**ORMS1020 - OPERAATIOANALYYSI/OPERATIONS RESEARCH**

MOCK EXAM'S SOLUTION

**Exercise 1:** a) The LP is in standard form when 1) the object function is a maximization, 2) all constraints are upper bound constraints and 3) the decision variables are all nonnegative. In the LP under consideration the objective function is a minimization. To make the problem into a maximization we multiply the objective function with minus one resulting in the following new objective function:

$$\max\ (-z) = x_1 - 6x_2.$$

As the second step the constraints are to be made into upper bound constraints. This procedure will be done in two steps. First the constraints will be split:

$$\begin{array}{lrcrcrcr}
\text{s.t.} & & & x_1 & + & x_2 & \leq & 8 \\
& 5 & \leq & x_1 & + & x_2 & & \\
& & & -x_1 & + & 2x_2 & \leq & 2 \\
& 8 & \leq & 2x_1 & + & 4x_2 & & \\
& & & 2x_1 & + & 4x_2 & \leq & 8
\end{array}$$

Secondly, the lower bound constraints need to be multiplied with minus one to make them into upper bound constraints:

$$\begin{array}{lrcrcr}
\text{s.t.} & x_1 & + & x_2 & \leq & 8 \\
& -x_1 & - & x_2 & \leq & -5 \\
& -x_1 & + & 2x_2 & \leq & 2 \\
& -2x_1 & - & 4x_2 & \leq & -8 \\
& 2x_1 & + & 4x_2 & \leq & 8
\end{array}$$

Since all decision variables are nonnegative in the given LP, the standard form of the LP is obtained by collecting all the material together. I.e., the standard form of the LP is:

$$\max\ (-z) = x_1 - 6x_2.$$

$$\begin{array}{lrcrcr}
\text{s.t.} & x_1 & + & x_2 & \leq & 8 \\
& -x_1 & - & x_2 & \leq & -5 \\
& -x_1 & + & 2x_2 & \leq & 2 \\
& -2x_1 & - & 4x_2 & \leq & -8 \\
& 2x_1 & + & 4x_2 & \leq & 8 \\
& & & x_1, x_2 & \geq & 0
\end{array}$$

b) There are several ways of doing this. The original LP can be solved directly with `glpk`:

```
octave:> c=[-1 6]'; A=[1 1 ; 1 1; -1 2; 2 4]; b = [8 5 2 8]';
octave:> [xmax,zmax,error,extra] = glpk(c,A,b,[0 0],[],"ULUS","CC",1)
```

If one wants to use `stu_lp_solver` or `simplex_lp_solver` one has to start from the obtained standard form:

```
octave:> c=[1 -6]'; A=[1 1 ; -1 -1; -1 2; -2 -4; 2 4]; b = [8 -5 2 -8 8]';
octave:> [zmax,xmax,status] = stu_lp_solver(c,A,b)
octave:> zmin = -zmax
```

c) The feasible region is empty because the first inequality in the LP and the equality in the LP conflict with the nonnegativity of the decision variables. More precisely, we have that

$$5 \leq x_1 + x_2 \qquad \text{and} \qquad 2x_1 + 4x_2 = 8.$$

Dividing the second equality by 2 and isolating $x_1$ yields

$$5 \leq x_1 + x_2 \qquad \text{and} \qquad x_1 = 4 - 2x_2.$$

Plugging in the equality into the inequality and simplifying yields

$$1 \leq -x_2 \quad \Longleftrightarrow \quad x_1 \leq -1.$$

This is in contradiction with the assumed nonnegativity of the decision variables.

**Exercise 2:** a) To model this problem define the decision variables precisely. One naturally choice would be:

$$y_1 = \#\text{cans of energy drink} \quad \text{and} \quad y_2 = \#\text{cups of herbal tea}.$$

Alternatively, you can also take $y_1$ and $y_2$ to be ml's of energy drink and herbal tea, respectively, but that unnecessarily introduces complications.As the energy drink and herbal tea can be sold in any quantity, we have obtained the following LP:

$$\begin{array}{lrcrcrl} & & & \multicolumn{3}{c}{min\ w = 3y_1 + 0.9y_2} & \\ \text{s.t.} & 100y_1 & + & 10y_2 & \geq & 250 & \text{(Sugar constraint)} \\ & 25y_1 & + & 1y_2 & \geq & 60 & \text{(Caffeine constraint)} \\ & & & y_1, y_2 & \geq & 0 & \text{(Sign constraints)} \end{array}$$

b) Since the LP obtained in part a is in the anti-standard form, its dual has the standard form:

$$\begin{array}{lrcrcrl} & & & \multicolumn{3}{c}{max\ z = 250x_1 + 60x_2} & \\ \text{s.t.} & 100x_1 & + & 25x_2 & \leq & 3 & \\ & 10x_1 & + & 1x_2 & \leq & 0.9 & \\ & & & y_1, y_2 & \geq & 0 & \text{(Sign constraints)} \end{array}$$

c) To present an economic interpretation of the dual, we have to start by giving the proper definition of the dual variables. Those variables are related to the constraints of the primal and are:

$$y_1 = \text{price of sugar in euros/gr} \quad \text{and} \quad y_2 = \text{price of caffeine in euros/gr}.$$

By means of the decision variables the dual can be interpreted economically. The dual objective function is the maximization of the profit which accrues from selling to the student his desired quantities of sugar and caffeine. This profit is obtained by setting the prices of the raw materials. Those prices are restricted by the two constraints. The first inequality says that the seller of sugar and caffeine can not ask more for his raw products than it would cost the student to obtain them by buying a can of energy drink. Similarly, the second inequality says that the seller of sugar and caffeine can not ask more for his raw products than it would cost the student to obtain them by buying a cup of herbal tea.

**Exercise 3:** a) The decision variables are to be interpreted as follows:

$$X_{ij} = \begin{cases} 1, & \text{if student } i \text{ does exercise} j; \\ 0, & \text{otherwise.} \end{cases}$$

b) Constraint 2 says that student 2 should do one of the exercises (1-4), and Constraint 8 means that Exercise 4 has to be done by one student.
c) If the decision variable $X_{ij}$ are nonnegative, then the Constraints 1-4 imply that they are bounded from above by 1. Thus $0 \leq X_{ij} \leq 1$. Consequently, if we also demand that $X_{ij}$ is integer, $X_{ij}$ can only be zero or one.

**Exercise 4:** a) Based on the given information, the shadow price of Constraint 3 is three and the reduced cost of the variable $x_2$ is four.
b) The informed estimate would be $9 + (6 - 3) * 3 = 9 + 9 = 18$.
c) It should be raised by at least the reduced cost of $x_2$. In other words, it should be raised by 4.