ORMS1020 - OPERAATIOANALYYSI/OPERATIONS RESEARCH

MOCK EXAM

This exam is closed-book. Pocket calculators are allowed, laptops are not. Answer in either English or in Finnish.

1) Consider the following LP:

$$\begin{array}{lccccccc} & & & \min z = -x_1 + 6x_2 & & & & \\ \text{s.t.} & 5 & \leq & x_1 & + & x_2 & \leq & 8 \\ & & & -x_1 & + & 2x_2 & \leq & 2 \\ & & & 2x_1 & + & 4x_2 & = & 8 \\ & & & & & x_1, x_2 & \geq & 0 \end{array}$$

a) Write the above LP in standard form.
b) Write down the Octave code that solves the above problem, i.e., by solving either the LP or its dual. You may use either the function `glpk`, `stu_lp_solver` or `simplex_lp_solver`.
c) Prove that the above LP has no optimal solution because its feasible region is empty.

2) A student tries to prepare himself as well as possible for the Operations Research exam by drinking energy drinks and herbal tea. A can (250 ml) of energy drink is costing €3 and contains 100 gr of sugar and 25 gr of caffeïne. A cup (50 ml) of herbal tea is costing €0.90 and contains, after adding five lumps of sugar, 10 gr of sugar and 1 gr of caffeïne. The energy drink and herbal tea can be sold to the student in any quantity. For his brain to work properly the student needs at least 250 gr of sugar and 60 gr of caffeïne. How can the student do this as cheaply as possible?

a) Model the above problem mathematically as an LP.
b) Give the dual of the LP found in part a.
c) Give the economic interpretation of the dual LP from part b.

3) Defying the rules, four students team up to do four of the five weekly Operations Research questions. The time (in minutes) needed for doing the exercises for each of the students is listed in the following table:

	Exercises			
Students	Ex 1	Ex 2	Ex 3	Ex 4
Student 1	6	14	20	30
Student 2	3	10	7	5
Student 3	9	6	10	14
Student 4	2	3	5	5

They decide that each of them should do exactly one exercise. To solve this problem they, correctly, formulated it as the following LP:

$$
\begin{array}{lllllllllll}
\min z = & 6X_{11} + & 14X_{12} + 20X_{13} + & 30X_{14} \\
& +3X_{21} + 10X_{22} + 7X_{23} + & 5X_{24} \\
& +9X_{31} + & 6X_{32} + 10X_{33} + & 14X_{34} \\
& +2X_{41} + & 3X_{42} + 5X_{43} + 5X_{44}
\end{array}
$$

$$
\begin{array}{lccccccccl}
\text{s.t.} & X_{11} & + & X_{12} & + & X_{13} & + & X_{14} & = 1 & \text{(Constraint 1)} \\
& X_{21} & + & X_{22} & + & X_{23} & + & X_{24} & = 1 & \text{(Constraint 2)} \\
& X_{31} & + & X_{32} & + & X_{33} & + & X_{34} & = 1 & \text{(Constraint 3)} \\
& X_{41} & + & X_{42} & + & X_{43} & + & X_{44} & = 1 & \text{(Constraint 4)} \\
& X_{11} & + & X_{21} & + & X_{31} & + & X_{41} & = 1 & \text{(Constraint 5)} \\
& X_{12} & + & X_{22} & + & X_{32} & + & X_{42} & = 1 & \text{(Constraint 6)} \\
& X_{13} & + & X_{23} & + & X_{33} & + & X_{43} & = 1 & \text{(Constraint 7)} \\
& X_{14} & + & X_{24} & + & X_{34} & + & X_{44} & = 1 & \text{(Constraint 8)} \\
& & & X_{ij} = 0 & & \text{or} & & X_{ij} = 1 & &
\end{array}
$$

a) Explain the meaning of the decision variables X_{ij}.
b) Explain "Constraint 2" and "Constraint 8".
c) The conditions $X_{ij} = 0$ or $X_{ij} = 1$ can be replaced by the conditions $X_{ij} \geq 0, X_{ij}$ integer. Give an argument for why this is true.

4) Consider the following LP:

$$
\begin{array}{lccccccl}
\max z = & \multicolumn{6}{l}{4x_1 + 2x_2 + 3x_3} & \\
\text{s.t.} & x_1 & + & x_2 & + & x_3 & \leq 5 & \text{(Constraint 1)} \\
& 3x_1 & + & 2x_2 & & & \leq 2 & \text{(Constraint 2)} \\
& 3x_1 & + & 2x_2 & + & x_3 & \leq 3 & \text{(Constraint 3)} \\
& & & & x_1, x_2, x_3 & & \geq 0 &
\end{array}
$$

By `glpk` the following (correct) information was obtained about the solution to the above LP:

```
octave:> c=[4 2 3]'; A=[1 1 1; 3 2 0; 3 2 1]; b = [5 2 3]';
octave:> [xmax,zmax,error,extra] = glpk(c,A,b,[],[],"UUU","CCC",-1)
xmax =   0   0   3       zmax =  9     error = 0
extra =  scalar structure containing the fields:
    lambda =   0   0   3    redcosts =   -5  -4  0    time = 0    status =  5
```

a) Based on the given information state the shadow price of Constraint 3 and the reduced cost of the variable x_2.
b) Make an informed estimation about the optimal solution (i.e., total profit) if the amount of resources available in Constraint 3 would increase from the current 3 to 6.
c) By how much should the coefficient of x_2 in the objective function be at least raised for x_2 to become a basic variable (in the new LP)?