ORMS1020 Operations Research — Final Exam
University of Vaasa — 20.12.2010

The exam is closed-book. Pocket-calculators are allowed, laptops are not. Answer in English or in Finnish.

1. Giapetto's Woodcarving Inc. manufactures wooden toy soldiers and trains. A soldier sells for €27 and uses €10 worth of raw materials. Each soldier built increases Giapetto's labor costs by €14. A train sells for €21 and uses €9 worth of raw materials. Each train built increases Giapetto's labor costs by €10. The manufacture of wooden soldiers and trains requires two types of skilled labor: carpentry and finishing. A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hour of finishing and 1 hour of carpentry labor. Each week, Giapetto can obtain all the needed raw material but only 100 finishing hours and 80 carpentry hours. Demand for trains is unlimited, but at most 40 soldier are bought each week. Giapetto wants to maximize weekly profits (revenues − costs).

   (a) Formulate the Giapettos's optimization problem as an LP.
   (b) Formulate the *dual* of the Giapettos's optimization problem.
   (c) Write the Octave code that solves the Giapetto's problem. You may use either the function `glpk` or `simplex_lp_solver`.

2. Consider the Octave function

```
function y = exclamation(n)
if ( n=0 | n=1 )
  y = 1;
  return;
endif
y = n*exclamation(n-1);
endfunction
```

   (a) The code has bugs (typos). Identify them.
   (b) What does the (corrected) code output with input parameter $n = 6$?
   (c) What does the (corrected) code output with input parameter $n = -1$?

3. Consider the following final simplex tableau of an LP

Row	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	BV	RHS
1	1	0	5	0	0	10	10	0	$z =$	280
2	0	0	−2	0	1	2	−8	0	$s_1 =$	24
3	0	0	−2	1	0	2	−4	0	$x_3 =$	8
4	0	1	1.25	0	0	−0.5	1.5	0	$x_1 =$	2
5	0	0	1	0	0	0	0	1	$s_4 =$	5

   (a) Why is this simplex tableau optimal?
   (b) What are the optimal decision and the optimal value of the objective function of the LP?
   (c) What are the optimal decision and the optimal value of the objective function of the *dual* of the LP?

4. (a) Explain briefly what is the traveling salesman problem.
   (b) Formulate the traveling salesman problem as an MILP.
   (c) Why is the traveling salesman problem considered "difficult", although it can be solved e.g. as a MILP?

Solutions

1. (a)

$$\begin{array}{lrcrcrcr} \max z & = & 3x_1 & + & 2x_2 & & & \\ \text{s.t.} & & 2x_1 & + & x_2 & \leq & 100 \\ & & x_1 & + & x_2 & \leq & 80 \\ & & x_1 & & & \leq & 40 \\ & & & & x_1, x_2 & \geq & 0 \end{array}$$

(b)

$$\begin{array}{lrcrcrcrcr} \min w & = & 100y_1 & + & 80y_2 & + & 40y_3 & & \\ \text{s.t.} & & 2y_1 & + & y_2 & + & y_3 & \geq & 3 \\ & & y_1 & + & y_2 & & & \geq & 2 \\ & & & & & & y_1, y_2, y_3 & \geq & 0 \end{array}$$

(c)

```
c = [3,2]';
A = [2,1; 1,1; 1,0];
b = [100,80,40]';
[x_max,z_max] = glpk(c,A,b,[0,0]',[],"UUU","CC",-1)
```

2. (a) in the second line `n=0` should be `n==0` and `n=1` should be `n==1`.
   (b) `720`.
   (c) Some kind of error message (indicating infinite recursion).

3. (a) Because there are only non-negative values in the first row for the NBV x_2, s_2, s_3.
   (b) The decision is $[2\ 0\ 8]'$ and the optimal value is 280.
   (c) The decision is $[0\ 10\ 10\ 0]'$ and the optimal value is 280.

4. (a) In the traveling salesman problem one has to find a tour around N points so that the the total distance traveled is as short as possible.
   (b)

$$\begin{array}{lrcll} \min z = & \sum_i \sum_j C_{ij} X_{ij}, & & & \\ \text{s.t.} & \sum_i X_{ij} & = & 1, & \text{for all } j \\ & \sum_j X_{ij} & = & 1, & \text{for all } i \\ & u_i - u_j + N X_{ij} & \leq & N-1 & \text{for all } i \neq j \geq 2, \\ & X_{ij} & = & 0 \text{ or } 1 & \text{for all } i \text{ and } j, \\ & u_i & \geq & 0 & \text{for all } i \geq 2. \end{array}$$

   (c) The MILP way (and all the known ways, so far) of solving the traveling salesman problem is computationally very demanding, because there are many decision variables and many constraints. So, there is, so far, no fast way of solving the problem.