cash during period 1)
$135000

Fixed cash expenditures in period 1
  $ 60000
  $ 60000

Maximum permissible outstanding loans
in period 1
  $140000
in period 2
  $140000

Required minimum closing cash for period 1
  $8000
  $8000

Interest rate on normal borrowing, adjusted for the length of the planning period (e.g. 45 days)
  1 %

Penalty charge on distress borrowing
  100 %

<table>
<thead>
<tr>
<th>q = 1</th>
<th>q = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>radio</td>
<td>tape r</td>
</tr>
<tr>
<td>cash portion of normal sales</td>
<td>50 %</td>
</tr>
<tr>
<td>cash portion of dumping sales</td>
<td>20 %</td>
</tr>
<tr>
<td>cash portion of variable cost</td>
<td>100 %</td>
</tr>
<tr>
<td>cash portion of purchase price</td>
<td>80 %</td>
</tr>
<tr>
<td>cash portion of interest</td>
<td>100 %</td>
</tr>
<tr>
<td>cash portion of penalty charge</td>
<td>0 %</td>
</tr>
</tbody>
</table>
42. Solution of the Numerical Example \(^{21}\)

The solution of the numerical example suggests the following production policy.

<table>
<thead>
<tr>
<th></th>
<th>period 1</th>
<th>period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>radios</td>
<td>2389</td>
<td>1948</td>
</tr>
<tr>
<td>tape recorders</td>
<td>1408</td>
<td>1701</td>
</tr>
</tbody>
</table>

The optimal borrowing policy suggested by the solution is to raise the maximum amount of loans, i.e. a two-period loan of $140,000. These suggestions form the basis of the current action.

Separate budgets can be prepared for each considered state of the world and for each period on the basis of these figures and the optimal values of the second-stage variables as given by the computer solution \(^{22}\).

The combined sales, production, inventory and purchase budgets are given below.

\[
\begin{array}{llllll}
\text{Period 1} & \text{radios} & \text{tape r} & \text{radios} & \text{tape r} \\
\text{Initial inventory} & 0 & 0 & 0 & 0 \\
\text{Production} & 2389 & 1408 & 2389 & 1408 \\
\text{Purchases} & 611 & 1092 & 0 & 92 \\
\text{Dumping} & 0 & 0 & 37 & 0 \\
\text{Demand} & 3000 & 3000 & 2000 & 1500 \\
\text{Closing inventory} & 0 & 0 & 352 & 0 \\
\end{array}
\]

\[
\begin{array}{llllll}
\text{Period 2} & \text{radios} & \text{tape r} & \text{radios} & \text{tape r} \\
\text{Initial inventory} & 0 & 0 & 352 & 0 \\
\text{Production} & 1948 & 1701 & 1948 & 1701 \\
\text{Purchases} & 1252 & 999 & 0 & 99 \\
\text{Dumping} & 3200 & 2700 & 2300 & 1800 \\
\text{Closing inventory} & 0 & 0 & 0 & 0 \\
\end{array}
\]

An interesting feature in the solution is the fact that in the occurrence of \(q = 2\) (the 'unfavorable future') radios are to be stored for the demand in the

---

\(^{21}\) The numerical example was solved by UNIVAC-1108 linear programming computer code ILONA. The computer listing is documented in an appendix of reference [18].

\(^{22}\) As is seen, the approach can thus be used to prepare flexible budgets. Consequently, the approach can be considered an extension of the linear programming approach to budgeting.
second period, and the excessive radios are to be sold at a reduced price in the first period. This model behavior is explained when it is recalled that the production policy is determined prior to the revelation of the true state of the world.

Also cash budgets can easily be prepared for each state and period.

<table>
<thead>
<tr>
<th>Period 1</th>
<th>q = 1</th>
<th>q = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL CASH</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td>CASH INFLOWS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales revenue</td>
<td>232500</td>
<td>128648</td>
</tr>
<tr>
<td>From accounts receivable</td>
<td>130000</td>
<td>130000</td>
</tr>
<tr>
<td>Normal borrowing</td>
<td>140000</td>
<td>140000</td>
</tr>
<tr>
<td>Distress borrowing</td>
<td>2680</td>
<td>0</td>
</tr>
<tr>
<td>Total cash available</td>
<td>515180</td>
<td>408648</td>
</tr>
<tr>
<td>CASH OUTFLOWS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable production cost</td>
<td>196890</td>
<td>196890</td>
</tr>
<tr>
<td>Purchases from competitors</td>
<td>113880</td>
<td>7360</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>60000</td>
<td>60000</td>
</tr>
<tr>
<td>Amortization</td>
<td>125000</td>
<td>125000</td>
</tr>
<tr>
<td>Interest</td>
<td>1400</td>
<td>1400</td>
</tr>
<tr>
<td>To accounts payable</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td>Total cash outflows</td>
<td>507170</td>
<td>400650</td>
</tr>
<tr>
<td>CLOSING CASH</td>
<td>8010</td>
<td>7998</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL CASH</td>
<td>8010</td>
<td>7998</td>
</tr>
<tr>
<td>CASH INFLOWS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales revenue</td>
<td>249500</td>
<td>150700</td>
</tr>
<tr>
<td>From accounts receivable</td>
<td>195000</td>
<td>109592</td>
</tr>
<tr>
<td>Normal borrowing</td>
<td>140000</td>
<td>140000</td>
</tr>
<tr>
<td>Distress borrowing</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total cash available</td>
<td>592510</td>
<td>408290</td>
</tr>
<tr>
<td>CASH OUTFLOWS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable production cost</td>
<td>190955</td>
<td>190955</td>
</tr>
<tr>
<td>Purchases from competitors</td>
<td>154080</td>
<td>7920</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>60000</td>
<td>60000</td>
</tr>
<tr>
<td>Amortization</td>
<td>142680</td>
<td>140000</td>
</tr>
<tr>
<td>Interest</td>
<td>1400</td>
<td>1400</td>
</tr>
<tr>
<td>To accounts payable</td>
<td>28470</td>
<td>0</td>
</tr>
<tr>
<td>CLOSING CASH</td>
<td>14925</td>
<td>8015</td>
</tr>
</tbody>
</table>
For the first state of the world distress borrowing of $2,680 is suggested in the first period. If the fat formulation of two-stage linear programming had been used, no solution would have been found for the numerical example. Thus in the present problem utilizing the slack formulation the $2,680 can be considered a kind of measure of potential infeasibility for the first state of the world.

In an actual decision-making situation the solution would naturally be a preliminary suggestion only. The questions of implementation are not, however, subjects of this paper. Nevertheless, it can be noted that adopting a formulation retaining the feasibility of the model even in planning situations with basically infeasible alternatives, makes analysing the infeasibilities and revising the model an easier task to perform.

5. Conclusion

This paper presented a model for joint production and short-term financial decisions of the firm under probabilistic conditions, and reviewed two-stage linear programming especially with managerial decision-making problems in focus. It was demonstrated that by adopting a relevant formulation with only a single revelation, a multi-period decision-making problem can be handled without an exponential growth of the size of the model. It was also suggested that forecasts of a less complex nature can be used than with the traditional two-stage linear programming approach to decision-making problems under a probabilistic future.

APPENDIX: List of Input Parameters of the Model

Left-Hand Side and Objective Function Constants

\begin{align*}
p(q) & = 	ext{the subjective probability of occurrence of the } q:\text{th state of the world.} \\
A1_{ik} & = 	ext{the capacity of the } k:\text{th capacity category required to produce one unit of product } i. \\
A2_{iq} & = 	ext{the normal sales price of product } i \text{ sold in period } t \text{ if the } q:\text{th state of the world occurs.} \\
A3_{iq} & = 	ext{the portion of the normal sales price of product } i \text{ sold in period } t \text{ received in cash in period } t \text{ if the } q:\text{th state of the world occurs. The remaining balance of the sales price is assumed to be received in cash in period } t+1. \\
A4_{iq} & = 	ext{the sales price for the units of product } i \text{ exceeding normal demand in period } t \text{ which are sold at a reduced price in the occurrence of the } q:\text{th state of the world.}
\end{align*}
A5_{itq} = the cash portion of A4_{itq}. The remaining balance is assumed to be received in cash in period t+1.

A6_{itq} = the variable cost of one unit of product in produced in period t predicted for the q:th state of the world.

A7_{itq} = the purchase price of product i bought from competitors to fulfil the demand in period t if the q:th state of the world occurs.

A8_{itq} = the cash portion of A7_{itq}. The remaining balance is to be paid in cash in period t+1.

A9_{itq} = the inventory carrying cost of product i in period t for the q:th state of the world. Inventory carrying cost is stated on the ending balances for simplicity.

A10_q = the absolute interest rate on normal borrowing if the q:th state of the world occurs. The interest is to be paid in the same period as the loan is raised.

A11 = the subjective penalty cost on distress loans stated as an absolute interest rate.

Right-Hand Side Constants

B1_{kt} = the capacity available in the k:th capacity category in period t.

B2_{iq} = the predicted number of units of product i sold in period t if the q:th state of the world occurs. (normal sales)

B3_{iq} = the fixed and predetermined cash outflows less the fixed and predetermined cash inflows in period t if the q:th state of the world occurs.

B4_t = the maximum permissible amount of outstanding loans in period t.

B5_t = the minimum closing cash required for period t.

REFERENCES


