STAT 3120 Probability and Stochastic processes **University of Vaasa**
Final Exam **2017-12-01**

The exam is closed-book. Pocket calculators are allowed.

1. Calculate the ultimate extinction probabilities for the branching processes having offspring distributions

   (a) $\mathbf{p} = [0.20 \;\; 0.30 \;\; 0.50]$,
   (b) $\mathbf{p} = [0.75 \;\; 0.15 \;\; 0.05 \;\; 0.00 \;\; 0.05]$.

2. Let X be Poisson distributed with parameter 2. Let Y be Poisson distributed with parameter 4. Suppose that X and Y are independent. Calculate

   (a) $\mathbf{P}[X+Y=2]$,
   (b) $\mathbf{P}[Y=0 \mid X+Y=1]$.

3. Suppose that the probability whether it rains tomorrow depends only on whether it has rained today. Let X_n, $n \in \mathbb{N}$, be the Markov chain modeling the weather: $X_n = 0$ if it rains at day n and $X_n = 1$ if it does not rain at day n. Let

$$\mathbf{P} = \begin{bmatrix} 0.90 & 0.10 \\ 0.50 & 0.50 \end{bmatrix}$$

   be the transition probability matrix of X_n, $n \in \mathbb{N}$.

   (a) Suppose that on Monday it rains. What is the probability that it rains on Thursday?
   (b) In the long run, how may rainy and non-rainy days would you expect in this model?

4. A Markovian queue with unlimited capacity is fed by 2 customers per minute on average, and the service times are, on average, 10 seconds.

   (a) What is the probability that a customer arriving to the system finds it empty?
   (b) What is the average time a customer spends in the system?