ORMS1020 Operations Research — Final Exam

University of Vaasa — 12.11.2010

The exam is closed-book. Pocket-calculators are allowed, laptops are not. Answer in English or in Finnish.

1. Manuel Eixample produces and sells three different labels of ice cream: Aragon, Castile and Catalonia. To produce one liter of each label Manuel needs egg, milk and sugar as follows:

	Product		
Resource	Aragon	Castile	Catalonia
Egg	8 liters	6 liters	1 liter
Milk	4 liters	2 liters	1.5 liters
Sugar	2 liters	1.5 liters	0.5 liters

Manuel has bought 48 liters of egg, 20 liters of milk, and 8 liters of sugar. A liter of Aragon sells for €60, a liter of Castile for €30, and a liter of Catalonia for €20. The demand for Aragon and Catalonia is unlimited, but at most 5 liters of Castile can be sold. Manuel wants to maximize total revenue.

(a) Formulate Manuel's optimization problem as an LP.
(b) Formulate the *dual* of Manuel's optimization problem.
(c) Write the code that solves Manuel's optimization problem with Octave function `glpk`.

2. Consider the following final simplex tableau of an LP

Row	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	BV	RHS
1	1	0	5	0	0	10	10	0	$z =$	280
2	0	0	-2	0	1	2	-8	0	$s_1 =$	24
3	0	0	-2	1	0	2	-4	0	$x_3 =$	8
4	0	1	1.25	0	0	-0.5	1.5	0	$x_1 =$	2
5	0	0	1	0	0	0	0	1	$s_4 =$	5

(a) Why is this simplex tableau optimal?
(b) What are the optimal decision and the optimal value of the objective function of the LP?
(c) What are the optimal decision and the optimal value of the objective function of the *dual* of the LP?

3. The Company Co. has 2 factories. Each factory produce two products: 1 and 2. The number of products produced (each working day) and number of workers in each factory is:

	Product 1	**Product 2**	**Workers**
Factory 1	11	65	10
Factory 2	49	70	20

(a) Formulate an LP that will solve the DEA efficiency of factory 1.
(b) Both factories, 1 and 2, are actually 100% DEA efficient. Why is this easy to see without any additional analysis?
(c) Suppose there is a new factory that produces 100 products 1 and 10 products 2 with 10 workers. Is factory 1 or factory 2 still 100% DEA efficient?

4. (a) Explain briefly what is a transshipment problem.
(b) Suppose you want to write Octave function that solves a balanced transshipment problem. What are the input and output parameters for such function?
(c) Suppose you want to write Octave function that solves a possibly unbalanced transshipment problem. What are the input and output parameters for such function?

Solutions

1. (a) Set the decision variables to be

$$\begin{aligned} x_1 &= \text{liters of Aragon produced} \\ x_2 &= \text{liters of Castile produced} \\ x_3 &= \text{liters of Catalonia produced} \end{aligned}$$

The the LP is

$$\begin{array}{lrcrcrcrcl} \max z & = & 60x_1 & + & 30x_2 & + & 20x_3 & & \\ \text{s.t.} & & 8x_1 & + & 6x_2 & + & x_3 & \leq & 48 \\ & & 4x_1 & + & 2x_2 & + & 1.5x_3 & \leq & 20 \\ & & 2x_1 & + & 1.5x_2 & + & 0.5x_3 & \leq & 8 \\ & & & & x_2 & & & \leq & 5 \\ & & & & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

(b)

$$\begin{array}{lrcrcrcrcrcl} \min w & = & 48y_1 & + & 20y_2 & + & 8y_3 & + & 5y_4 & & \\ \text{s.t.} & & 8y_1 & + & 4y_2 & + & 2y_3 & & & \geq & 60 \\ & & 6y_1 & + & 2y_2 & + & 1.5y_3 & + & y_4 & \geq & 30 \\ & & y_1 & + & 1.5y_2 & + & 0.5y_3 & & & \geq & 20 \\ & & & & & & & & y_1, y_2, y_3, y_4 & \geq & 0 \end{array}$$

(c)

```
c = [60 30 20]'
A = [8 6 1; 4 2 1.5; 2 1.5 0.5; 0 1 0]
b = [48 20 8 5]'
[xmax,zmax] = glpk(c,A,b,[0 0 0]',[],"UUUU","CCC",-1)
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2. (a) Because there are only non-negative values in the first row for the NBV x_2, s_2, s_3.
   (b) The decision is $[2\ 0\ 8]'$ and the optimal value is 280.
   (c) The decision is $[0\ 10\ 10\ 0]'$ and the optimal value is 280.

3. (a)

$$\begin{array}{lrcrcrcl} \max \theta & = & 11u_1 & + & 65u_2 & & & \\ \text{s.t.} & & & & & 10v_1 & = & 1 \\ & & 11u_1 & + & 65u_2 & & \leq & 10v_1 \\ & & 49u_1 & + & 70u_2 & & \leq & 20v_1 \\ & & & & & u_1, u_2, v_1 & \geq & 0 \end{array}$$

   (b) Factory 1 excels in product 2, and factory 2 excels in product 1, when their inputs (workers) are normalized.
   (c) Factory 1 will still be 100% DEA efficient, but factory 2 will not.

4. (a) I a transshipment problem one has to ship product from ports with given supplies to markets with given demands via possible transshipment points with given capacities. The objective is to minimize the total shipping cost while still meeting the markets' demands.
   (b) (There are many possible solutions. Here is one possibility) Inputs: Matrices $PMcost$, $PTcost$, $TTcost$, and $TMcost$ for the costs from ports to markets, ports to transshipment points, transshipment points to transshipment points, and transshipment points to markets; vectors s, c, and d for the supplies, capacities, and demands.
   Output: Matrices $PMopt$, $PTopt$, $TTopt$, and $TMopt$ for the optimal schedule and number $cost$ for the total cost.
   (c) Outputs could be as in the case (b). The inputs could also be the same, but in addition one needs a penalty vector p for the penalties associated with case when the markets demands are not met.