

The exam is closed-book. Pocket calculators are allowed.

1. Calculate the ultimate extinction probabilities for the branching processes having offspring distributions

(a) $\mathbf{p} = [0.20 \ 0.30 \ 0.50]$,

(b) $\mathbf{p} = [0.75 \ 0.15 \ 0.05 \ 0.00 \ 0.05]$.

2. Explain briefly what is

(a) the exponential distribution,

(b) the gamma distribution.

3. Suppose that the probability whether it rains tomorrow depends only on whether it has rained today. Let X_n , $n \in \mathbb{N}$, be the Markov chain modeling the weather: $X_n = 0$ if it rains at day n and $X_n = 1$ if it does not rain at day n . Let

$$\mathbf{P} = \begin{bmatrix} 0.95 & 0.05 \\ 0.30 & 0.70 \end{bmatrix}$$

be the transition probability matrix of X_n , $n \in \mathbb{N}$.

(a) Suppose that on Monday it rains with probability 0.25. What is the probability that it rains on Wednesday?

(b) In the long run, how many rainy and non-rainy days would you expect in this model?

4. Explain briefly the following queueing concepts:

(a) Palm probability.

(b) PASTA.